Eikonal QNMs of black holes beyond GR

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Context: BH spectroscopy

- Testing deviations from GR: extraction of QNMs from ringdown signal ("BH spectroscopy"). This approach requires modelling of QNMs of BHs beyond GR.

- The relevant QNM perturbation wave equations have been derived for a number of theories, in most cases under the assumption of spherical symmetry (i.e. non-rotating BHs).

- Typically, we have a system of coupled equations between the metric perturbation and the extra fields. **Very few QNM solutions have been obtained so far**.

- **This work:** consider a parametrised "theory agnostic" system of equations, assuming spherical symmetry. **We derive QNM solutions using the eikonal/geometric optics approximation**.

The coupled QNM wave equations

- We assume that our QNMs are described by a coupled system of wave equations for the tensor and scalar field degrees of freedom.

$$\frac{d^2\psi}{dx^2} + [\omega^2 - V_\psi(r)]\psi = \beta_\psi(r)\Theta$$

$$\frac{d^2\Theta}{dx^2} + g(r)\frac{d\Theta}{dx} + [\omega^2 - V_\Theta(r)]\Theta = \beta_\Theta(r)\psi$$

ψ= tensor perturbation

Θ= scalar perturbation

x= tortoise coordinate

$$V_\psi = \{V_{\rm RW}, V_{\rm Z}\}, \qquad V_\Theta = f\left[\frac{\ell(\ell+1)}{r^2}\alpha + \frac{2M}{r^3}\zeta\right]$$

Tensor potential is assumed to be either Regge-Wheeler or Zerilli.

$$f = 1 - \frac{2M}{r}$$

Eikonal approximation

- The eikonal approximation consists in assuming solutions of the form,

$$\psi(x) = A(x)e^{iS(x)/\epsilon}, \qquad \Theta(x) = B(x)e^{iH(x)/\epsilon}$$

and taking the double limit $\epsilon \ll 1$ and $\ell \gg 1$ $\quad \epsilon\ell = \mathcal{O}(1)$

- The QNM's complex frequency is expanded as:

$$\omega = \underbrace{\omega_R^{(0)}}_{\mathcal{O}(\ell)} + \underbrace{\omega_R^{(1)} + i\omega_I}_{\mathcal{O}(1)} + \mathcal{O}(\ell^{-1})$$

- Expansion of the coupling functions:

$$\beta_{\psi\Theta} \equiv \beta_\psi \beta_\Theta = \beta_{\psi\Theta}^{(0)} + \beta_{\psi\Theta}^{(1)} + \mathcal{O}(\ell^2), \qquad \beta_{\psi\Theta}^{(0)} = \ell^4 \tilde{\beta}_{\psi\Theta}.$$

Leading order eikonal results

- At leading order:

$$\omega^4 - \omega^2\left[\ell^2\tilde{V}(1+\alpha) + \frac{1}{\epsilon^2}\{(S_{,x})^2 + (H_{,x})^2\}\right] + \ell^2\tilde{V}\left[\alpha\left\{\ell^2\tilde{V} + \frac{(S_{,x})^2}{\epsilon^2}\right\} + \frac{(H_{,x})^2}{\epsilon^2}\right] + \frac{(S_{,x}H_{,x})^2}{\epsilon^4} = \beta_{\psi\Theta}.$$

$$\tilde{V}(r) = \frac{f}{r^2},$$

- We assume a radius $r = r_m$ where the phase functions have an extremum

$$S_{,x} = H_{,x} = 0$$

$$\omega^4 - \omega^2\ell^2\tilde{V}_m(1+\alpha_m) + \ell^4\tilde{V}_m^2\alpha_m - (\beta_{\psi\Theta})_m = 0$$

- We obtain the QNM's leading order *Re* part:

$$\omega_R^{(0)} = \omega_\pm \qquad \omega_\pm^2 = \frac{\ell^2}{2}\left[\tilde{V}_m(1+\alpha_m) \pm \sqrt{\tilde{V}_m^2(1-\alpha_m)^2 + 4(\tilde{\beta}_{\psi\Theta})_m}\right]$$

The effective potential

- As it turns out, the radius r_m corresponds a "peak" of an effective potential:

$$V_{\rm eff}(r,\omega) \equiv \ell^2 \tilde{V}[\omega^2(1+\alpha) - \ell^2 \tilde{V}\alpha] + \ell^4 \tilde{\beta}_{\psi\Theta}, \qquad V'_{\rm eff}(r_m) = 0$$

- The association with a potential peak implies that the eikonal approximation captures the *fundamental* QNM (as in BHs in GR).

- Unlike the case of a single wave equation, the coupled system does *not* appear to admit a correspondence to a geodesic photon ring.

Sub-leading order analysis

- At sub-leading order we obtain the leading-order *Im* part of the QNM (plus the correction to the *Re* part). The relevant equations are:

$$2\omega_{\pm}\omega_R^{(1)}[2\omega_{\pm}^2-\ell^2\tilde{V}_m(1+\alpha_m)]-\ell\tilde{V}_m[\omega_{\pm}^2(1+\alpha_m)-2\ell^2\tilde{V}_m\alpha_m]-(\beta_{\psi\Theta}^{(1)})_m=0$$

$$2\omega_{\pm}\omega_I[2\omega_{\pm}^2-\ell^2\tilde{V}_m(1+\alpha_m)]+(H_{,xx})_m(\omega_{\pm}^2-\ell^2\tilde{V}_m)+(S_{,xx})_m(\omega_{\pm}^2-\ell^2\tilde{V}_m\alpha_m)=0.$$

assume $(S_{,xx})_m = (H_{,xx})_m$

$$\rightarrow \quad \boxed{\omega_I=-\frac{(S_{,xx})_m}{2\omega_{\pm}}=-\frac{(dr/dx)_m}{2\sqrt{2}\omega_{\pm}\ell}\frac{|V''_{\text{eff}}(r_m,\omega_{\pm})|^{1/2}}{[\tilde{V}_m^2(1-\alpha_m)^2+4(\tilde{\beta}_{\psi\Theta})_m]^{1/4}}}$$

Example: QNMs of BHs in dCS gravity

- As an application of our eikonal formalism, we consider Schwarzschild BHs in *dynamical Chern-Simons* gravity.

- The *axial* QNMs are described by a coupled system of tensor-scalar field equations which has a form consistent with our eikonal analysis.

- We find:

Coupling function →

$$\beta_{\psi\Theta} = \frac{576\pi M^2}{\beta} \frac{f^2}{r^{10}} [\ell^4 + 2\ell^3 - \ell^2 + \mathcal{O}(\ell)]$$

QNM's leading order real part →

$$\omega_{\pm}^2 = \frac{\ell^2 f_m}{r_m^2} \left[1 + \frac{288\pi M^2}{\beta r_m^6} \left(1 \pm \sqrt{1 + \frac{\beta r_m^6}{144\pi M^2}} \right) \right]$$

where β is the theory's coupling constant

Example: QNMs of BHs in dCS gravity

coupling strength | Multipole | Potential peak radius | eikonal QNM | Numerical QNM | Fractional error

$M^4\beta$	ℓ	$+/-$	r_m/M	$M\omega_{\rm eik}$	$M\omega_{\rm num}$	$\Delta\omega$ [%]
0.005	2	−	9.96	$0.133-0.0642i$	$0.186-0.0606i$	$39.8+5.6i$
	2	+	2.25	$15.7-0.148i$		
	10	−	9.96	$0.665-0.0642i$	$0.696-0.0636i$	$4.7+0.9i$
	10	+	2.25	$78.3-0.14i$		
0.04	2	−	7.24	$0.178-0.0793i$	$0.220-0.0760i$	$23.6+4.1i$
	2	+	2.25	$5.55-0.148i$		
	7	−	7.24	$0.624-0.0793i$	$0.662-0.0687i$	$6.1+13.4i$
	7	+	2.25	$19.4-0.148i$		
0.5	2	−	5.07	$0.244-0.0936i$	$0.276-0.0936i$	$13.1+0i$
		+	2.26	$1.620-0.144i$	$1.97-0.144i$	$21.6+0i$
1	2	−	4.65	$0.263-0.0959i$	$0.292-0.0971i$	$11.0+1.3i$
		+	2.28	$1.183-0.141i$	$1.43-0.142i$	$20.9+0.7i$
100	2	−	3.23	$0.359-0.0968i$	$0.367-0.092i$	$2.2+4.9i$
		+	2.77	$0.421-0.0976i$	$0.501-0.0954i$	$19.0+2.3i$
10^4	2	−	3.02	$0.382-0.0962i$	$0.374-0.0889i$	$2.1+7.6i$
		+	2.98	$0.388-0.0962i$	$0.484-0.0968i$	$24.7+0.6i$

Outlook

- The eikonal approximation can be easily applied to system of coupled wave equation describing QNMs of non-rotating BHs in alternative theories of gravity.

- The accuracy of the eikonal results is similar to that in GR, i.e. ~ 5-20 % for the $\ell = 2$ multipole.

- The work presented here has been recently extended to systems with more general couplings and first-order rotation [see arXiv:1912.09286].

- The QNM formulas discussed here reduce to familiar results in the GR limit.