Quantum gravity predictions for black hole interior geometry

Daniele Pranzetti

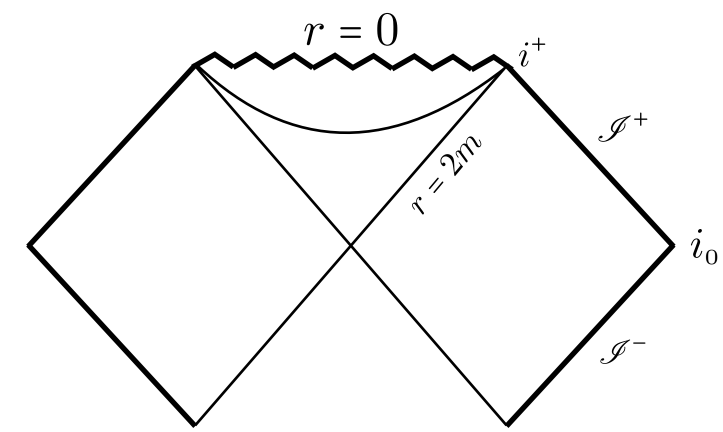
Based on work in collaboration with Emanuele Alesci and Sina Bahrami

- Phys. Rev. D98, 046014 (2018), [gr-qc/1807.07602];
- Phys. Lett. B797 (2019) 134828, [gr-qc/1904.12412];
- Work in progress…

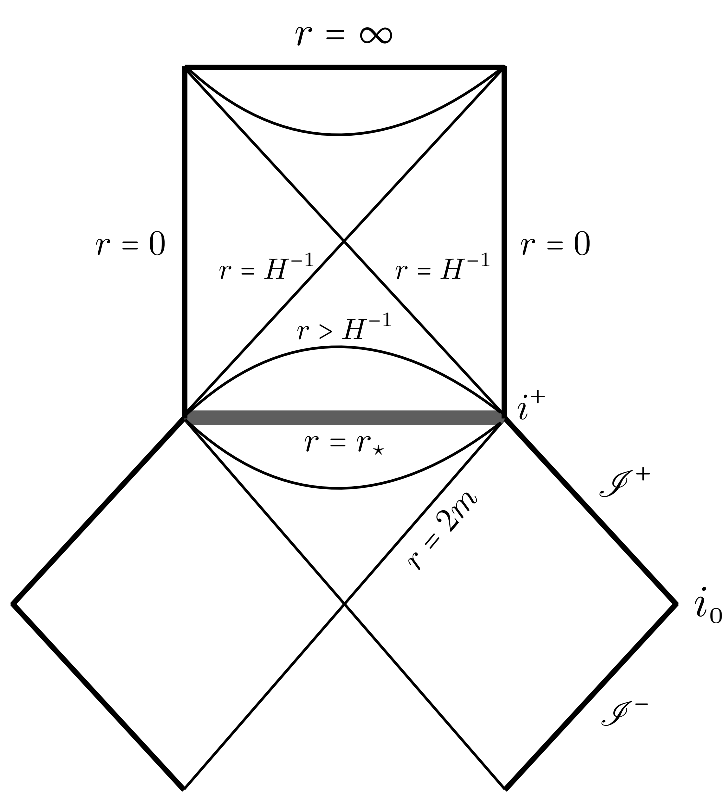
PERIMETER
INSTITUTE

Black hole singularity

[Penrose PRL 1965] and [Hawking, Penrose 1970] singularity theorems:
Singularities are generic predictions of general relativity!

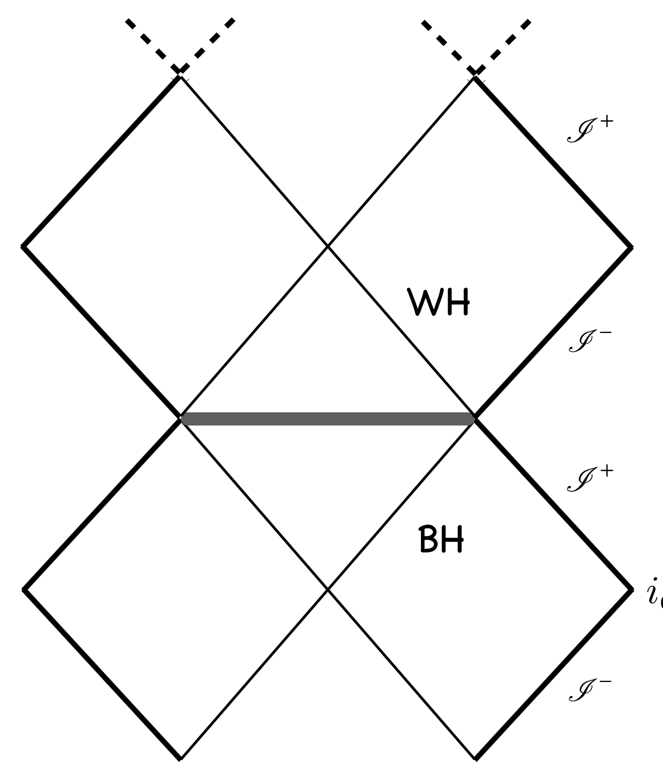


Despite their robustness, singularity theorems are reliable only in the regime where spacetime geometry is classical:
Quantum gravitational effects are expected to smooth out spacetime singularities



Closed de Sitter Universe inside

[Frolov, Markov, Mukhanov, PLB 1988];
[Smolin, CQG 1982];
[Poplawski, PLB 2010];



Black hole / White hole bounce

[Hajicek, Kiefer, IJMPD 2001];
[Barcelo, Carballo-Rubio Garay, IJMPD 2014];
[Haggard, Rovelli, PRD 2015];

Moreover, conservative solutions to restore unitary evolution rely on elimination of singularities [Hossenfelder, Smolin, PRD 2010].

At the end of the day, only a full quantum gravity calculation can discriminate between different scenarios.

Loop Quantum Gravity (LQG) provides a non-perturbative framework to investigate BH singularity resolution.

However, the analysis is strongly affected by an important choice:

Reduction or Quantization first?

The two in general do NOT commute!

General Relativity in
Ashtekar variables

$$A_a^i(t), E_i^a(t)$$

Classical symmetry reduction

[Ashtekar, Boehmer, Bojowald, Brahma, Campiglia, Corichi, Gambini, Kastrup, Modesto, Olmedo, Pullin, Singh, Swiderski, Vandersloot,...]

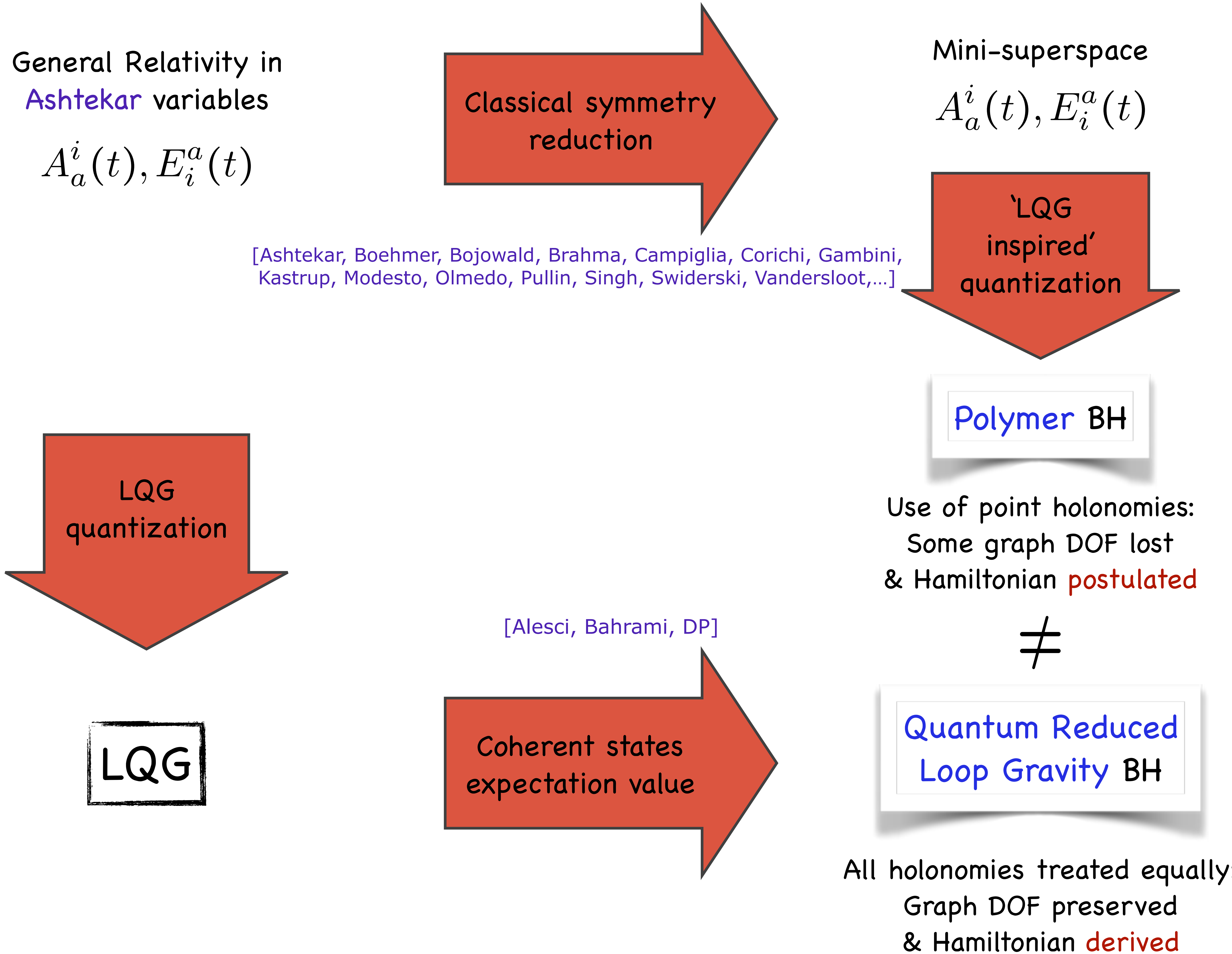
Mini-superspace

$$A_a^i(t), E_i^a(t)$$

'LQG inspired' quantization

Polymer BH

Use of point holonomies:
Some graph DOF lost
& Hamiltonian postulated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$\neq$
LQG
quantization
LQG
[Alesci, Bahrami, DP]
Coherent states
expectation value
Quantum Reduced
Loop Gravity BH
All holonomies treated equally:
Graph DOF preserved
& Hamiltonian derived

Quantum symmetry reduction

Partial Gauge fixing

1. Impose the second class constraints weakly in the Full Hilbert Space: Selects the reduced states i.e. the quantum reduced phase space.
2. Project the constraints defined in the full theory to represent the classical gauge unfixed constraints (preserving the gauge fixing).

Symmetry reduction

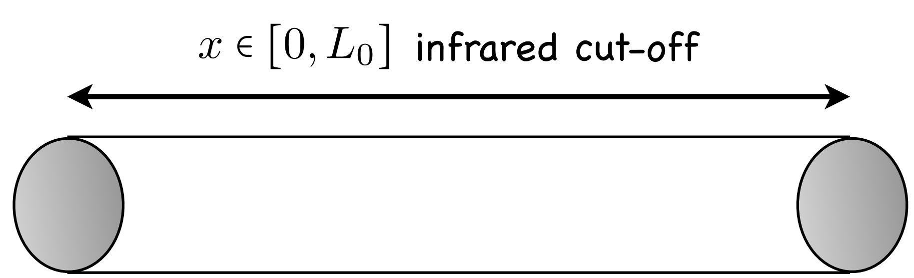
3. Impose the symmetry reduction on the reduced states using coherent states.
4. Define the effective constraints by taking the expectation value of the quantum reduced constraints on the symmetry reduced states.

Deriving the effective dynamics

We are interested in the effective description for the interior geometry of a spherically symmetric black hole, namely we restrict our search for quantum geometries to metrics in the minisuperspace of the form

$$ds^2 = -N(\tau)^2 d\tau^2 + \Lambda(\tau)^2 dx^2 + R(\tau)^2 d\Omega^2$$

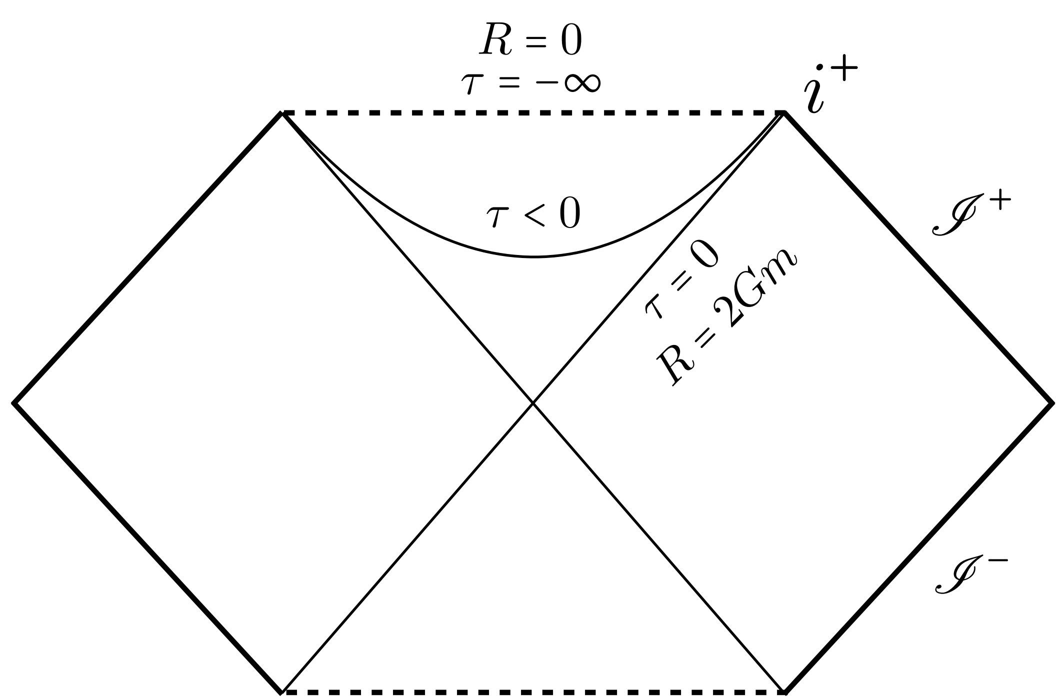
Homogeneous Cauchy slices with topology $\mathbb{R} \times S^2$



$$N_c = -\frac{R^2}{2G^2 m P_\Lambda}\,, \qquad -\infty < \tau < 0$$

$\tau = 0 \rightarrow$ BH horizon

$\tau = -\infty \rightarrow$ classical singularity



ADM phase space:

$$E^r = R^2\,, \quad E^1 = R\Lambda\,,$$

$$A_r = -\frac{\gamma G}{R}\left(P_R - \frac{\Lambda P_\Lambda}{R}\right)\,, \quad A_1 = -\frac{\gamma G}{R} P_\Lambda\,,$$

$$\rightarrow \quad \{R, P_R\} = \{\Lambda, P_\Lambda\} = 1/L_0$$

☆ Effective Hamiltonian:

$$H_{\rm eff} = -\frac{L_0}{4\gamma^2 G\epsilon_x\epsilon^2}\left[\epsilon R\sin\left(\frac{\gamma G\epsilon_x[P_R R - P_\Lambda\Lambda]}{R^2}\right)\left\{2\sin\left(\frac{\gamma G\epsilon P_\Lambda}{R}\right) + \pi H_0\left(\frac{\gamma G\epsilon P_\Lambda}{R}\right)\right\} + \epsilon_x\Lambda\left\{8\gamma^2\cos(\epsilon)\sin\left(\frac{\epsilon}{2}\right)^2 + \pi\sin\left(\frac{\gamma G\epsilon P_\Lambda}{R}\right)H_0\left(\frac{\gamma G\epsilon P_\Lambda}{R}\right)\right\}\right]$$

Classical limit: $\hbar \to 0\,, \quad \epsilon, \epsilon_x \to 0 \quad \Rightarrow \quad H_{\rm eff} \to H_c$

Struve function of order 0:

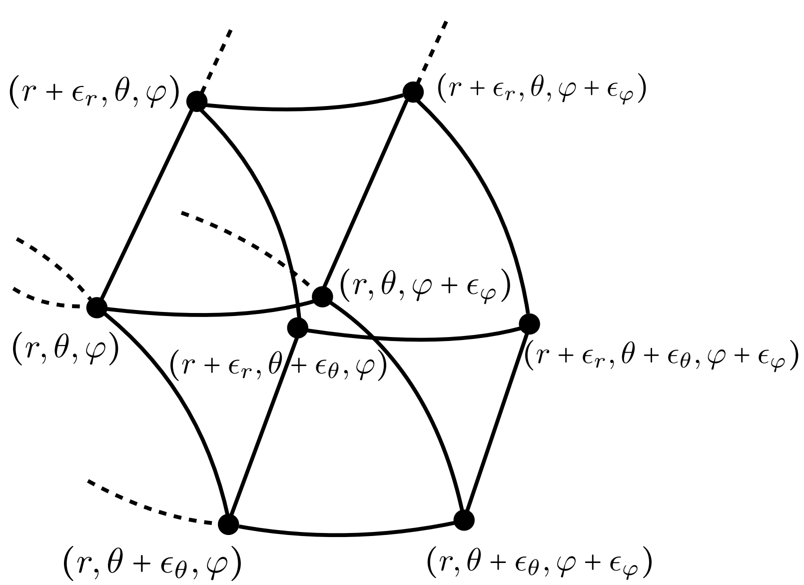
$$H_0[z] = \sum_{k=0}^{\infty}\frac{(-1)^k}{\left(\Gamma\left[k+\frac{3}{2}\right]\right)^2}\left(\frac{z}{2}\right)^{2k+1}$$

main departure from the minisuperspace quantization models: encodes the DOF associated with the 2-sphere graph structure which are frozen in all the previous treatments relying on the use of point holonomies.

⇨ No White Hole horizon in the effective interior geometry

like, e.g., [Ashtekar, Olmedo, Singh, PRL 2018]

Quantum parameters:



$$\epsilon_\theta = \epsilon_\varphi := \epsilon = \frac{2\pi}{\mathcal{N}}\,, \quad \epsilon_x \equiv \frac{L_0}{\mathcal{N}_x}\,, \quad \mathcal{N}, \mathcal{N}_x \gg 1$$

$$A(R) = 4\pi R^2 \simeq 4\pi\gamma\ell_P^2 j_0\mathcal{N}^2$$

$$V(\Sigma) = 8\pi L_0 R^2\Lambda \simeq 4(8\pi\gamma\ell_P^2)^{3/2} j\sqrt{j_0}\mathcal{N}_x\mathcal{N}^2$$

➥

$$\epsilon = \frac{\alpha}{R}\,, \quad \alpha := 2\pi\sqrt{\gamma j_0}\,\ell_p\,,$$

$$\epsilon_x = \frac{\beta}{\Lambda}\,, \quad \beta := \frac{4\sqrt{8\pi\gamma}\,j\,\ell_p}{\sqrt{j_0}}$$

Effective Hamilton evolution eq.s:

$$N = -\frac{\gamma\epsilon R}{Gm\left[\sin\left(\frac{\gamma G\epsilon P_\Lambda}{R}\right) + \frac{\pi}{2}h_0\left(\frac{\gamma G\epsilon P_\Lambda}{R}\right)\right]}$$

$$\dot{R} = \frac{1}{2L_0}\frac{\partial H_{\text{eff}}[N]}{\partial P_R} = \frac{R}{2Gm}\cos\left[\gamma G\beta\left(\frac{P_R}{R\Lambda} - \frac{P_\Lambda}{R^2}\right)\right],$$

$$\dot{\Lambda} = \frac{1}{2L_0}\frac{\partial H_{\text{eff}}[N]}{\partial P_\Lambda} = \frac{\Lambda}{2Gm}\Bigg\{-\cos\left[\gamma G\beta\left(\frac{P_R}{R\Lambda} - \frac{P_\Lambda}{R^2}\right)\right]$$

$$+\frac{\pi h_{-1}\left[\frac{\gamma G\alpha P_\Lambda}{R^2}\right]\left(2\sin^2\left[\frac{\gamma G\alpha P_\Lambda}{R^2}\right] - 8\gamma^2\cos\left[\frac{\alpha}{R}\right]\sin^2\left[\frac{\alpha}{2R}\right]\right) + \cos\left[\frac{\gamma G\alpha P_\Lambda}{R^2}\right]\left(\pi^2 h_0^2\left[\frac{\gamma G\alpha P_\Lambda}{R^2}\right] - 16\gamma^2\cos\left[\frac{\alpha}{R}\right]\sin^2\left[\frac{\alpha}{2R}\right]\right)}{\left(2\sin\left[\frac{\gamma G\alpha P_\Lambda}{R^2}\right] + \pi h_0\left[\frac{\gamma G\alpha P_\Lambda}{R^2}\right]\right)^2}\Bigg\}$$

$\vdots$

The solutions to the evolution equations above present three different regimes labelled uniquely by the parameter:

$$\eta \equiv \frac{\alpha}{\beta} = \frac{\sqrt{2\pi}}{8\gamma} \qquad \eta < 1, \eta = 1, \eta > 1$$

In the asymptotic region where $\tau \to -\infty$:

$$N = const\,, \quad R(\tau) \propto e^{\cos\left(\frac{\kappa}{\eta}\right)\frac{\tau}{2Gm}}\,, \quad \Lambda(\tau) \propto e^{\mathcal{F}(\eta)\frac{\tau}{2Gm}}$$

where

$$\mathcal{F}(\eta) \equiv -\cos\left(\frac{\kappa}{\eta}\right) + \frac{\left[2\pi h_{-1}(\kappa)\sin^2(\kappa) + \pi^2\cos(\kappa)h_0(\kappa)^2\right]}{\left[2\sin(\kappa) + \pi h_0(\kappa)\right]^2}$$

and $\kappa(\eta)$ is implicitly defined by the relation

$$\eta\sin\left(\frac{\kappa}{\eta}\right) + \frac{\pi\sin(\kappa)h_0(\kappa)}{\left[2\sin(\kappa) + \pi h_0(\kappa)\right]} = 0$$

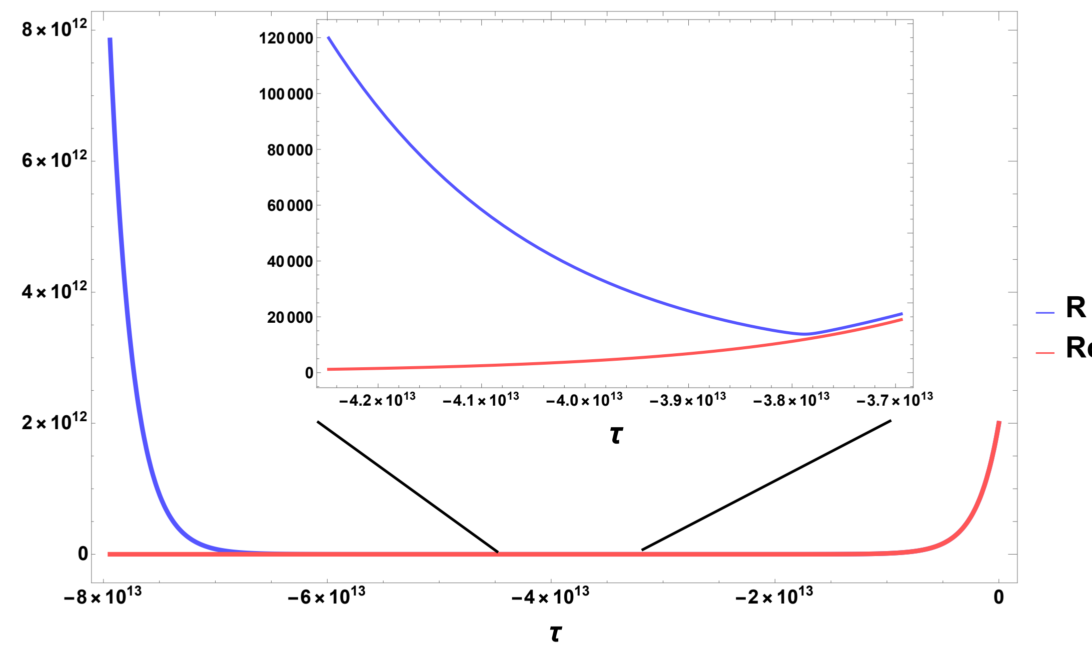
Kretschmann scalar

$$\mathcal{K}_c := R_{abcd}R^{abcd} = \frac{3\, e^{-\frac{3\tau}{Gm}}}{4(Gm)^4}$$

QG regime: $\mathcal{K}_c \sim 1/\ell_p^4\,,$ for $\tau_\star = \frac{Gm}{3}\log\left[\frac{3\ell_p^4}{(4G^4m^4)}\right]$

- The effective metric function $R(\tau)$ has a qualitatively similar behavior in all three regimes:

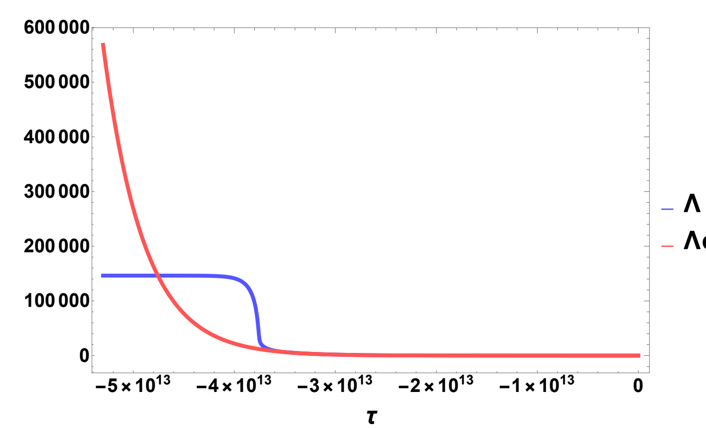
It follows the classical trajectory Rc till the quantum region $\tau \sim \tau_\star$ is reached, at which point a bounce occurs and it starts increasing exponentially



Numerical Solution: $m = 10^{12}m_p\,,\quad \tau_\star \approx -3.7\times 10^{13}$

- The effective metric function $\Lambda(\tau)$ has a has a different asymptotic behavior in each sector:

  - It vanishes exponentially for $\eta < 1$;
  - It approaches a constant value for $\eta = 1$;
  - It grows exponentially for $\eta > 1$



All curvature invariants have a mass-independent bound ☞ Singularity resolution

Is there a numerical value of the Immirzi parameter such that a de Sitter Universe is recovered in the post-bounce asymptotic region?

Desiderata: $$ds^2 = -N_0^2 d\tau^2 + \sinh^2\left(\frac{\tau}{\ell}\right) dx^2 + N_0^2 \ell^2 \cosh^2\left(\frac{\tau}{\ell}\right) d\Omega^2 \,, \quad \lambda = \frac{3}{N_0^2 \ell^2}$$

★ The strategy

Start with:

$z := \exp(-\tau/\ell)$

$$\lim_{z\to\infty} N(z) = N_0 + \mathcal{O}(z^{-1}),$$
$$\lim_{z\to\infty} \Lambda(z) = \frac{1}{2}\left(z - \frac{1}{z}\right) + \mathcal{O}(z^{-2}),$$
$$\lim_{z\to\infty} R(z) = \frac{N_0 \ell}{2}\left(z + \frac{1}{z}\right) + \mathcal{O}(z^{-2})$$

→ derive P_R, P_Λ by means of two Hamilton's equations

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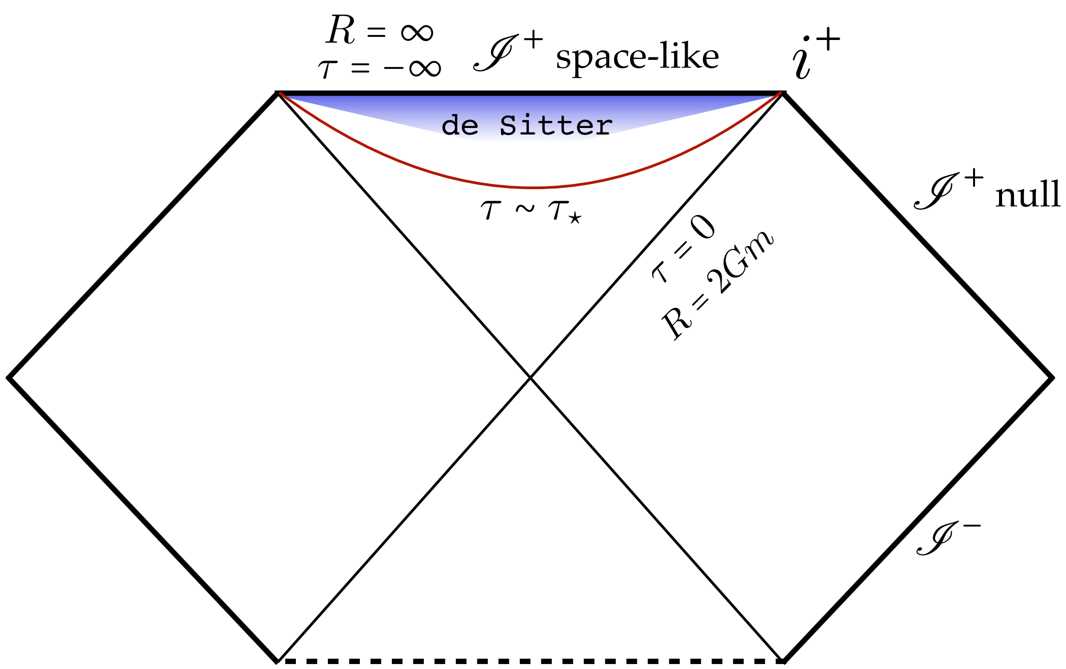
Same numerical value as from the SU(2) black hole entropy calculation!

[Agullo, Barbero, Borja, Diaz-Polo, Villasenor, PRD 2009];
[Engle, Noui, Perez, DP, PRD 2010]

After conformal completion



We recover an asymptotically homogenous, isotropic, spatially flat de Sitter Universe…

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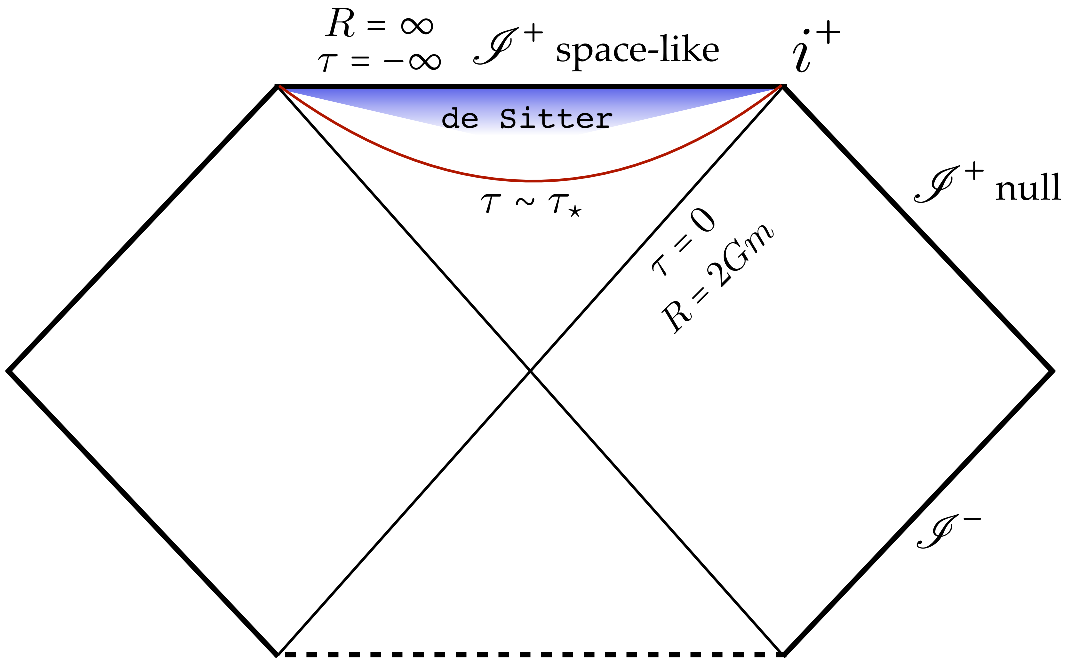

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Black hole cosmology as an alternative to cosmic inflation?

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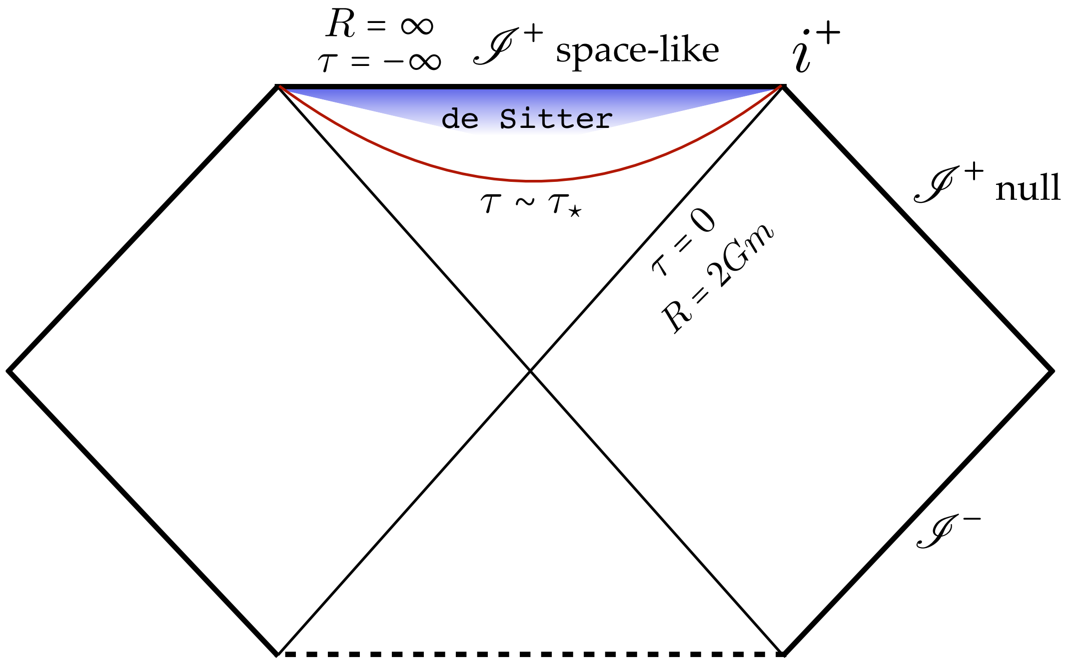

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Comparison of (approximate) Dirac observables pre- and post-bounce suggests $\ell_p\sqrt{j} \approx Gm \quad \rightarrow \quad \lambda \approx \frac{3}{4G^2m^2}$

FRWL metric with Hubble parameter: $H_0 = \frac{1}{2Gm\sqrt{\Omega_\lambda}}$, where effects of a matter dominated phase encoded in a $\Omega_\lambda < 1$.

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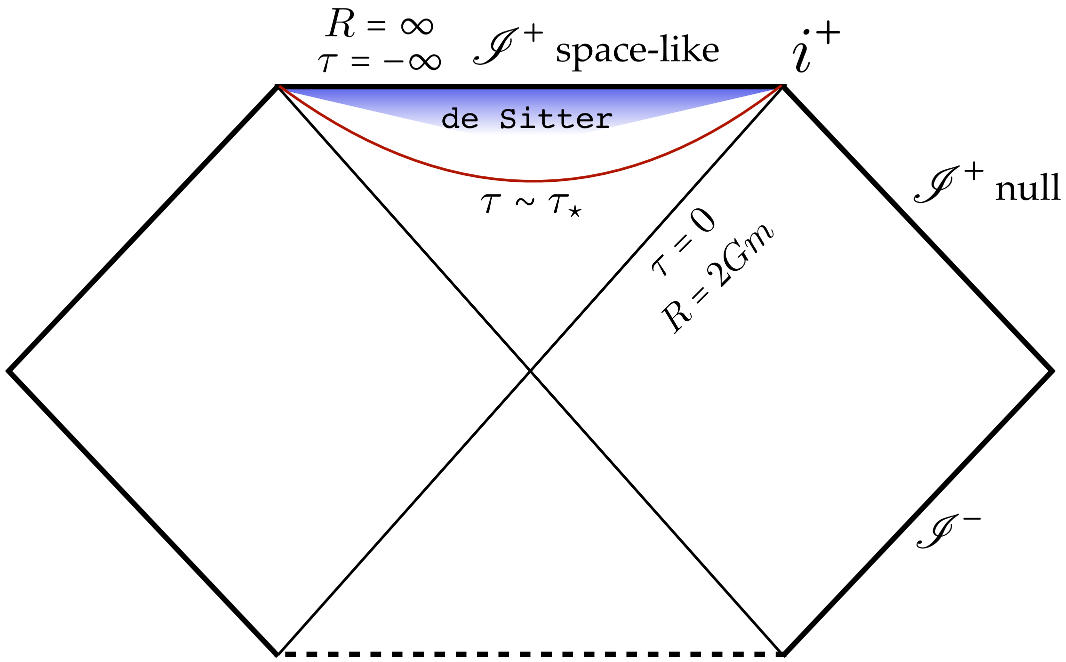

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BH ADM mass — Baryonic matter of the observable Universe

$$m = m_b \quad \rightarrow \quad \frac{1}{\sqrt{\Omega_\lambda}} = \Omega_b \zeta^3$$

comoving particle horizon $\chi \simeq \zeta H_0^{-1}$

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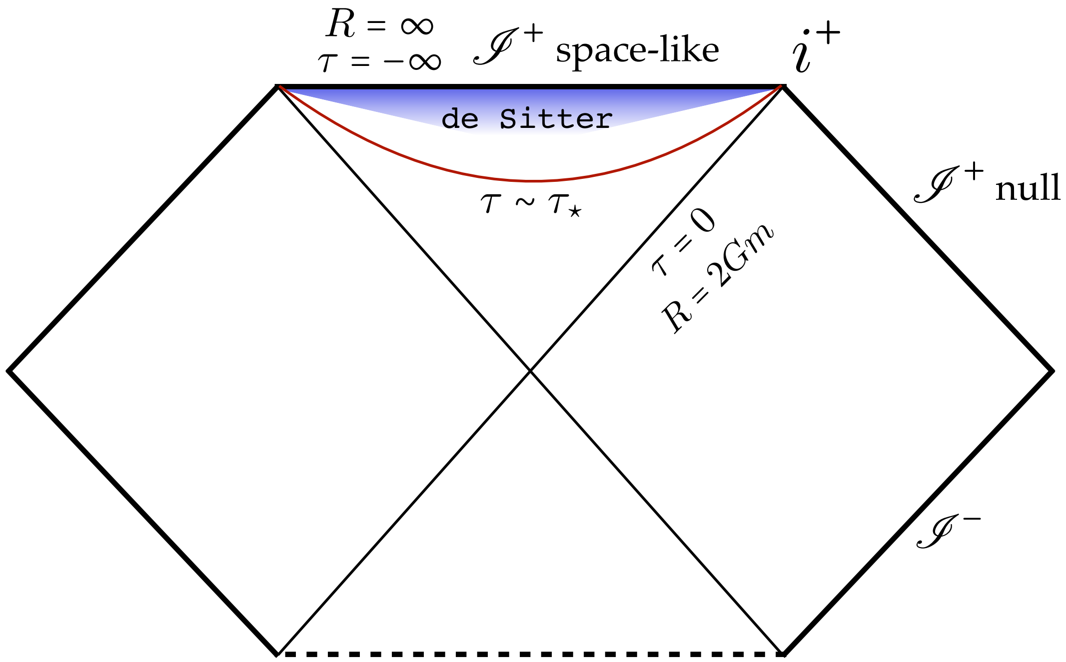

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$$m = m_b \quad \rightarrow \quad \frac{1}{\sqrt{\Omega_\lambda}} = \Omega_b \zeta^3 \quad \Rightarrow \quad \Omega_\lambda^{\mathrm{BH}} = 0.636 \quad \rightarrow \quad H_0^{\mathrm{BH}} = H_0^{\mathrm{CMB}} \sqrt{\frac{\Omega_\lambda^{\mathrm{CMB}}}{\Omega_\lambda^{\mathrm{BH}}}} = 70.6 \ \frac{\mathrm{km}}{\mathrm{s\ Mpc}}$$

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Summary & Outlook

- ☑ By performing the symmetry reduction at the quantum level all relevant DOF are encoded in the effective dynamics and the Hamiltonian can be derived for the first time:

  -> Crucial modifications w.r.t. minisuperspace quantization models:

  No BH / WH transition, but a baby Universe inside

- ☑ Geometric considerations to fix the most relevant dynamics ambiguities:

  -> Asymptotically de Sitter effective metric for the same Immirzi parameter value as in SU(2) BH entropy calculation.

- ☑ Emerging cosmological constant due to quantum gravity effects.

- ☐ Inclusion of matter: Does the gravitational collapse encode the history of the Universe??

- ☐ Horizon penetrating foliation: exterior and interior dynamics together for the first time:

  - Algebra of effective constraints.

  - Development of numerical techniques to solve non-local ODE's.

  - Compute expectation value of the shear operator in the near horizon region, expand in terms of spherical harmonics and compare its modes to the QNM of the (luminosity of the) outgoing radiation of a GW flux.