MoRePaS 2015

Model Reduction of Parametrized Systems III

October 13–16, 2015

SISSA, International School for Advanced Studies, Trieste, Italy

BOOK OF ABSTRACTS

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EU-MORNET European Network for Model Reduction

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Venue

SISSA, International School for Advanced Studies
Aula Magna Paolo Budinich and Building A
Via Bonomea 265, 34136 Trieste, Italy
Contact: `morepas2015@sissa.it`

MoRePaS 2015 Organization

Themes, Committees and Partners

The workshop aims at an international exchange of new concepts and ideas with respect to the following topics:

- Reduced basis methods
- Proper orthogonal decomposition
- Proper generalized decomposition
- Approximation theory related to model reduction
- Learning theory and compressed sensing
- Stochastic and high-dimensional problems
- System-theoretic methods
- Nonlinear Model Reduction
- Reduction of coupled problems/multiphysics
- Optimization and optimal control
- State estimation and control
- Reduced order models and domain decomposition methods
- Krylov-subspace and interpolatory methods
- Application to real, industrial and complex problems

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Support

COST initiative EU-MORNET

The event is supported and organized in the framework of COST (European Cooperation in Science and Technology) initiative EU-MORNET: European Union Model Reduction Network (TD1307).

This Action brings together all major groups in Europe working on a range of model reduction strategies with applications in many domains of science and technology. The increasing complexity of mathematical models used to predict real-world systems, such as climate or the human cardiovascular system, has led to a need for model reduction, which means developing systematic algorithms for replacing complex models with far simpler ones, that still accurately capture the most important aspects of the phenomena being modelled. The Action emphasizes model reduction topics in several themes: 1. design, optimization, and control theory in real-time with applications in engineering; 2. data assimilation, geometry registration, and parameter estimation with a special attention to real-time computing in biomedical engineering and computational physics; 3. real-time visualization of physics-based simulations in computer science; 4. the treatment of high-dimensional problems in state space, physical space, or parameter space; 5. the interactions between different model reduction and dimensionality reduction approaches. The focus of the Action is methodological; however, a wide range of both scientific and industrial problems of high complexity is anticipated to motivate, stimulate, and ultimately demonstrate the meaningfulness and efficiency of the Action. The main objective of the Action is to significantly bringing down computation times for realistic simulations and co-simulations of industrial, scientific, economic and societal models by developing appropriate 'model reduction' methods.

Website: `http://eu-mor.net`

SISSA, International School for Advanced Studies, Trieste, Italy, University of Ulm and University of Münster, Germany provide support and sponsorship for the event as well.

Previous MoRePaS editions were held in Germany: Münster (2009) and Gunzburg (2012).
Website: `http://www.morepas.org`

Welcome

Dear Participants, dear Colleagues, dear Friends,

welcome to the Third MoRePaS workshop in Trieste, after Münster (2009) and Gunzburg (2012). As for past editions we hope to provide you a pleasant and stimulating experience within a growing international community focused on reduced order methods with several and important themes.

We thank all the invited speakers for accepting to provide a talk on their topics of current recognized research and expertise, as well as all the contributors for talks and posters, and all the delegates. We acknowledge and thank MoRePaS Executive and Scientific Committees for all the work carried out, as well as all the tasks faced Local organizing team and by several administrations (SISSA, Ulm, COST).

This new edition is characterized by the support provided through COST EU-MORNET action with the goal of shaping a stronger and structured research network on model order reduction. The action is organized in work-groups focusing from methodological developments to industrial applications in model order reduction.

We hope you could enjoy also the city of Trieste and, last but not least, SISSA!

We wish you all the best for a fruitful workshop,

Gianluigi Rozza, Chair

Karsten Urban, Co-chair

Wil Schilders, COST EU-MORNET action chair

Invited Talks

Adjoint-Based PDE-Constrained Robust Optimization of Aircraft Systems Using Reduced-Order Models

D. Amsallem[1], C. Bou-Mosleh[2], P. Avery[1], Y. Choi[1], and C. Farhat[1]

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[2]Notre Dame University, Beirut, Lebanon

Design optimization under PDE constraints is usually a computationally intensive task due to the repeated computations associated with solving the PDE. Projection-based model reduction aims at alleviating that cost by reducing the dimensionality of the system of equations to be solved by considering solutions confined to a subspace of much smaller dimension.

In this work, a comprehensive approach is developed for the robust optimization of full aircraft systems with a large number of design variables. In addition, the operating conditions for the system of interest are also parameterized. The proposed approach relies on the definition of two reduced-order models: the first one for the forward system and the second one for the adjoint system. Both reduced-order models are constructed by a greedy approach in the space of operating conditions parameters. As the design of the aircraft evolves, the reduced-order models are also updated using a Trust-Region Model Management framework.

An application to the robust design of an aircraft system will highlight the capability of the proposed approach to accelerate the design process.

Issues in the Loewner framework for the reduction of parametric and bilinear systems

A. C. Antoulas[1]

[1]Rice University, Houston, TX, US and Jacobs University, Bremen, Germany

The purpose of this talk is to discuss two aspects of the Loewner framework for model reduction. The first has to do with linear parametrized systems and the second with linear as well as bilinear systems. We will show namely how the central concept of shifted Loewner matrices can be generalized in the parametric case and how this leads to an explicit description of reduced parametrized systems. Furthermore the issue of one-sided interpolation and the ensuing parametrizations will be discussed, focusing on bilinear systems.

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Model reduction for integro-differential equations

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Model reduction approaches for general linear Volterra integro-differential systems are studied. A structure-preserving method based on generalized system Gramians is introduced. It is shown that these Gramians can be characterized as solutions of certain delay Lyapunov equations similarly arising for other classes (e.g. finite delay) of systems. The usual energy interpretation of the Gramians is provided and a reduced-order model is obtained by truncation of a balanced system. The new approach allows to reduce coupled as well as time fractional systems. Numerical examples demonstrating the applicability of the method are given.

Structure-preserving Model Reduction

C. Beattie[1]

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Dynamical systems form the basic modeling framework for an enormous variety of complex systems. Direct numerical simulation of the correspondingly complex dynamical systems is one of few means available for accurate prediction of the associated physical phenomena. However, the ever increasing need for improved accuracy requires the inclusion of ever more detail in the modeling stage, leading inevitably to ever larger-scale, ever more complex dynamical systems that must be simulated.

Simulations in such large-scale settings can be overwhelming and make unmanageably large demands on computational resources; this is the main motivation for model reduction, which has as its goal production of much simpler dynamical systems retaining the same essential features of the original systems (high fidelity emulation of input/output response and conserved quantities, preservation of passivity, etc.).

I will describe briefly the objectives and methodology of systems-theoretic approaches to model reduction, focussing for the most part on interpolatory projection methods that are both simple and capable of providing nearly optimal reduced order models in many circumstances. Interpolatory methods provide a framework for model reduction that allows for the retention of special structure such as parametric dependence, port-Hamiltonian structure, and internal delays. Nonintrusive “data-driven” approaches will be discussed as well.

Model Reduction in PDE-Constrained Optimization

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The numerical solution of optimization problems governed by partial differential equations (PDEs) requires the repeated solution of coupled systems of PDEs. Model reduction can be used to substantially lower the computational cost at various stages of the optimization algorithm, for example, to construct Hessian approximations for use within a traditional gradient based optimization algorithms, or to generate surrogate models for use within a trust-region method. I will review recent approaches and their convergence properties, and demonstrate their performance on example problems. The emphasis is on nonlinear PDE constrained optimization problems, where precomputations of reduced order models that are valid over the entire optimization parameter range are typically impossible.

Reduced Order Models in the context of System Level Simulation

S. Kher[1]

[1]ANSYS Inc., Canonsburg, PA, US

The goal of system level simulation is to simulate and analyze virtual prototypes of the complete product being designed. This requires the ability to combine detailed physics based models, with behavioral models (such as those developed using languages like Modelica and VHDL-AMS), compact device models (typically developed using C++) and controls and embedded software. ROMs are particularly useful in this context.

This talk will cover some definitions of model types and ROMs in the context of system level simulation. We will look at a typical model reduction/ROM flow in the context of commercial software from ANSYS. We will then look at examples and applications where existing techniques work well - for example S-parameter fitting for electronics and linear network fitting for LTI thermal systems. Finally, we will present current areas of research and highlight key challenges - including the need and motivation for standardization.

Preconditioners for parameter-dependent equations and projection-based model reduction methods

A. Nouy[1] and O. Zahm[1]

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We present a method for the construction of preconditioners for large systems of parameter-dependent equations [1], e.g. arising from the discretization of PDEs with uncertain coefficients. The proposed preconditioner is an interpolation of the matrix inverse obtained by a projection of the identity matrix with respect to the Frobenius norm. The use of randomized linear algebra allows us to handle large matrices, with the guaranty of obtaining quasi-optimal interpolations with high probability. Adaptive interpolation strategies are then proposed for different objectives in the context of projection-based model order reduction methods: the improvement of residual-based error estimators, the improvement of the projection on a given reduced approximation space, or the recycling of computations for sampling-based model reduction methods.

References

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HIerarchical MODel (HI-MOD) reduction: towards haemodynamics applications

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A hierarchical model (Hi-Mod) reduction provides surrogate models suited to describe phenomena with a dominant dynamics, even though locally featuring relevant transverse components. Istances of such phenomena are the blood flow in arteries, the hydrodynamics in river networks, the gasdynamics in an internal combustion engine.

The driving idea of a Hi-Mod reduction is represented by a different discretization of the dominant and of the transverse dynamics, in the spirit of a separation of variables. In particular, according to the original formulation [1, 2], the mainstream is tackled by affine finite elements, while a modal approximation solves the transverse directions. This approach leads to solve along the principal direction a "psicologically" one-dimensional model, with coefficients automatically including the effect of the transverse dynamics. Moreover, the number of modes can be locally tuned along the mainstream, according to the meaningfulness of the transverse information [3]. The rationale is that relatively few modes are enough to capture the transverse dynamics of interest with an overall reduction of computational costs.

In this presentation, we focus on the most recent advances in the Hi-Mod setting. The reference application is the computational haemodynamics. This goal has led us to apply the Hi-Mod reduction to 3D cylindrical geometries, in case with a curvilinear supporting fiber, by assuming the Stokes equations as first reference model.

Another interesting issue is represented by the generalization of the Hi-Mod procedure to a parameter dependent setting, with a view to an estimation of the parameters involved in the haemodynamics models.

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Sparse-Grid, Reduced-Basis Bayesian Inversion

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We report on reduced basis accelerated Bayesian inversion methods for affine-parametric, linear operator equations, from joint work with Peng Chen [2, 1]. We allow a general class of non-affine parametric, nonlinear operator equations. We present an analysis of sparsity of parametric forward solution maps and of Bayesian inversion in to the fully discrete setting. The analysis includes Petrov-Galerkin high-fidelity (HiFi) discretization of the forward maps. We develop adaptive, stochastic collocation based reduction methods for the efficient computation of reduced bases on the parametric solution manifold. The nonlinearity with respect to the distributed, uncertain parameters and the unknown solution is collocated; specifically, by the so-called Generalized Empiricial Interpolation Method (GEIM). For the corresponding Bayesian inversion problems, computational efficiency is enhanced in two ways: first, expectations with respect to the posterior are computed by adaptive quadratures with dimension-independent convergence rates, following [4, 3]. Our analysis accounts for the impact of the PG discretization in the forward maps on the expectation of the Quantities of Interest (QoI). Second, we perform the Bayesian estimation only with respect to a parsimonious, reduced basis approximation of the posterior density. In [5], under general conditions on the forward map, the infinite-dimensional parametric, deterministic Bayesian posterior was shown to admit N-term approximations which converge at rates which depend only on the sparsity of the parametric forward map. We present dimension-adaptive collocation algorithms to build finite-dimensional parametric surrogates which realize these rates. In several numerical experiments, the proposed algorithms exhibit parameter dimension-independent convergence rates which equal, at least, the currently known rate estimates for N-term approximation. We propose to accelerate Bayesian estimation by offline computation of reduced basis surrogates of the Bayesian posterior density. The parsimonious surrogates can be employed for online data assimilation and for Bayesian estimation. They also open a perspective for optimal experimental design.

Supported in part by Swiss National Science Foundation (SNF) and by the European Research Council (ERC) under AdG 247277.

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Towards automated model reduction: exact error bounds and simultaneous finite-element reduced-basis refinement

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We develop a reduced basis method for parametrized coercive partial differential equations (PDEs) with two objectives: providing an error bound with respect to the exact weak solution of the PDE as opposed to the typical finite-element "truth" solution; providing automatic adaptivity in both physical and parameter spaces. The error bound builds on two key ingredients: a minimum-residual mixed formulation which provides an upper bound of the dual-norm of the residual computed in an infinite-dimensional function space; an extension of the successive constraint method which provides a lower bound of the stability constant computed in the infinite-dimensional function space. The automatic adaptivity is provided in the offline stage which combines a spatial mesh adaptation for finite elements and a greedy parameter sampling strategy for reduced bases to yield a reliable online system in an efficient manner. We demonstrate the effectiveness of the approach for parametrized elasticity problems.

Contributed Talks

A subspace projection-based method for stabilization and enhancement of projection-based ROMs of the Navier–Stokes equations

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Computational fluid dynamics (CFD) has become an indispensable tool for many engineering applications. Unfortunately, high-fidelity CFD is often too expensive for parametric, time-critical and many-query applications such as design optimization, control and uncertainty quantification. Reduced order models (ROMs) are a promising tool for bridging the gap between high-fidelity, and real-time, multi-query applications. Despite recent advances in the field, model reduction is still in its infancy, particularly for high-Reynolds number turbulent flows.

For a projection-based ROM to be stable and accurate, the dynamics of the truncated subspace must be taken into account. In fluid flow applications, the traditional approach involves the addition of empirical energy-absorbing eddy-viscosity terms to the ROM. The drawback of this approach is that empirical turbulence models destroy consistency between the Navier-Stokes equations and the ROM. Accurately identifying and matching free coefficients of the turbulence models is another challenge.

In this work, an alternative approach for stabilizing and enhancing ROMs is proposed. Instead of adding an empirical turbulence model term to the ROM, we derive a transformation of the projection subspace that accounts for truncated modes [1]. Because only the projection subspace is modified, consistency between the ROM and the Navier-Stokes equations is retained. The proposed approach can be formulated mathematically as a trace minimization problem on the Stiefel manifold. The reproductive as well as predictive capabilities of the method are evaluated on several compressible flow problems, including a problem involving laminar flow over an airfoil with a high angle of attack (Figure 1), and a channel-driven cavity flow problem.

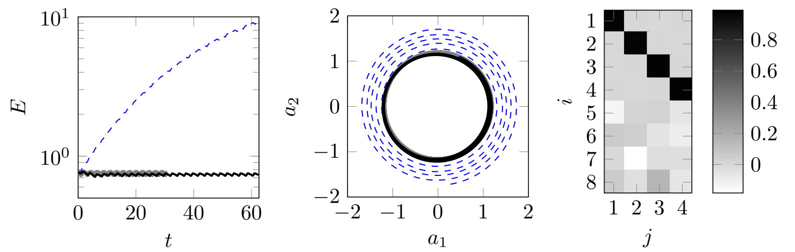

Figure 1: Nonlinear model reduction of the laminar airfoil. Evolution of modal energy (left), and phase plot of the first and second temporal basis, $a_1(t)$ and $a_2(t)$ (middle); DNS (thick gray line), standard 4 DOF ROM (dashed blue line), stabilized 4 DOF ROM (solid black line). Stabilizing transformation matrix (right)

References

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POD-Galerkin methods for parametrized problems in flow control and fluid-structure interaction

F. Ballarin[1] and G. Rozza[1]

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We discuss a computational reduction framework based on POD-Galerkin projections for parametrized problems in computational fluid dynamics with an offline/online computational splitting between a high fidelity model (offline) and a reduced order one (online). Special attention is paid to the stabilization of the resulting online system, and shape parametrization maps to efficiently handle deformation of the computational domain. The results focus on some applications of the proposed framework to optimal flow control problems, and to a fluid-structure interaction formulation. A simultaneous reduction of both fluid and structural equations is sought; coupling of the multiphysics system on the interface is handled by means of shape parametrization maps. In this context, domain-decomposition based reduced order models are exploited and take advantage of the existing parametrized framework for optimal control problems. They enforce the minimization of the jump of the velocity on either the physical interface (between fluid and structure) or computational interfaces (between different partitions of the domain). Cardiovascular flows problems provide a real-life computational challenge for the developed methodology.

Time-dependent Parametric Model Order Reduction for Material-Removal Simulations

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Machining of thin and lightweight structures is a crucial manufacturing step in industries ranging from aerospace to power engineering. Practical problems include machining of frame components, milling of blades for aircraft propellers or turbines and many more. The applications demand high geometric accuracy and excellent surface finish. The elastic workpieces, however, deform due to acting cutting or clamping forces. For numerical simulations, typically, Finite-Element-(FE)-based simulations are required to accurately predict the deflection of complex workpieces during machining.

To consider elastic effects as well as rigid body motions, elastic multibody systems (EMBS) are used. In order to enable efficient simulations and solve typical tasks, like the prediction of process stability, reduced elastic models have to be determined by linear model order reduction, [2]. Thereby, the system matrices need to be constant, which cannot be assumed for elastic bodies with varying geometry due to material removal. In [3], parametric model order reduction methods based on the interpolation of reduced system matrices [1, 4] are extended and applied for systems with moving loads, with the restriction that the FE mesh remains constant.

In this presentation we propose a technique to generate reduced elastic bodies for systems with varying geometry and their application in time-domain simulations. Therefore, the interpolation of the reduced system matrices, which are calculated for certain parameter samples individually, is applied. Based on these systems, interpolated reduced systems are calculated and used during the time-domain transient simulation. Due to the fact that the integration scheme in the multibody code has to be adapted, the software package Neweul-M^2 [5] is used. The adaption enables the generation of time-dependent parametric system matrices and the solution of the nonlinear differential equation of the EMBS. Results of the simulation of the material removal are illustrated for the model of a T-shaped plate.

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ArbiLoMod: Communication Avoiding Localized Reduced Basis Methods for Problems with Arbitrary Local Modifications

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During the development of today's computer architectures in the last decade, communication capabilities did not grow at the same speed as computation capabilities. This trend was accelerated by the rise of accelerator devices, multiplying the available computing power in each node of a cluster. Cloud environments pushed this trend to the extreme, allowing any user to request a high number of workstations, connected to the user by an internet connection capable of transferring usually only a few megabytes per second. To leverage these computing potentials for engineers working with finite element based simulation software, we designed the localized reduced basis method "ArbiLoMod" which is tailored to this computing environment. In all design decisions, avoiding of communication was the primary goal, often trading computation in for less communication.

ArbiLoMod employs reduced basis techniques to find problem adapted low dimensional spaces in which the solutions of a given parametrized partial differential equation can be quickly approximated for arbitrary parameters. The reduced space is localized by a decomposition of the solution space into overlapping local subspaces. Elements of other existing localized reduction methods are incorporated [4, 5, 1, 2]. The use of localized reduced spaces not only allows for a good parallelization, but also for a fast recomputation of the solution after arbitrary local modifications of the underlying problem, a situation often occurring in engineering environments, hence the name "ArbiLoMod". To guarantee the quality of the solution, a localized a-posteriori error estimator is employed, based on the global residual of the problem. When necessary, the reduced local spaces are enriched adaptively.

We will show numerical results, obtained by an implementation of the method in our model reduction framework pyMOR [3], for elliptic model problems.

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Time-parallel reduced-order models via forecasting

K. Carlberg[1], L. Brencher[2], B. Haasdonk[2], and A. Barth[2]

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Many-query and real-time scenarios demand reduced-order models (ROMs) for tractability. For the many-query case, the relevant notion of simulation cost is *core-hours*, which dictates how many queries are possible given fixed computing resources. In this sense, nonlinear ROMs have already demonstrated significant savings, as ROMs can be simulated using a 'sample mesh' that necessitates far fewer computing cores. In contrast, the relevant cost for real-time scenarios is *wall time*. Nonlinear ROMs have been less effective at reducing this cost, as spatial parallelism is quickly saturated. For example, on a compressible flow problem, the GNAT ROM yielded a 452X improvement in core-hours, but only a 6.86X improvement in wall time [1]. Spatial parallelism was saturated with only 12 cores.

Time-parallel methods (e.g., parareal [3], PITA) constitute one approach to improve wall-time performance. However, speedups are often modest (i.e., less than five), in part because typical time integrators are often employed for the coarse propagator. As the coarse time step is usually outside the asymptotic range of convergence, the coarse propagator can be inaccurate, yielding slow convergence.

The main idea of this work is to enable efficient time parallelism for ROMs by employing an accurate coarse propagator that exploits *time-evolution data* available from offline training simulations. In particular, we propose adopting the forecasting method introduced in Ref. [2] as a data-driven coarse propagator. As shown in Fig. 1, the forecasting method can provide a much more accurate coarse propagation than traditional time integrators, leading to significantly faster convergence.

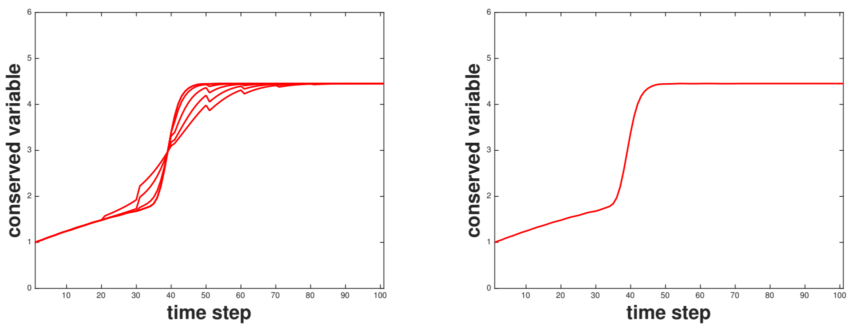

Figure 1: Time evolution of a state variable across time-parallel iterations for a ROM applied to the inviscid Burger's equation for two coarse propagators (left: backward Euler; right: forecasting).

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*Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed Martin Company, for the United States Department of Energy under contract DE-AC04-94-AL85000.

Time-Varying Parametric Model Order Reduction by Matrix Interpolation

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Model Order Reduction of parametric linear time-invariant systems has been extensively investigated over the last ten years. For this reason, there exist many different parametric model order reduction (pMOR) approaches, which can be classified in either global or interpolatory methods. The interpolatory technique presented in [4] – based on the interpolation of reduced system matrices – is not restricted to a certain type of parameter dependency and can be applied to efficiently obtain a parametric reduced order model from the precomputed reduced system matrices at different grid points in the parameter space.

In many engineering applications the underlying high-dimensional system may depend not only on different parameters, but on parameters which *vary with time*. Such systems show a time-varying input-output behaviour which is caused by the time variability of the parameters. In this contribution we consider model order reduction for large-scale linear parameter-varying (LPV) systems of the form

$$\begin{aligned}\mathbf{E}(\mathbf{p}(t))\dot{\mathbf{x}}(t) &= \mathbf{A}(\mathbf{p}(t))\mathbf{x}(t) + \mathbf{B}(\mathbf{p}(t))\mathbf{u}(t),\\ \mathbf{y}(t) &= \mathbf{C}(\mathbf{p}(t))\mathbf{x}(t).\end{aligned} \tag{1}$$

Since the system matrices explicitly depend on the time-varying parameter vector $\mathbf{p}(t)$ we develop a projection-based, time-varying parametric model order reduction approach, which we call *p(t)MOR*, to obtain a reduced order model of a large-scale LPV system. Based on this approach, we adapt the reduction method of matrix interpolation to the time-varying case, whereby new time-derivative terms emerge which must be considered during the reduction process.

In this talk, we first present the aforementioned time-varying parametric model order reduction approach by matrix interpolation and then apply this method for the reduction of a system with *moving load*. Unlike the publications [1, 3, 2], the varying load location is here considered as a time-dependent parameter $p(t)$ of the system model with $\dot{p} \neq 0$. Both the standard and the adapted matrix interpolation with additional time-derivative terms will be employed for the reduction and their performance compared to each other.

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Simultaneous Empirical Interpolation and Reduced Basis method for non-linear problems

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In this talk, we will focus on the reduced basis methodology in the context of non-linear and non-affinely parametrized partial differential equations in which affine decomposition necessary for the reduced basis methodology are not obtained.

To deal with this issue, it is now standard to apply the Empirical Interpolation Method (EIM) methodology [1, 4] before deploying the Reduced Basis (RB) methodology. The EIM building step can be costly and require many (hundreds) finite element solutions when the terms are non-linear that forbids its application to large non-linear problems.

In this talk, we will introduce a Simultaneous EIM Reduced basis algorithm (SER) [2] based on the use of reduced basis approximations into the EIM building step. Enjoying the efficiency offered by reduced basis approximation, this method provides a huge computational gain and can require as little as $N + 1$ finite element solves where N is the dimension of the RB approximation.

We will start this talk with a brief overview of the EIM and RB methodologies applied to non-linear problems. The identification of the main issue, discussing the changes to be made in the EIM offline step for such problems will then introduce the SER method detailed in the first part of the talk with some of its variants.

The second part of the talk will first illustrate our method with preliminary results obtained on a benchmark introduced in [4]. We will then assess its performances for large scale problems it is designed for, through a 3D non-linear multi-physics reduced model used in high field magnet optimization context introduced in [3].

The SER method is now available in the generic and seamlessly parallel reduced basis framework of the opensource library Feel++ (Finite Element method Embedded Language in C++, `http://www.feelpp.org`).

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MOR-based Uncertainty Quantification in Transcranial Magnetic Stimulation

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Electromagnetic computations rely on the perfect knowledge of material parameters. However, for a wide range of examples in electrical and biomedical engineering some uncertainty should be associated in that knowledge in the modeling process. In order to quantify the uncertainty of the output quantities of interest coming from the lack of knowledge of the input material parameters, the spectral stochastic finite element method based on Polynomial Chaos Expansion (PCE) can be applied. This method, in both the intrusive and non-intrusive forms [2], allows to dramatically reduce computational time with respect to Monte Carlo (MC) methods. Nevertheless, in many situations, computational complexity can still be prohibitively large. This is the case of Transcranial Magnetic Stimulation (TMS), a non-invasive technique to stimulate cortical regions of the human brain by the principle of electromagnetic induction. In TMS there is an essential need for more effective techniques of uncertainty quantification due to the increasing model complexity [1]. For such problems, a novel approach based on Model Order Reduction (MOR) is proposed here.

The electromagnetic problem at hand is simplified due to the low electrical conductivities not exceeding 10 S/m and moderate excitation frequencies which are in the range of $2-3$ kHz so that the secondary magnetic field from the induced eddy currents are neglected. We use a realistic head model which contains five different tissues, namely scalp, skull, cerebrospinal fluid (CSF), grey matter (GM) and white matter (WM). The electrical conductivities of scalp and skull are modeled as deterministic, while the conductivities of CSF, GM, and WM, which show a wide spread across individuals and measurements, are modeled as uniform distributed random variables.

The starting point of the new MOR-based approach is a non-intrusive PCE method, in which the PCEs of variables are estimated from the solutions of the deterministic problems for all values of the random tissue parameters belonging to a sparse grid. The main idea of the proposed algorithm is that of reducing the number of solutions of such deterministic problems by constructing a parametric reduced order model, which is used to approximate the solution of the deterministic problems. Such parametric reduced order model is tailored to approximating with chosen accuracy the deterministic problems for the values of the random tissue parameters in the chosen sparse grid. The parametric reduced order model is generated in an efficient way by solving a reduced number of deterministic problems with respect to the non-intrusive PCE approach. The computational cost to construct the parametric reduced order model is optimized by exploiting the relatedness among the solutions to the deterministic problems for different values of the random material parameters.

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Reduced Basis Landweber method for nonlinear ill-posed inverse problems

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The numerical solution of nonlinear inverse problems such as the identification of a parameter in a partial differential equation (PDE) from a noisy solution of the PDE via iterative regularization methods, e.g. the Landweber method or Newton-type methods, usually requires numerous amounts of forward solutions of the respective PDE. One way to speed up the solution process therefore is to reduce the computational time of the forward solution, e.g. via the reduced basis method.

The reduced basis method is a model order reduction technique which constructs a low-dimensional subspace of the solution space. Galerkin projection onto that space allows for an approximative solution. An efficient offline/online decomposition enables the rapid computation of the approximative solution for many different parameters.

The simple and intuitive approach of weaving reduced basis methods into the solution process of inverse problems is the replacement of the forward solution by a global reduced basis approximation in a given regularization algorithm. The limitations of this approach in the context of imaging (very high-dimensional parameter spaces) will be shortly discussed in this talk.

The main topic is the new Reduced Basis Landweber method. It combines the key concepts of the reduced basis method with the Landweber method and is inspired by [1]. The general idea is to adaptively construct a small, problem-oriented reduced basis space instead of constructing a global reduced basis space like it is normally the case in reduced basis methods. This will be done in an iterative procedure: the inverse problem will be solved up to a certain accuracy with a Landweber method that is projected onto the current reduced basis space. The resulting parameter then is utilized to enrich the reduced basis space and therefore fit it to the given problem. This iteration is performed until an iterate is accepted as the solution of the inverse problem. Numerical results will show a significant speed-up and the possibility to reconstruct very high-dimensional parameters.

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Reduced basis method for noncoercive variational inequalities

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We consider variational inequalities with different trial and test spaces and a possibly noncoercive bilinear form. Well-posedness has been shown under general conditions that are e.g. valid for the space-time formulation [3] of parabolic variational inequalities. Using space-time formulations, we do not have a time-stepping scheme anymore, but take the time as an additional variable in the variational formulation of the problem. As an example for a parabolic variational inequality, we may think about time-dependent obstacle problems or option pricing, e.g. for American Options.

Fine discretizations for such problems resolve in large scale problems and thus in long computing times. To reduce the size of these problems, we use the Reduced Basis Method (RBM)[2]. The objective of the RBM is to efficiently reduce discretized parametrized partial differential equations. Problems are considered where not only a single solution is needed but solutions for a range of different parameter configurations.

Combining the RBM with the space-time formulation, a residual based rigorous error estimator has been derived in [1]. In this talk, we will provide new numerical results for a computational realization of the model.

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Balanced Truncation Model Reduction for Quadratic-Linear Systems

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We discuss balanced truncation for model reduction of continuous time quadratic-linear systems. Balanced truncation for linear systems mainly involves the computation of Gramians of the system, namely the controllability and observability Gramians. These Gramians were extended to the general nonlinear setting by Scherpen (SCL,1993) where it was shown that the Gramians are the solutions of nonlinear Hamilton-Jacobi equations. These solutions, in general, depend on the state vector which makes it hard to utilize them in the model reduction framework. In this talk, we aim to determine approximate Gramians for the quadratic-linear system which can be used to balance quadratic-linear systems in order to identify a reduced-order quadratic-linear system. We also investigate the important properties of the reduced-order system such as the Lyapunov stability of the system. The efficiency of the reduced-order system obtained by the proposed method is demonstrated for various semi-discretized nonlinear partial differential equations.

POD model reduction for constrained optimal control problems

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We consider an optimal control problem for a parabolic partial differential equation (PDE) subject to control and state constraints. Since the optimality conditions are nonlinear, model order reduction provided by proper orthogonal decomposition (POD) is applied [2] to limit the numerical effort. Classical POD basis constructions usually contain *heuristic* information of the expexted optimal state solution to the PDE and therefore lack of a-priori error estimators; these are available only if the *optimal* state dynamics are included into the basis elements.

One way to compensate this drawback is to augment the reduced optimal control problem by interpreting the POD basis functions as optimization variables as well, introducing the information about the optimal state trajectory by postulating the state equation as a side condition: The optimality system proper orthogonal decomposition strategy (OSPOD). We will present a-priori [3] and a-posteriori [4] error estimations for this augmented reduced-order optimal control problem, propose an efficient solution algorithm [1] and illustrate our results by numerical tests.

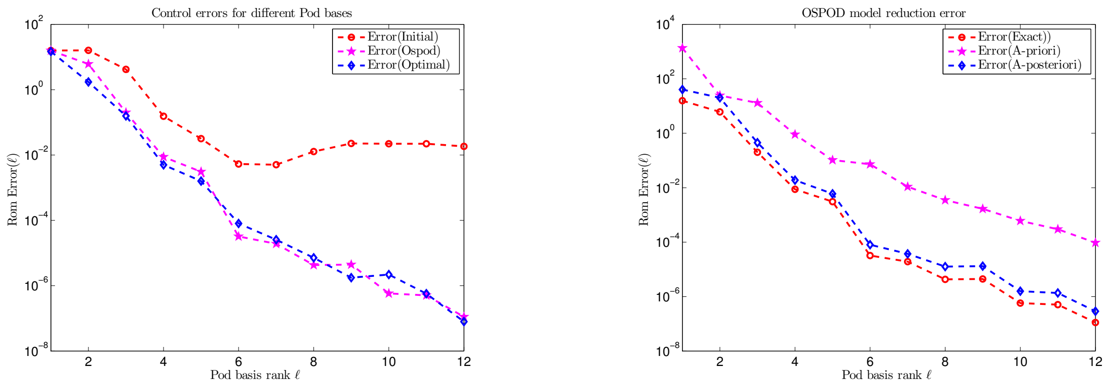

Figure 1: On the left, we see that the initial reference trajectory does not capture enough dynamics of the optimal state to build up a sufficiently accurate reduced-order model $\circ$ while the quality of the OSPOD basis $\star$ matches the accuracy of the optimal one $\diamond$. On the right, we estimate the OSPOD model error $\circ$ by a-priori bounds $\star$ based on eigenvalue estimations and by a-posteriori bounds $\diamond$ derived by a perturbation argument.

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A New Selection Operator for the Discrete Empirical Interpolation Method

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This paper introduces a new framework for constructing the Discrete Empirical Interpolation Method (DEIM) projection operator. The interpolation nodes selection procedure is formulated using a QR factorization with column pivoting. This selection strategy leads to a sharper error bound for the DEIM projection error and works on a given orthonormal frame U as a point on the Stiefel manifold, i.e., the selection operator does not change if U is replaced by $\mathsf{U}Q$ with arbitrary unitary matrix Q. The new approach allows modifications that, in the case of gargantuan dimensions, use only randomly sampled rows of U but are capable of producing equally good approximations.

A Reduced Basis Kalman Filter for Certified and Rapid State Estimation of Parametrized PDEs

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The Kalman filter is a widely known tool in control theory for estimating the state of a linear system disturbed by noise. However, when applying the Kalman filter on systems described by parametrized partial differential equations (PPDEs) the calculation of state estimates can take an excessive amount of time and real-time state estimation may be infeasible. In a recent article [1] we presented a low dimensional representation of a parameter dependent Kalman filter for PPDEs via the reduced basis method. This allows rapid state estimation, and in particular the rapid estimation of a linear output of interest. It is also possible to derive a posteriori error bounds for evaluating the quality of the output estimations. Additionally, the stability of the filter can be verified using an observability condition. We will demonstrate the performance of the reduced order Kalman filter and the error bounds with a numerical example modeling the heat transfer in a plate.

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Simultaneous Reduced Basis Approximation of Parameterized Eigenvalue Problems

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Reduced basis methods for parameterized partial differential equations have been studied for many problem classes like linear and non-linear equations, compliant and non-compliant outputs as well as static and time-dependent problems. They are also used in the context of component mode synthesis by static condensation reduced basis methods.

In many applications, e.g., in the field of vibro-acoustics during the planning process of large timber buildings, it is necessary to simulate the same large eigenvalue problem numerous times, using different material parameter values in each simulation. Furthermore, one is interested in approximating a certain number of the smallest eigenvalues and eingenvectors rather than only one eigenvalue or eigenvector and the eigenvalues of interest usually have multiplicities that depend on the parameters. One is interested in optaining the optimal parameter values for the components of the building. Reduced basis approximations allow to handle these problems in a fast way that is nevertheless accurate.

This talk is about a model reduction framework for parameterized eigenvalue problems by a reduced basis method [1], which, in contrast to the standard single output case, allows to approximate several outputs simultaneously. The outputs considered in this case are a certain number of the smallest eigenvalues with multiplicities. We analyze the corresponding a posteriori asymptotically error estimators for the eigenvalues, in order to achieve a fast and reliable evaluation of these input-output relations. Moreover, we present different greedy strategies and systematically study their performances, paying special attention to the multiple eigenvalues in the cases of the analysis of the estimator and the development of the greedy part of the algorithm.

We introduce a method to build a single reduced space for the simultaneous variational approximation of the eigenvalue problem and compare the performance of this space to the performance of a space built with a standard POD method.

Furthemore an extension of our reduced basis method to a component mode synthesis is shown, which allows to establish a component library that can be used to simulate large composed timber buildings.

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Large Eddy Simulation Reduced Order Models

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This talk proposes several large eddy simulation reduced order models (LES-ROMs) based on the proper orthogonal decomposition (POD). To develop these models, explicit POD spatial filtering is introduced. Two types of spatial filters are considered: A POD projection onto a POD subspace and a POD differential filter. These explicit POD spatial filters allow the development of two types of ROM closure models: phenomenological and approximate deconvolution. Furthermore, the explicit POD spatial filters are used to develop regularized ROMs in which various ROM terms are smoothed (regularized). The new LES-ROMs are tested in the numerical simulation of a three-dimensional flow past a circular cylinder.

Model Order Reduction for Magneto-Quasistatic Equations

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Design of complex integrated circuits with electromagnetic devices involves the numerical simulation of Maxwell's equations coupled with the network equations. In magneto-quasistatic problems, the contribution of the displacement currents is neglected compared to the conductive currents. A finite element discretization of the resulting Maxwell equations in the magnetic vector potential formulation leads to a high dimensional nonlinear system of differential-algebraic equations (DAEs)

$$E\dot{x} = A(x)x + Bu, \quad y = Cx$$

with the structured matrices

$$E = \begin{bmatrix} M & 0 \\ X^T & 0 \end{bmatrix}, \qquad A(x) = \begin{bmatrix} -K(a) & X \\ 0 & -R \end{bmatrix}, \qquad B = C^T = \begin{bmatrix} 0 \\ I \end{bmatrix}.$$

In model reduction of DAEs, special care should be taken while approximating the algebraic components and algebraic constraints which restrict the solution to a manifold.

By employing the system structure, we develop an efficient model reduction approach for the magneto-quasistatic DAE system. Our approach is based on transforming this system into the ODE form and applying the proper orthogonal decomposition (POD) [2] combined with the discrete empirical interpolation method (DEIM) [1] for efficient evaluation of the nonlinearity in the reduced-order model. A further reduction in computational complexity can be achieved by using the matrix DEIM for the approximation of the Jacobi matrix. We present an efficient implementation of the matrix DEIM which avoids the vectorization of the snapshot matrices of the Jacobian and significantly reduces the computational cost of the offline phase. Furthermore, we investigate the passivity of the infinite-dimensional problem and the preservation of this property in the semidiscretized system and the POD-DEIM reduced model. Numerical examples will demonstrate the properties of the developed model reduction method.

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Model reduction for a coupled nearwell and reservoir models using multiple space-time discretizations.

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Proper reservoir management often becomes challenging to be performed due to the intrinsic uncertainties and complexities associated with the reservoir properties. To this end, accurate results for reservoirs are obtained if fine grid discretization is induced into the model. This leads to large-scale system of nonlinear equations that needs to be solved every time step. The importance of obtaining a simpler model that can represent the physics of the full system is paramount to speed up the workflows that require several (from dozen to thousands) calls of the forward model.

Over the past decade, numerous techniques have been applied in the context of porous media flow simulation to reduce the computational effort associated with the solution of the underlying coupled nonlinear partial differential equations. In many cases, reduced-order modeling techniques have shown to be a viable way of mitigating computational complexity in simulation of the large-scale model, while they maintain high level of accuracy when compared with here high fidelity models. Many of these simulation models treat the reservoir as a whole model. In many cases, the nearwell accuracy is very important because it controls the production rate. In fact, researchers often use more complex near well models to achieve a high accuracy. In this talk, I will describe model reduction techniques that consider near well and reservoir regions separately and use different spatial grids and temporal accuracy to achieve efficient and accurate reduced order models.

Detection of parameter-dependent regimes in complex flows via compressed sensing and dynamic mode decomposition

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Complex flows are parameter dependent and are sensitive to changes in flow topology. Designing controllers and observers for such systems is challenging due to the shear size of the state-space as well as the parametric dependence. Reduced order models combined with an online estimate of the current parameters can be used for observer and controller design for such complex systems.

Using sparse sensing and reconstruction techniques, we present a framework to detect dynamical phenomena such as bifurcations and changes in flow topology in thermo-fluid dynamical systems. First, a library of reduced order models of dynamic regimes is constructed, and online measurements are used to identify the dynamic regime the flow resides at. The proposed method is based on dynamic mode decomposition [2, 3] to obtain a low dimensional representation of the dynamics, and compressed sensing [1], which allows for the use of few sensors to achieve the classification task. Moreover, by processing time-sequential information from the sparse sensor array, the algorithm shows improved robustness to noise. The method is purely data-driven, and can therefore be used in conjunction with experimental or simulation data. This framework can be combined with adaptive reduced order models for improved observation and control. Numerical results for the case of Navier-Stokes and Boussinesq equations are presented.

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Nonlinear Model Reduction for Complex Systems using Sparse Sensor Locations from Learned Nonlinear Libraries

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We demonstrate the synthesis of sparse sampling and dimensionality-reduction to characterize and model complex, nonlinear dynamical systems over a range of bifurcation parameters. First, we construct modal libraries using the classical proper orthogonal decomposition in order to expose the dominant low-rank coherent structures. Here, libraries of the nonlinear terms are also constructed in order to take advantage of the discrete empirical interpolation method and projection that allows for the approximation of nonlinear terms from a sparse number of grid points. The selected grid points are shown to be effective sensing/measurement locations for characterizing the underlying dynamics, stability, and bifurcations of complex systems. The use of empirical interpolation points and sparse representation facilitates a family of local reduced-order models for each physical regime, rather than a higher-order global model, which has the benefit of physical interpretability of energy transfer between coherent structures. The method advocated also allows for orders-of-magnitude improvement in computational speed and memory requirements.

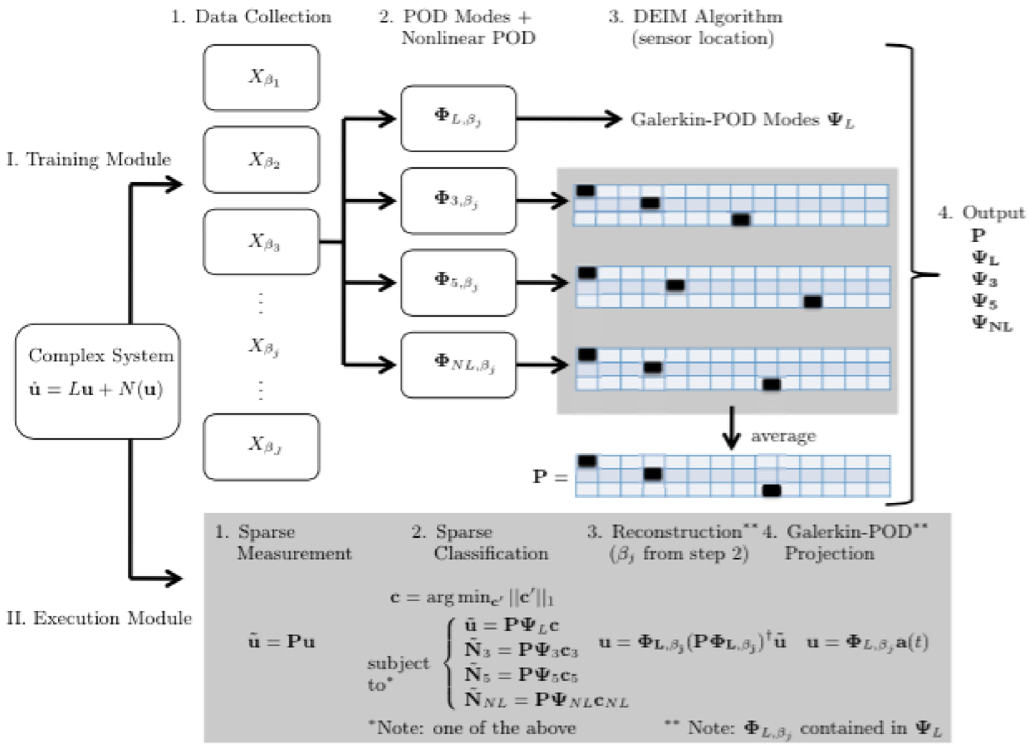

Figure 1: The training module samples the various dynamical regimes $(\beta_1, \beta_2, \cdots, \beta_J)$ through snapshots. For each dynamical regime, low-rank libraries are constructed for the nonlinearities of the complex system $(\mathbf{\Phi}_{L,\beta_j},\ \mathbf{\Phi}_{3,\beta_j},\ \mathbf{\Phi}_{5,\beta_j},\ \mathbf{\Phi}_{NL,\beta_j})$. The DEIM algorithm is then used to select sparse sampling locations and construct the projection matrix $\mathbf{P}$. The execution module uses the sampling locations to classify the dynamical regime β_j of the complex system, reconstruct its full state $(\mathbf{u} = \mathbf{\Phi}_{\mathbf{L},\beta_\mathbf{j}}(\mathbf{P}\mathbf{\Phi}_{\mathbf{L},\beta_\mathbf{j}})^{\dagger}\tilde{\mathbf{u}})$, and provide a ROM (Galerkin-POD) approximation $(\mathbf{u} = \mathbf{\Phi}_{L,\beta_j}\mathbf{a}(t))$.

Certified Reduced Basis Approximation for the Coupling of Viscous and Inviscid Parametrized Flow Models

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We present a model order reduction approach for parametrized laminar flow problems including viscous boundary layers. The viscous effects are captured by a Navier-Stokes model in the vicinity of the boundary layer, whereas a potential model is used in the outer region [4]. By this, we provide an accurate model avoiding the usage of the Kutta condition for potential flows as well as an expensive numerical solution of a global Navier-Stokes model. The domain decomposition approach is combined with the reduced basis method, which takes account of the parametrized nature of the heterogeneous coupled system.

We avoid the more involved ansatz of posing localized, decoupled problems on the subdomains [2] and instead consider a monolithic approach for this problem [3]. For this, we can apply recent elements of the reduced basis methodology for non-coercive and nonlinear partial differential equations [1, 5]. The accuracy of the reduced order model is ensured by computable a-posteriori error bounds. Numerical experiments are conducted with a parametrized flow around a NACA airfoil, including geometry variations. A considerable reduction of the computational times is obtained. Different methods to approximate the inf-sup constant of the global Fréchet-derivative are compared.

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Space-Time RBM for parabolic PDEs with Parameter Functions

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We consider reduced basis methods for parabolic partial differential equations (PDEs) in space-time variational formulation. The space-time approach avoids the (costly) time-stepping scheme in the RB-online phase and provides a posteriori error bounds (cf. [2]). The PDE solution is a function in space and time in an appropriate Bochner space.

We do not only allow parameters in the coefficients (for e.g. model calibration) but choose the initial condition as a parameter (function) as well. A reduced basis method is introduced that handels the parameter function and the corresponding infinite dimensional parameter space in a two-step greedy procedure (cf. [1]). The space-time variational formulation is splitted. A reduced basis for the initial condition is constructed first using a greedy procedure or proper orthogonal decompostion. In the second step a greedy procedure leads to a basis for the evolutionary part of the solution. Here, the reduced basis for the initial condition is involved. A posteriori error estimates are available. Online we solve two small linear equation systems.

In option pricing, the method offers the possibility to use the same reduced basis for different types of options as they differ only by their initial value (assuming the same model approach). Using the reduced basis does not only reduce the computational effort in pricing but can also be used in the PDE-constrained optimisation of the model calibration process.

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Goal-oriented error estimation for the reduced basis method Application to sensitivity analysis

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The reduced basis method is a powerful model reduction technique designed to speed up the computation of multiple numerical solutions of parametrized partial differential equations. We consider a quantity of interest, which is a linear functional of the PDE solution. A new probabilistic error bound for the reduced model is proposed[1]. It is efficiently and explicitly computable. We show, on two different practical examples, that this bound is clearly better than the naive Lipschitz bound and that, at the expense of a slight, controllable risk, the performances of this new bound are better than the ones of the existing dual-based output error bound[2].

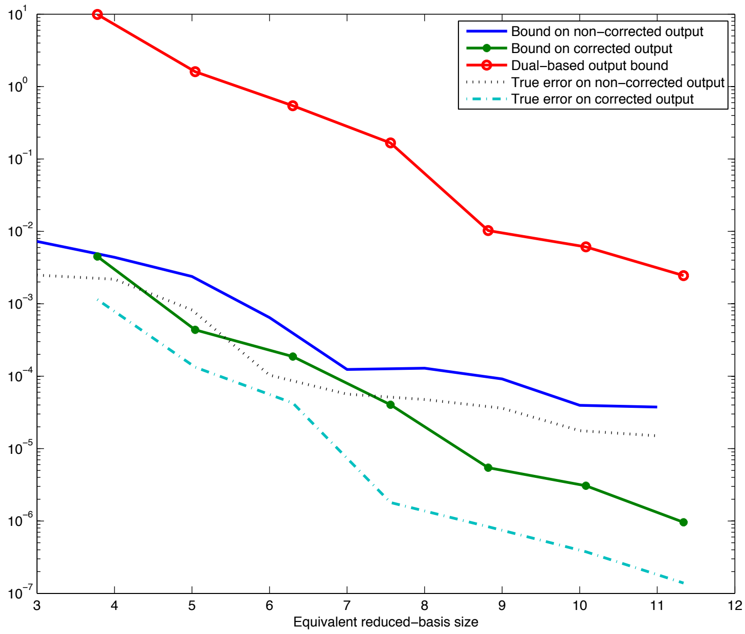

Figure 1: Comparison of the mean error bound on the non-corrected output, the mean dual-based error bound (ϵ_{cc}) and the mean error bound on the corrected output (for risk $\alpha = 0.0001$). The "equivalent" reduced basis sizes are in abscissae.

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Offline Error Bounds for the Reduced Basis Method

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Error quantification is of vital importance in reduced-basis modeling. A reduced model is only useful if it matches the high-dimensional model to within some tolerance. Even before it is time to evaluate the error in a simulation, the reduced-basis method uses a posteriori error bounds to build the model in an intelligent manner.

Reduced-basis models are most often generated iteratively using the greedy algorithm [3]. Each time the basis needs to be expanded, the low-dimensional solution $u_N(\mu)$ and the a posteriori error bound $\Delta_N(\mu)$ are calculated for all parameters μ in some Ξ, which is a finite subset of the parameter domain $\mathcal{D}$. The basis is then enhanced using the high-dimensional solution $u(\mu_{N+1})$ associated with the parameter defined by

$$\mu_{N+1} = \arg\max_{\mu\in\Xi} \frac{\Delta_N(\mu)}{\|u_N(\mu)\|}.$$

The greedy algorithm guarantees that the relative error is small for all $\mu \in \Xi$. In practice, it is expected that if Ξ is sufficently large and properly distributed, then the relative error will also be small everywhere in D. For $\mu \in \mathcal{D} \setminus \Xi$ the only way to verify that a tolerance has been met is to use a posteriori error bounds. The difficulty is in dealing with error bounds that do not meet the tolerance: either precomputed basis elements must be added to the basis [2] or new basis elements need to be computed.

In real-time problems, which have been mentioned by many authors as possible applications of reduced-basis modeling [1, 2], solutions are needed quickly and there is often no time to rerun simulations with additional basis elements.

In this talk we propose a solution to this problem. The idea is to prove that the error will be sufficiently small for all $\mu \in \mathcal{D}$. That cannot be achieved with traditional a posteriori error bounds, but we present new error bounds that make it possible. We also demonstrate these ideas with numerical results for some example problems.

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Using sensitivity analysis in the framework of proper orthogonal decomposition with application to cake filtration

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We are considering a problem resulting from the mathematical modeling of an industrial process called cake filtration [3]. It is described by a moving boundary problem involving a system of convection-diffusion-reaction equations and kinetic equations. In our case the moving boundary is evolving in time by a constant rate. The model exhibits various parameters, which influence the solution. The aim of this study is to derive a reduced order model for a reference configuration and to adjust this model under parameter change. Therefore we have to get rid of the time dependence of our computation domain. To this end we apply a pullback operator. This results in a transformed formulation involving a system of PDEs with time-dependent coefficients. As model reduction technique we choose proper orthogonal decomposition (POD). It is well-known that the application of POD involves the solution of an eigenvalue/eigenvector problem (cf. e.g. [5, 6]) using a snapshot matrix. Since we want to account for the parameter sensitivity of our reduced model, a sensitivity analysis is performed. First of all the sensitivity of the solution, i.e. of the snapshot matrix, is needed, in addition also the sensitivity of the eigenvalue/eigenvector problem for computing the reduced basis [2, 4]. To derive an improved approximation of the POD basis we combine POD and sensitivity analysis. Therefore we assume that for at least one reference parameter configuration μ_0 the POD basis and the corresponding sensitivities are available. Then various methods can be used to approximate the POD for an arbitrary parameter value μ, compare [1, 2, 4]. To solve the problem efficiently an offline/online decomposition is used, due to the time dependence of the coefficients.
It is shown that the quality of the methods strongly depend on the underlying problem. In our case the time horizon of the process has a large influence.

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Multifidelity methods for uncertainty quantification

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A classical and common remedy to reduce the runtime of uncertainty quantification is to replace the high-fidelity (full) model by a cheap surrogate model of lower fidelity. Such a surrogate model can be derived in various ways, including data-driven and projection-based model reduction, interpolation and regression approaches from machine learning, and different modeling and physics assumptions. A surrogate model is traditionally constructed with one-time high computational costs in an offline phase and then replaces the high-fidelity model in an online phase to conduct the uncertainty quantification. In this talk, we introduce multifidelity methods for uncertainty quantification that use in the online phase a combination of the high-fidelity model with multiple surrogate models. Our multifidelity methods leverage surrogate models to accelerate the computation, but explicitly allow occasional recourse to the high-fidelity model to establish accuracy guarantees on the overall uncertainty quantification result. These guarantees exist even in the absence of error bounds and error estimators for the surrogate models themselves.

The key component of our multifidelity methods is model management that balances the model evaluations across the high-fidelity and the surrogate models, and that combines the high-fidelity and low-fidelity solutions. We discuss three cases: (1) Model management based on online adaptivity, where a surrogate model (data-driven projection-based reduced model) is corrected online with sparse/partial solutions of the high-fidelity model. (2) Model management based on importance sampling. (3) Model management based on control variates, where we introduce an optimization problem to distribute model evaluations between the high-fidelity model and an arbitrary number of surrogate models of any type, including projection-based reduced models, data-fit interpolation models, and support vector machines.

We show mathematically and demonstrate numerically that combining the high-fidelity model with surrogate models of different approximation quality and costs is often more beneficial than combining the high-fidelity model with accurate surrogate models only. In this sense, surrogate models that inform different aspects of the high-fidelity model are better than surrogate models that are accurate but lack a rich diversity. Numerical results with linear and nonlinear examples show that our multifidelity methods achieve speedups by orders of magnitude compared to methods that invoke a single model only.

A Modularized Modelling, Discretization and Model Order Reduction Workflow for the Simulation of Li-ion Batteries

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A major cause for performance degradation and failure of rechargeable Li-ion batteries is the disposition of a metallic Li-phase at the negative electrode of the battery cell (Li-plating). The process leading to the formation of this additional phase, however, is still poorly understood. It is the aim of the MULTIBAT research project to gain a better insight into the causes of Li-plating with the help of mathematical modelling and numerical simulation. However, since Li-plating is initiated on a micrometer scale at the interface between electrode and electrolyte, battery models which resolve the porous electrode geometry [2] are needed to accurately describe this effect. This in turn leads to high-dimensional nonlinear finite volume discretizations [4] which require substantial computational effort for their solution.

In this contribution we present the simulation workflow that has been established by MULTIBAT to tackle this challenging application problem. Model order reduction plays an integral role in this workflow in order to make the microscale modelling approach computationally feasible for parameter studies and responsive simulation tools. Our workflow is designed from ground up to be modular, allowing different groups to contribute their expertise in the different parts of the modelling and simulation pipeline, and allowing easy adaption of the developed algorithms and tools to similar application problems based on transport processes in materials with microscale structure.

For the model order reduction we have implemented reduced basis approximation algorithms in conjunction with empirical operator interpolation [1] for the nonlinear parts in the system's space differential operator as generic algorithms in our freely available model reduction software library `pyMOR` [3]. Due to `pyMOR`'s interface-based approach, these algorithms can be automatically applied to newly developed battery models in the battery simulation software `BEST`. At the same time, improved reduction algorithms can be used without requiring changes in `BEST`. Moreover, `pyMOR`'s algorithms can be easily reused with other PDE solvers to tackle new application problems.

Besides a presentation of the MULTIBAT workflow, we will also cover the battery model and its reduction in more detail and discuss our current results.

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Localization and adaptivity in Reduced Basis methods

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Reduced Basis (RB) methods are a powerful methodology to significantly reduced the computational complexity of solving parameterized partial differential equation problems. They allow for speedups of several orders of magnitude, compared to traditional discretization methods (such as Finite Element methods), by splitting the computational process into an offline and an online part. However, for interesting real-world problems, such as parametric multi-scale flow in porous media, the offline part of the computation can become unbearably costly, thus limiting the usefulness of RB methods in these scenarios. In the past years, several methodologies arose incorporating localization ideas from domain decomposition or numerical multi-scale methods into RB methods, such as the localized reduced basis multi-scale method (LRBMS) [1, 2, 3].

The main idea of the LRBMS is to build several local reduced bases (associated with subdomains of the physical domain) which are coupled in the spirit of discontinuous Galerkin methods. The resulting local reduced bases consist of a locally varying number of local basis functions, representing possible local influences of the parameterization. Since less global solution snapshots are required than with traditional RB methods and all offline work can be executed in parallel on local quantities, the LRBMS allows to balance the computational effort between the offline and the online part of the computation by choosing an appropriate number of subdomains (see [1]).

We present a recent extension of the LRBMS, based on an offline/online decomposable localized a posteriori estimate on the full error (including the discretization as well as the model reduction error, see [2]). This estimate allows to efficiently identify subdomains where the local reduced basis is insufficient during the online part of the computational process. By relaxing the traditional offline/online approach the *online adaptive LRBMS* allows for local high-dimensional computations in order to adaptively enrich these insufficient local reduced bases online, where required (see [3]).

The online adaptive LRBMS is thus suitable for complex real-world problems where we lack sufficient resources to fully prepare a suitable reduced basis offline. We demonstrate the methodology in the context of heterogeneous Darcy flow, compare it to related methods and discuss adaptivity in the context of RB methods in general.

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Reduced Basis Approximation for Parametric Large Scale Algebraic Riccati Equations

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The Algebraic Riccati Equation (ARE) is a quadratic matrix valued equation with a variety of important applications in the field of dynamical systems, such as optimal feedback control, filtering and state estimation, see e.g. [1]. As the modelling of many complex technical phenomena is usually performed by means of partial differential equations, which after semidiscretization in space yield very large time dependent ODE systems, the question arises how the resulting AREs can be solved efficiently. Although many very efficient solution algorithms exist, real-time applications or many-query scenarios in a parametric case can lead to infeasible calculation times. For that reason we are interested in extending the reduced basis (RB) method (see e.g. [2]) for obtaining approximate solutions to the ARE rapidly and with reliable error statements. RB methods aim at splitting the calculation in two parts: A potentially expensive offline phase, where a suitable problem adapted basis is constructed, followed by a projection of the original problem onto the low dimensional subspace. This expensive procedure then pays off in the online phase, where approximate solutions can be calculated rapidly for varying parameters, along with error bounds that quantify the quality of the approximation.

In order to apply the RB method for parametric AREs, we formulate a new procedure called "Low Rank Factor Greedy" algorithm, which exploits a low rank structure in the solution matrices to build a suitable low dimensional basis, cf. [4, 3]. We show how a new residual based error estimator can be derived and how a computable version can be formulated by stating upper bounds for the constants occuring in the error estimator. Furthermore, parameter separability of the data matrices will allow a calculation completely independent of the original (high) dimension.

The solution to the ARE directly determines a so called gain matrix which solves an optimal feedback control problem for linear systems (linear quadratic regulator, LQR). As this is certainly among the most interesting applications of the ARE, we state error bounds on the suboptimality imposed by using the approximation from the proposed RB method as a surrogate for the true (expensive) solution of the ARE, including bounds on the cost functional, the state and the output of the control systems.

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Data-driven interpolation of dynamical systems with after-effect

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Simulation of complex physical, chemical or biological processes described by mathematical models is a standard tool in research and industry. Besides the computational cost for solving high fidelity models, it might even be challenging to develop the underlying dynamical system due to its complex nature. Data-driven model order reduction is a promising approach to construct low-dimensional models directly from measurements. The rate of change of realistic models often depends not only on the current time point, but also on the configuration at previous time instances and we wish to preserve this delay structure in the reduced model.

In this talk, we present a data-driven realization methodology for descriptor systems with retarded argument and unknown delay, which is a generalization of the Loewner framework [1]. This is accomplished by using ideas from moment matching. The realization is obtained with low computational cost directly from measured data of the transfer function. The internal delay is estimated by solving a least-square optimization over some sample data. Our approach is validated by numerical examples, which indicate the need for preserving the delay structure in the reduced model.

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Optimal local approximation spaces for component-based static condensation procedures

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In this talk we introduce local approximation spaces for component-based static condensation (sc) procedures that are optimal in the sense of Kolmogorov [2].

To facilitate simulations for large structures such as aircraft or ships it is crucial to decrease the number of degrees of freedom on the interfaces, or "ports", in order to reduce the size of the statically condensed system. To construct optimal port spaces we consider a (compact) transfer operator that acts on the space of harmonic extensions on the component pair associated with the respective port and maps the traces (of the harmonic extensions) on the boundary ports to the trace on the shared port. Solving the "transfer eigenproblem" for the composition of the transfer operator and its adjoint yields the optimal space. For a related work in the context of the generalized finite element method we refer to [1]. Next we introduce a spectral greedy algorithm to generalize the transfer eigenproblem procedure to the parameter dependent setting and construct a quasi optimal parameter independent port space. Moreover, we show that given a certain tolerance and an upper bound for the ports in the system, the spectral greedy constructs a port space that yields a sc approximation error on a system of arbitrary configuration which is smaller than this tolerance for all parameters in a rich train set. Numerical experiments demonstrate the very rapid and exponential convergence both of the eigenvalues and the sc approximation based on spectral modes also for non-separable and irregular geometries.

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A Model Reduction Approach to Structural Health Monitoring

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We present a model reduction approach to the problem of Structural Health Monitoring of systems modeled by parametric PDEs. The approach generates, by exploiting machine learning algorithms and numerical simulations, a classifier that monitors the state of damage of the system.

Our approach is based on an offline/online computational decomposition. In the offline stage, the field associated with many different system configurations, corresponding to different states of damage, are computed and then employed to teach a classifier. In the online stage, the classifier is used to associate measured data to the relevant diagnostic class. In addition, outlier detection schemes can be applied to assess the relevance of the offline model and sampling assumptions.

In order to explore the high-dimensional parameter space associated with the possible system configurations of the undamaged and damaged system, a component-based model reduction technique based on static condensation Reduced Basis Element (scRBE) method is employed. The latter is particularly effective in permitting large variations in geometry (and properties) and also variations in topology.

We illustrate our method through an acoustic duct example: damage is represented as a side hole of variable location and size; both the undamaged and damaged states are subject to uncertainty in wavenumber; input impedance as a function of frequency is chosen as the classification feature.

POD-Galerkin for finite elements with dynamic mesh adaptivity

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Spatial adaptivity has recently gained interest in the context of snapshot-based reduced-order modeling. Typically, one relies on the same spatial mesh for all snapshots, which allows only static adaptivity [3]. Independent snapshot meshes have been investigated so far for wavelet-based [2] and one-dimensional [1] discretization schemes. In this study, we consider POD-Galerkin modeling for two-dimensional unsteady problems with dynamically adapted finite element snapshots.

We concentrate on snapshots obtained with newest vertex bisection based on some fixed initial mesh. For any subset of such snapshots, by refinement one can find a common mesh on which all snapshots in the subset can be represented exactly. Therefore, one way of creating a POD-Galerkin model is to construct the common mesh of all snapshots, interpolate the snapshots onto this mesh, and proceed with standard techniques. Alternatively, for PDEs with polynomial non-linearities one can work with the common grids of all $(N+1)$-tuples of snapshots, where N is the maximum polynomial degree. This approach can be useful if the common mesh of all snapshots contains much more nodes than each individual snapshot mesh.

As an example we study a viscous Burgers equation with smooth initial data. Figure 1 presents adapted meshes at different solution times. Figure 2 shows the error of the projected snapshots (POD) and the error of the POD-Galerkin solution (ROM) with respect to the original snapshots. The solution of the reduced-order model stops converging when the basis functions start reproducing mainly spatial discretization effects. At this point, however, the finite element error of the snapshots (FEM) already suggests snapshot refinement in order to converge to the true solution.

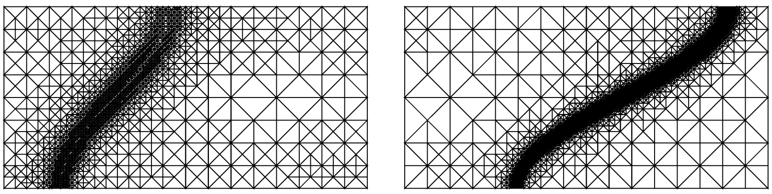

Figure 1: Adaptive meshes at times $t = 0.15$ and $t = 0.3$.

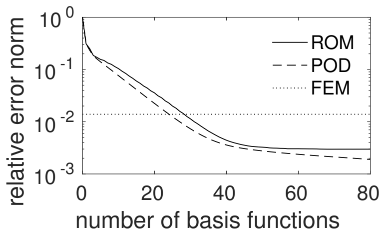

Figure 2: Convergence plot.

A necessary ingredient for our approach to POD-Galerkin modeling with adaptive finite element snapshots is the ability to construct common meshes of sets of snapshots. While this may be difficult to accomplish for arbitrary mesh structures, our work is a proof of concept for nested meshes, which readily generalizes to projection-based methods other than POD.

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A Certified Reduced Basis Method for Variational Inequalities and Optimal Control Problems

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In this talk, we present a method for computing reduced order models and associated a posteriori error estimates for two problem classes: variational inequalities and optimal control.

In the first part of the talk, we present a minimum residual, slack–variable reduced basis approach for variational inequalities of the first kind. The strict feasibility of the resulting approximations leads to two significant improvements upon existing methods. First, it provides a posteriori error bounds which are significantly sharper than existing bounds. Second, it enables a full offline–online computational decomposition in which the online cost to compute the error bound is completely independent of the dimension of the original (high–dimensional) problem.

In the second part of the talk, we discuss the use of reduced order models as surrogate models for PDE–constrained optimal control problems. In this context, we develop rigorous and efficiently computable a posteriori error bounds for both the optimal control and the associated optimal cost functional.

Numerical results compare the performance of the proposed and existing approaches.

Element-based model reduction for parameter dependent parabolic PDE's on networks

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We consider parameter dependent parabolic PDE's on one-dimensional networks. For such networks as well as component-based problems Maday and Rønquist [3] developed the reduced basis element method (RBE). It is based on the idea of constructing a reduced basis for every network edge and afterwards coupling the reduced components by a mortar-like method. This way, large networks can be reduced without calculating a full solution of the given problem.

However, this decomposition procedure leads to a big problem for large parameter dependent networks. Especially on edges far away from the boundary the solution is hardly predictable. Therefore, the required boundary conditions for the edge problem in the POD process are not known. Based on this, the computation of a reduced basis for this edge is not possible or leads to poor basis functions. Initial approaches concerning this problem are given in the context of the static condensation reduced basis element method (SCRBE) [2].

We present a method to calculate basis functions which remedies this problem and provides a good basis representation for the single edges. Our approach uses piecewise linear functions to approximate the unknown boundary conditions. Hereby, a new set of parameters is introduced which is additionally considered in the POD method. Furthermore, we introduce an error analysis which substantiates the good basis representation of the global solution.

We demonstrate our method on an one-dimensional network of heat equations with varying thermal conductivity. The numerical treatment of the basis construction is based on the greedy-POD. Due to the high number of parameters the method of Bui-Thanh et. al. [1] is applied which extends the greedy by an optimization algorithm.

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An adaptive sampling approach for nonlinear dimensionality reduction based on manifold learning

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In [2], the authors and colleagues had proposed a non-intrusive dimensionality reduction method for nonlinear parametric flow problems governed by the Navier-Stokes equations. The approach is based on the manifold learning method of Isomap [3] combined with an interpolation scheme and will be referred to hereafter as as Isomap+I. Via this method, a low-dimensional *embedding space* is constructed that is approximately *isometric* to the manifold that is assumed to be formed by the high-fidelity Navier-Stokes flow solutions under smooth variations of the inflow conditions.

As with almost all model reduction methods, the offline stage for the Isomap+I approach requires a suitable *design of experiment*, i. e., a well-chosen sampling of high-fidelity flow solutions, the so-called *snapshots*. The online stage, however, might be considered as an adaptive way for choosing for each low-order prediction the most suitable local snapshot neighborhood rather than using all available snapshot information in a brute-force way. The notion of locality is based on the Isomap metric.

This talk will focus on an adaptive construction and refinement of the underlying design of experiment. Since Isomap comes with a natural non-Euclidean metric for measuring snapshot distances, we make use of this metric to detect gaps in the embedding space. By the (approximate) isometry between the embedding space and the manifold of flow solutions, we obtain in this way a *manifold filling* design of experiment. In contrast, standard approaches like the Latin Hypercube method [1] aim at a parameter-space filling design of experiment. The performance of the proposed manifold filling method will be illustrated by numerical experiment, where we consider nonlinear parameter-dependent steady-state Navier-Stokes flows in the transonic regime.

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Control of an elastic structure based on Galerkin approximations

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In this talk, we consider an elastic shell model described by the following variational formulation:

$$\int_D \left\{ \rho \left(\frac{\partial^2 \tilde{r}}{\partial t^2}, \delta r \right) + \sum_{i,j=1}^{2} \left(\frac{1}{2} \frac{\partial \Phi}{\partial \varepsilon_{ij}} \delta g_{ij} + \frac{\partial \Phi}{\partial \varkappa_{ij}} \delta q_{ij} \right) - (\tilde{W}, \delta r) \right\} |\mathring{G}|^{1/2} d\xi^1 d\xi^2 - \int_\Gamma (\tilde{F}, \delta r) ds = 0, \quad (1)$$

for each admissible variation $\delta r(\xi^1, \xi^2, t)$. Here ρ is the surface density, $\tilde{r}(\xi^1, \xi^2, t)$ is the radius vector describing the median surface of the shell, $(\xi^1, \xi^2) \in D \subset \mathbb{R}^2$ are the Lagrangian coordinates, $\Phi(\varepsilon, \varkappa)$ is the energy density, tensors $\varepsilon = (\varepsilon_{ij})$ and $\varkappa = (\varkappa_{ij})$ are introduced to measure the strain and bending, $\mathring{G}$ is the metric tensor, g_{ij} and q_{ij} are components of the first and the second quadratic forms of the median surface, respectively. We assume that the shell is controlled by the force $\tilde{F}$ on the boundary Γ of D, and that external disturbances $\tilde{W}$ are distributed on the shell surface. Variational formulation (1) is obtained by applying Hamilton's principle within the framework of classic shell theory [1].

In order to derive a reduced model, we consider a finite set of basis functions $r_1(\xi^1, \xi^2)$, $r_2(\xi^1, \xi^2)$, ..., $r_N(\xi^1, \xi^2)$ and assume that

$$\tilde{r}(\xi^1, \xi^2, t) = \sum_{j=1}^{N} q_j(t) r_j(\xi^1, \xi^2) \text{ and } \delta r(\xi^1, \xi^2, t) \in \operatorname{span}\{r_1(\xi^1, \xi^2), r_2(\xi^1, \xi^2), ..., r_N(\xi^1, \xi^2)\}.$$

As a result, we get the following Galerkin system for (1):

$$M\ddot{q}(t) + Kq(t) = \bar{B}u + d(t), \quad q(t) = \begin{pmatrix} q_1(t) \\ q_2(t) \\ \vdots \\ q_N(t) \end{pmatrix}, \; u = \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \end{pmatrix}, \quad (2)$$

where $u \in \mathbb{R}^m$ is the control (represented in terms of the integral of $\tilde{F}$ over Γ) and $d \in \mathbb{R}^N$ is the disturbance vector (represented via the integral of $\tilde{W}$ over D). The procedure for computing the components of matrices M, K, and $\bar{B}$ is described in the paper [1].

We address the following problems for the Galerkin system in this talk:

1. for a given disturbance d, find a control u that minimizes the total stress in the shell;
2. identification of the disturbance vector d;
3. observer design problem for system (2) with the output from strain gauges;
4. stabilization of system (2) by a state feedback law and with an observer-based controller.

We also show some simulation results for reduced system (2) with two degrees of freedom where the basis functions $r_1(\xi^1, \xi^2)$ and $r_2(\xi^1, \xi^2)$ are computed by a finite element method.

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Poster Presentations

Error Estimation in Frequency Domain and its Use for Model Reduction of Quadratic Bilinear Systems

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We consider interpolatory techniques for model reduction of quadratic bilinear systems. The input-output representation of these nonlinear systems in frequency domain involves multivariate generalized transfer functions, each representing a subsystem of the original system. Existing interpolatory techniques [3, 1] for model reduction of quadratic bilinear systems interpolate these generalized transfer functions at some random set of interpolation points or by using the corresponding linear iterative rational Krylov algorithm. The goal here is to propose an approach that identifies a good choice of interpolation points based on the error bound expressions derived recently in [2]. We extend the use of error estimators to quadratic bilinear systems by identifying error bounds for the generalized transfer functions. This allows us to iteratively update the interpolation points in a predefined sample space by selecting the points corresponding to the maximal error bound. The approach results in a greedy type algorithm for model reduction of the generalized transfer functions and therefore for the quadratic bilinear system. Numerical results show the importance of choosing the interpolation points for some benchmark examples.

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A Certified Model Reduction Approach for robust optimal control with PDE constraints.

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We investigate the optimal placement of a permanent magnet in the rotor of a synchronous machine. The goal is to optimize the volume and position while maintaining a given performance level. These quantities are computed from the magnetic vector potentials obtained by the magnetostatic approximation of Maxwell's equation. Our optimization problem is governed by an elliptic partial differential equation and due to manufacturing, there are uncertainties in material and production precision. We introduce a robust optimization problem that accounts for uncertain model and optimization parameters. The resulting optimization problem, utilizing the robust worst-case formulation, is of bi-level structure. The solution of this problem may involve the solution of several Partial Differential Equations and, for this reason, we introduce a model reduction technique in order to approximate the problem efficiently. In particular, we propose a goal-oriented model order reduction approach in order to avoid expensive offline stages and to provide a certified reduced order surrogate model for the parametrized PDE which then is utilized in the numerical optimization. Numerical results are presented to validate the presented approach.

RBniCS – reduced order modelling in FEniCS

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`RBniCS` [1] is a python-based library, developed on top of `FEniCS` [3], aimed at the developement of reduced order models in the `FEniCS` environment. In particular, reduced order techniques such as the certified reduced basis method and proper orthogonal decomposition-Galerkin methods are implemented. The `FEniCS` project allows `RBniCS` to take advantage of the high-level (e.g., human readable) code used for the automated solution of partial differential equations. Thanks to the features of `FEniCS` the final user needs to prepare a short code (around 100 lines) to carry out a reduced order simulation.

It is ideally suited for novice users willing to learn reduced basis methods and reduced order modelling, thanks to an object-oriented approach and an intuitive and versatile python interface. Indeed, it is a companion of the introductory reduced basis handbook [2], and has been already used in doctoral classes within the "Mathematical Analysis, Modelling, and Applications" PhD course at SISSA, as well as for courses within the "Master in High Performance Computing" held by SISSA and International Centre for Theoretical Physics (ICTP).

`RBniCS` can also be used as a basis for more advanced projects that would like to assess the capability of reduced order models in their existing `FEniCS`-based software, thanks to the availability of several reduced order methods and algorithms in the library.

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PMOR for Nanoelectronic Coupled Problems

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Designs in nanoelectronics often lead to problems that are very large and that include strong feedback couplings. Industry demands to include variability to guarantee quality and yield. These parameter variations maybe due to material properties, system configurations, etc. It also requests to incorporate higher abstraction levels to allow for system simulation in order to shorten design cycles, while preserving accuracy. The transient circuit modelling and space discretized full Maxwell equations yield parameterized differential-algebraic systems (DAEs). In this work, we consider those DAEs that are quadratic in state space $\mathbf{x} = \mathbf{x}(\mu, t)$ and parameterized by a vector $\mu \in \mathbb{R}^d$,

$$\begin{aligned} \mathbf{E}(\mu)\mathbf{x}' &= \mathbf{A}(\mu)\mathbf{x}(\mu) + \mathbf{x}^T\mathbf{F}(\mu)\mathbf{x} + \mathbf{B}(\mu)\mathbf{u}(t), \\ \mathbf{y} &= \mathbf{C}(\mu)\mathbf{x} + \mathbf{D}(\mu)\mathbf{u}(t), \end{aligned} \tag{1}$$

where d is the number of parameters, $\mathbf{x} \in \mathbb{R}^n$ is the state vector, the matrix $\mathbf{E}(\mu) \in \mathbb{R}^{n\times n}$ is singular for every parameter μ, $\mathbf{A}(\mu) \in \mathbb{R}^{n\times n}, \mathbf{B}(\mu) \in \mathbb{R}^{n\times m}, \mathbf{C}(\mu) \in \mathbb{R}^{\ell\times n}, \mathbf{D}(\mu) \in \mathbb{R}^{\ell\times m}$. The tensor $\mathbf{F}(\mu) \in \mathbb{R}^{n\times n\times n}$ is a 3-D array of n matrices $\mathbf{F}_i(\mu) \in \mathbb{R}^{n\times n}, i = 1,\dots,n$, and $\mathbf{x}^T\mathbf{F}(\mu)\mathbf{x} := \left[\mathbf{x}^T\mathbf{F}_1(\mu)\mathbf{x},\dots,\mathbf{x}^T\mathbf{F}_n(\mu)\mathbf{x}\right]^T \in \mathbb{R}^n$. We assume that the matrices $(\mathbf{E}(\mu),\mathbf{A}(\mu),\mathbf{B}(\mu),\mathbf{C}(\mu),\mathbf{D}(\mu))$ and the tensor $\mathbf{F}(\mu)$ can be written as an affine parameter dependence, that is $\mathbf{M}(\mu) = \mathbf{M}_0 + \sum_{i=1}^{m} f_i(\mu)\mathbf{M}_i$, where the scalar functions f_i determine the parametric dependency, which can be nonlinears function of μ, and $\mathbf{M}_i$ can either be a constant matrix or a constant tensor. $\mathbf{u} = \mathbf{u}(t) \in \mathbb{R}^m$ and $\mathbf{y} = \mathbf{y}(\mathbf{t},\mu) \in \mathbb{R}^\ell$ are the inputs (excitations) and the desired outputs (observations), respectively. We note that system (1) includes electro-thermal couplings. The scientific challenges are to develop efficient and robust techniques for fast simulation of strongly coupled systems, that exploit the different dynamics of subsystems, and that can deal with signals that differ strongly in the frequency range and parameter variations. This calls for the application of parameterized model order reduction (PMOR) techniques. However, there exist no such PMOR techniques for quadratic coupled DAEs. In [1], a PMOR method for Electro-Thermal Package Models is proposed. This PMOR method involves first decoupling the parameterized quadratic DAE (1) into quadratic differential (thermal part) and algebraic (electrical) parts. Then the standard PMOR techniques such as PMOR based on implicit moment-matching [2] can be used to reduce both parts which leads to reliable and accurate PROM. We intend to extend these ideas to other coupled problems from nanoelectronics such as Power-MOS devices.

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Stability Preserving, Adaptive Model Reduction of DAEs by Krylov Subspace Methods

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Model order reduction based on *Krylov subspace methods* stands out due to its generality and low computational cost, making it a predestined candidate for the reduction of *truly-large-scale* systems. Even so, the inherent flexibility of the method can lead to quite unsatisfactory results as well. In particular, the preservation of stability is not guaranteed per se, attaching even more importance to the careful selection of free design parameters. Whenever a given system is modeled by a set of linear *ordinary differential equations* (ODE), some remedies for stability preservation are available, such as the one presented in [4] for *strictly dissipative* realizations or the $\mathcal{H}_2$-*pseudooptimal* reduction strategy introduced in [3, 5].

Oftentimes the object oriented, computerized modelling of dynamical systems yields a system of *differential algebraic equations* (DAE), which present characteristics not covered by standard ODE theory. In particular, the transfer behavior might be improper and in general, model reduction involves the approximaton of the dynamical and preservation of the algebraic part [1]. Even though in recent years many publications addressed *DAE-aware* reduction strategies for different indices and structures, the problem of stability preservation is hardly covered.

In this contribution, we consider index-1 DAEs in *semiexplicit form* and propose two reduction strategies that guarantee the stability of the reduced model. In this context, we will take special care in effectively reducing the underlying ODE while operating on the DAE. We will show in theory and through numerical examples that this is not always granted when extending the DAE-aware procedure described in [1] to the case of one-sided reduction. Moreover, we will show that also in the DAE case $\mathcal{H}_2$-pseudooptimal reduction has a series of advantages. The resulting stategy, adapted from [2], will preserve stability and select adaptively both the expansion points and the order of the Krylov subspace. The case of *improper* DAEs retaining an *implicit feedthrough* will be considered both in theory and examples.

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Reduced Basis method applied to large scale non linear multiphysics problems

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The Laboratoire National des Champs Magnétiques Intenses (LNCMI) is a French large scale facility [2] enabling researchers to perform experiments in the highest possible magnetic field. The design and optimization of such magnets require the prediction of performance metrics which can be the magnetic field in the center, maximum stresses, or maximum and average temperatures. These outputs are expressed as functionals of field variables associated with a set of coupled parametrized PDEs involving materials properties as well as magnet operating conditions. These inputs are not exactly known and form uncertainties that are essential to consider, since existing magnet technologies are pushed to the limits.

Solutions of a multi-physics model involving electro-thermal, magnetostatics and mechanics are requested to evaluate these implicit input-output relationships, but represent a huge computational time when applied on real geometries. The models typically include mesh (*resp.* finite element approximations) with (tens of) millions of elements (*resp.* degrees of freedom) requiring high performance computing solutions. Moreover, the non affine dependance of materials properties on temperature render these models non linear and non affinely parametrized.

The reduced basis (RB) method offers a rapid and reliable evaluation of this input-output relationship in a real-time or many-query context for a large class of problems among which non linear and non affinely parametrized ones. This methodology is well adapted to this context of many model evaluations for parametric studies, inverse problems and uncertainty quantification.

In this talk, we will present the RB method applied to the 3D non-linear and non affinely parametrized multi-physics model used in a real magnet design context. This reduced model enjoys features of reduced basis framework ([3, 1]) available with opensource library Feel++. (Finite Element method Embedded Language in C++, `http://www.feelpp.org`). Validations and examples will be presented for small to large magnet models, involving parametric studies and uncertainty quantifications.

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Adaptive Preconditioning of Fast Frequency Sweeps by ROMs

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Adaptive multi-point methods of model-order reduction (MOR) provide a highly efficient way of computing the broadband frequency response of electromagnetic structures by the finite-element (FE) method. When the considered model is many wavelengths in size, however, the dimension of the FE system is typically so large that it must be solved by iterative methods. Since the number of expansion points for the reduced order model (ROM) tends to grow as well, generating the ROM becomes computationally expensive.

To alleviate this problem, the present paper proposes to employ the ROM already available at a given adaptive step for constructing an efficient two-level preconditioner [1]. As the preconditioner is built in an adaptive manner, we use a domain-decomposition (DD) method [2] as a one-level preconditioner. To demonstrate the benefits of the suggested approach, we consider a linear array of 50 Vivaldi antennas in the frequency band $f \in [1, 4]$ GHz. The geometry of a single radiator is shown in Figure 1. FE discretization results in a linear system of dimension $8 \cdot 10^6$. It is solved by the restarted GMRES(30) iterative method [3], with stopping criterion $\delta = 10^{-6}$. As shown in Figure 2, the ROM-based two-level approach proposed in this paper reduces iteration count up to a factor of 20, compared to a standard DD preconditioner.

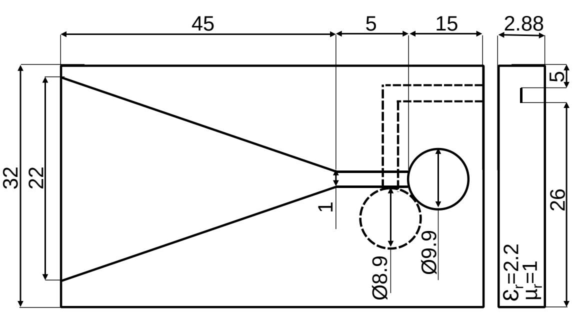

Figure 1: Antenna geometry of a single radiator. All dimensions are in mm.

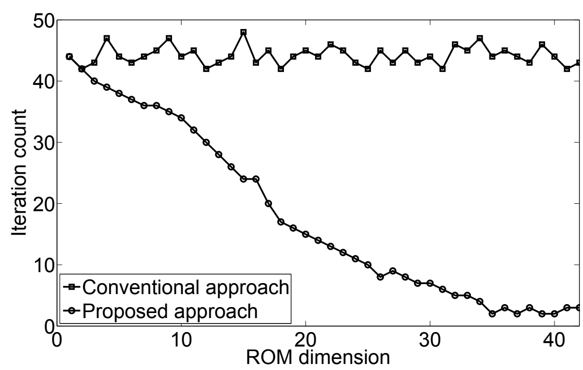

Figure 2: Iteration count versus ROM dimension for the standard one-level and the proposed two-level preconditioners.

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Simulating rigid body motion occurring in eddy current problems by parametric model-order reduction

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In the numerical modeling of eddy current problems, such as wireless power transfer (WPT) systems, one commonly encounters geometrical parameters, especially rigid body motion. One way of tackling this class of problems is by a coupled finite-element boundary-element (FE-BE) scheme [2], which does not require a mesh between the rigid bodies. The FE-BE method involves assembling and solving a large system of equations. Projection-based methods of parametric model-order reduction (MOR) greatly reduce computational times and thus enable tasks that require large numbers of function evaluations, like numerical optimization or response surface modeling. However, such MOR methods require the underlying model to exhibit affine parameterization, which is not the case for the boundary-element part of the FE-BE method. To make the model accessible to MOR, we propose to approximate the Green's function in affine form by means of the empirical interpolation method [1].

We consider an inductive WPT system where the lateral position t and angle α of the receiver coil are variable; see Figure 1. The resulting FE-BE model has about 250,000 degrees of freedom (DoF). Figure 2 shows the inductive coupling factor k versus the geometrical parameters at $f = 1$ Hz, using a reduced-order model (ROM) of 232 DoFs. The absolute error in k with respect to the full model, depicted on Figure 3, confirms that the ROM is of sufficient accuracy compared to typical errors of the underlying discretization methods. Online computational speed of the proposed method is more than 4000 times faster than with the conventional approach.

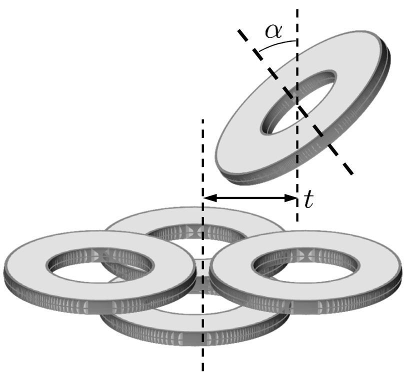

Figure 1: Geometric parameters.

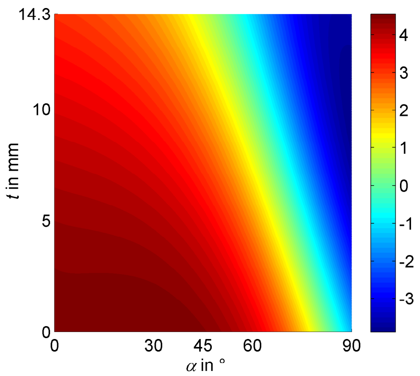

Figure 2: Coupling factor.

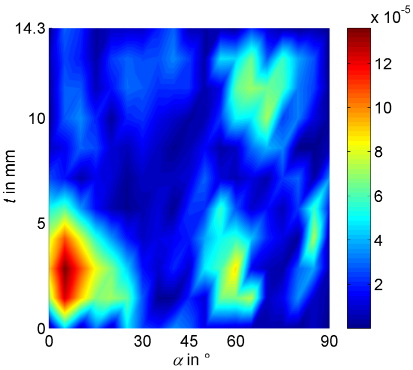

Figure 3: Absolute error.

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Model Reduction of a Nonlinear Crash Model of a Racing Kart

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Nowadays, crash simulation with commercial Finite-Element (FE) software is a core area of vehicle development. Bearing in mind that crash simulation is among the most calculation-time consuming task in car design, the usage of model reduction for speed-up and data reduction is a logical consequence. The dynamical behavior of a racing kart, depicted in Fig. 1, crashing into a rigid pole and exhibiting a plastic deformation, see Fig. 2, should be approximated by a reduced order model in LS-DYNA. The racing kart is a simple automobile. Hence, its crash behavior is strongly influenced by the nonlinear plastic deformation characteristics of the frame. Therefore, the kart frame is an ideal example to discuss the performance of various model reduction methods for structures in crashes.
In a crash scenario, some parts of the automobile exhibit large deformations, whereas others experience merely small linear vibrations [1]. In order to determine which parts of the model exhibit deformations and which can be considered as linear, it is necessary to run multiple simulations with varying parameters, e.g. impact velocity, in an offline step. Similar to passenger cars in frontal crashes, the plastic deformation in the rear of the kart is rather small. As a consequence, this part is an area where an approximation with linear ansatz functions is suitable. Subsequent to the substructure decision, the correct modeling and handling of the interfaces between the linearly approximated and nonlinear part, see e.g. [2], is decisive for good reduction results. For the calculation of the linear ansatz functions, the nonlinear FE equation $\boldsymbol{M}_e \cdot \ddot{\boldsymbol{q}}(t) + \boldsymbol{k}_e(\boldsymbol{q}, \dot{\boldsymbol{q}}, t) = \boldsymbol{f}(t)$ is linearized with the help of the implicit solver option of LS-DYNA to a second order linear time invariant system $\boldsymbol{M}_e \cdot \ddot{\boldsymbol{q}}(t) + \boldsymbol{K}_e^{\text{tang}} \cdot \boldsymbol{q}(t) = \boldsymbol{f}(t)$ where the elastic stiffness vector $\boldsymbol{k}_e(\boldsymbol{q}, \dot{\boldsymbol{q}}, t)$ is approximated by the tangential stiffness matrix $\boldsymbol{K}_e^{\text{tang}}$ times the deformations. Afterwards, the linearized model of the nonlinear LS-DYNA model is imported into MatMorembs and the linear ansatz functions are calculated by model based projection methods, e.g., Krylov, Gram, modal. Finally, the reduced model consisting of a linear reduced part and a nonlinear part is simulated with LS-DYNA in the online step. To evaluate the performance of the applied approach, the accelerations of the driver calculated with various reduction and parameter settings are compared with the accelerations measured when the original, unreduced nonlinear model is simulated.

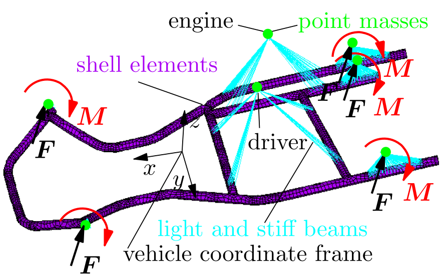

Figure 1: Racing kart used as a surrogate model.

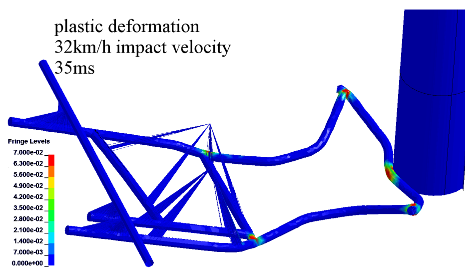

Figure 2: Plastic strain after 35 ms.

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Chebyshev Interpolation for Parametric Option Pricing

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In finance, for calibration, risk assessment and quotation tasks, the same pricing problem has to be computed for a large range of parameter values on a day-to-day or even high-frequency basis. Given the recurrent nature of the pricing problem, we exploit the way prices cohere for different parameters so as to develop fast and accurate approximation methods for option prices.
To that extent, we propose a polynomial interpolation method for Parametric Option Pricing (POP). The core idea of this method is to interpolate the prices in the parameter space with (tensorized) Chebyshev polynomials. For parameters p the option price $Price^p$ is then approximated by

$$Price^p \approx \sum_{j\in J} c_j T_j(p)$$

for so-called Chebyshev polynomials T_j and known, precomputed coefficients c_j. A demonstration of the approximation capability of the method is given by Figure 1.
The procedure naturally splits into two phases. In the precomputational phase the prices are computed for some fixed parameter configurations, the interpolation points. Here, any appropriate pricing method for instance based on Monte Carlo, PDE or Fourier techniques can be chosen, which reveals the universal applicability of the method. As the resulting prices may be conveniently stored, computing the price for arbitrary parameter constellations in the second phase simply amounts to the evaluation of a polynomial whose coefficients are explicitly known. The method thereby provides a significant improvement in computation time for the derivation of option prices compared to alternative pricing methods mentioned above. A thorough analysis of the Chebyshev approximation method and detailed numerical studies are provided in [1].

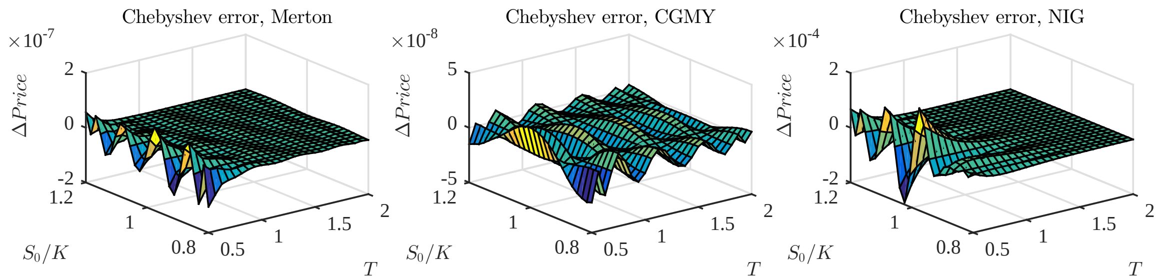

Figure 1: Absolute pricing error for a European call option in different models. The depicted error surfaces are generated by a Chebyshev approximation for the two free variables S_0/K and T. The model parameters have been fixed to reasonable values. The interpolation operator has been set up with a very low number of only $N = 11$ interpolation points in each of the two dimensions.

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Parametric Option Pricing with Fourier Methods

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We propose an interpolation method tailored to parametric option pricing of liquid options in finance. In most of the relevant asset models, prices of liquid options have a representation in terms of the Fourier transform of the distribution of the underlying random variable and of the payoff function. These types of options and methods are used for model calibration, where the same pricing problem needs to be solved for a large set of different parameters. In literature on parametric option pricing the main focus is on Fast Fourier techniques, which typically enables efficient computation of the price for a large set of option's strike and the initial asset value – which is a specific parameter dependence. Additionally, reduced basis for the related parametric Kolmogorov equations, which often are of parabolic type, has been proposed in the literature. In contrast, to benefit from both the recurrent nature of the option pricing problem and the explicitly given Fourier transforms of the option prices, we present a new interpolation method for Fourier prices. For a large variety of option types and models, we provide theoretical error estimates. Our numerical experiments confirm the theoretical findings and show a significant gain in efficiency. Compared to our recent work, [1], where we explore (tenzorized) Chebyshev interpolation for POP, the new interpolation method has two major advantages:

– The error decay can be estimated independently of the dimension of the parameter space.

– The interpolation yields an expansion that is *explicit in the model parameters*.

Our experimental results are illustrated in Figure 1.

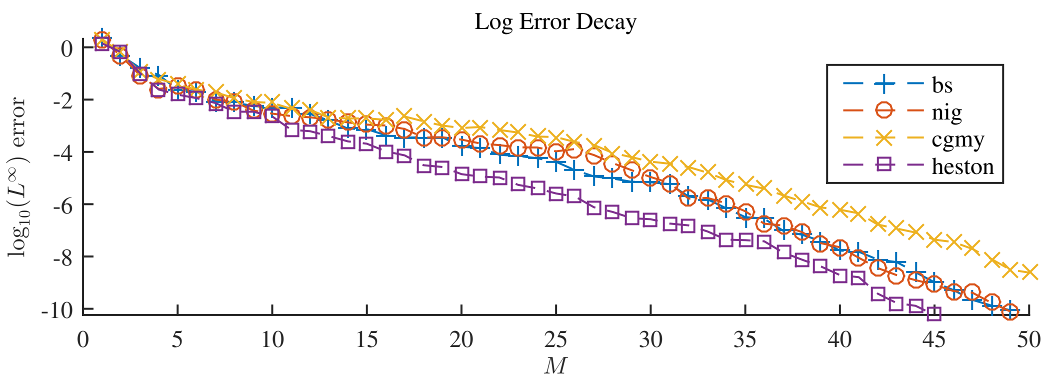

Figure 1: $\log_{10}$ error decay for increasing degree of freedom M during the construction of the interpolation operator on a representative parameter training set for the following models. *bs*: Black & Scholes, *nig*: Normal inverse Gaussian, *cgmy* CGMY also known as KoBoL, *heston*: Heston's model.

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Optimal snapshot location for POD model order reduction in optimal control

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In this work we present the approximation of an optimal control problem for linear parabolic PDEs. The method is based on a model reduction technique using Proper Orthogonal Decomposition (POD-MOR). POD-MOR is a Galerkin approach where the basis functions are obtained upon information contained in time snapshots of the parabolic PDE related to given input data. In the present work we show that it is important to have knowledge about the controlled system at the right time instances. Several works focus their attention on the choice of the snapshots in order to approximate either dynamical systems or optimal control problems by suitable surrogate models [2, 3, 4].
In our work, we address the question of optimal snapshot location by means of an a-posteriori error control approach proposed in [1], where the optimality system is rewritten as a second order in time and a fourth order in space elliptic equation. This equation then is approximated with a space-time finite element approach whose advantage is the possibility to perform a space-time grid adaptivity based on a-posteriori error estimates. In particular, the time adaptivity will turn out relevant to build the optimal grid which should be used to solve the optimal control problem. Here the contribution for the reduced control problem is twofold: we directly obtain snapshots from the optimal control problem related to an approximation of the optimal control and, at the same time, information about the time grid. Finally, we present numerical tests to illustrate our approach and to show the effectiveness of the method in comparison to existing approaches.

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Rank-optimal approximations of higher-order tensors for low-dimensional space-time Galerkin approximations of parameter dependent dynamical systems

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We interpret a dynamical system of type

$$\dot{y} = f(y;\mu), \quad \text{on } (0,T], \quad y(0) = y_0, \qquad \text{as a map} \qquad \mathbb{G}\colon \mu \mapsto y$$

that maps an input or a parameter μ onto the state y which is a function of space $x \in \Omega$ and time $t \in [0,T]$. We assume that the state is observed in a time-space tensor space, i.e. $y(t,x) = \tau(t) \cdot \xi(x)$, with $\tau \in S := L^2(0,T)$ and $\xi \in Y := L^2(\Omega)$. Thus, the state observations can be approximated using the tensor product of some bases $\{\nu_1, \dots, \nu_q\} \subset Y$ and $\{\psi_1, \dots, \psi_s\} \subset S$ of finite dimensional spaces of S and Y. If we also consider the inputs in a finite dimensional space $\mathsf{span}\{\mu_1, \dots, \mu_r\}$, then an approximation of the Input/Output (I/O) map $\mathbb{G}$ is a mapping $\mathbf{G}\colon \mathbb{R}^r \to \mathbb{R}^q \cdot \mathbb{R}^s$, which can be interpreted as a tensor $\mathbf{H} \in \mathbb{R}^{r \times q \times s}$.

Using a higher-order SVD of $\mathbf{H}$, cf. [2], we propose some low-dimensional bases for subspaces of Y and S optimized with respect to measurements in the space and time domain and sampling in the input space. In [1], we have shown that this approach can be seen as a generalization of the well-known proper orthogonal decomposition (POD) method. Basically, instead of using snapshots of the solution trajectory at discrete time instances, time information is sampled by testing against test functions in $L^2(0,T)$. The space dimension Y of the I/O map is readily defined by the expansion of the state in a *Finite Element* basis. The benefits of this approach are discussed and illustrated in [1].

In this contribution, we extend the I/O map based approach to control or parameter dependent setups in two steps. Firstly, we also sample the parameter space which adds to the space and time sampling and, thus, leads to a three dimensional input-to-output tensor. Secondly, we employ the higher-order SVD [2] to define new space and time bases that are optimized with respect to inputs and time or inputs and space, respectively. After a space-time Galerkin projection, the solution of the reduced systems can be obtained as the solution of a small system of algebraic equations.

By means of *Burgers'* equation for varying viscosity parameters, we compare this space-time-parameter Galerkin approach with the standard POD method and discuss it's relation to *Proper Generalized Decomposition* [3] and computational issues such as online-offline decompositions and stability of the approximations with respect to changes in the reduced bases.

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Reduced Basis Approximations for Maxwell's Equations in Dispersive Media

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The simulation of the propagation of an electromagnetic pulse through linear, temporally dispersive media (like water) or systems (like a dielectric waveguide) is a typical problem of electromagnetics.

Consider the Maxwell-Debye model formulated in second-order form in the electric field E

$$\frac{1}{\mu_0}\nabla \times \nabla \times E + \epsilon_0\epsilon_\infty \partial_t^2 E = f - \partial_t^2 P,$$
$$\partial_t P + \frac{1}{\tau}P = \frac{\epsilon_0(\epsilon_s - \epsilon_\infty)}{\tau}E,$$

with polarization P, relaxation time τ, relative permittivity at low-frequency limit ϵ_s and relative permittivity at high-frequency limit ϵ_∞, and a broadband input source f, which is modeled as a Gaussian pulse, [2]. The equations are discretized with Nédélec finite elements of first order over a 2D unit square. Dirichlet zero boundary conditions (i.e. PEC, perfectly electric conducting) are imposed on all boundaries.

The model is parametrized by τ and $\Delta_\epsilon = \epsilon_s - \epsilon_\infty$, defining the 2-dimensional parameter domain D. Using a POD-greedy sampling driven by an error indicator, we seek to generate a reduced model which accurately captures the dynamics in the parameter domain D, [3]. Typically, the reduced basis model reduction reduces the model order by a factor of more than 100, while maintaining an approximation error of less than 1%.

Since the POD step in each iteration of the greedy sampling is computationally expensive, we test and compare an interpolatory decomposition, cf. [1], in place of the POD. First numerical results indicate that the interpolative decomposition gives at least comparable or even better results in computational time and approximation quality to the POD for this model.

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The Versatile Cross Gramian

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The cross gramian matrix [1] is a tool for model order reduction and system identification of linear control systems with special focus on symmetric systems. We illustrate the cross gramian's relation to the controllability and observability gramian, the Hankel operator and balanced truncation, which can be used for model reduction. For system identification, the cross gramian can be utilized for sensitivity analysis [7] and decentralized control [6]. Furthermore, the cross gramian can boast with relations to the associated transfer function such as the Cauchy-index and system gain.

Beyond linear systems the cross gramian has been extended to nonlinear systems by a generalization to gradient systems [5]. Alternatively, an empirical cross gramian [7, 2] has been developed, which can also be used for parametrized systems [3]. Alongside, an extension of the cross gramian for parameter identification and thus combined state and parameter reduction [2] is available. More recently the cross gramian has been extended to non-symmetric systems [4] with a suprising result.

We give a tour through the results and applications of this system gramian uniting controllability and observability as well as recent developments and the parallel empirical computation.

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Model Order Reduction for Pattern Formation in FitzHugh-Nagumo Equation

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We investigate the formation of Turing patterns in excitable media described by the diffusive FitzHugh-Nagumo (FHN) equation. Different set of parameters satisfying Turing condition lead to labyrinth or spot like patterns. The FHN equation consisting of one activator and one inhibitor is discretized in space by the discontinuous Galerkin (DG) method [2] and by the Average Vector Field (AVF) method in time [3]. Applying the POD-DEIM to the full order model (FOM) we show that using few POD and DEIM modes, the dynamical behavior of the FHN equation and Turing patterns can be detected accurately. Due to the local nature of the DG discretization, the POD-DEIM requires less number of connected nodes for the nonlinear part of the FHN compared with the continuous finite element POD-DEIM [1]. This leads to a significant reduction of the computation cost for DG POD-DEIM in the reduced order mode (ROM).

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Model Order Reduction via a Balanced Truncation-Interpolation Approach for Gas Networks

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This work deals with the Gramian based model order reduction (MOR) [1, 2] of nonlinear, parametric systems of partial differential-algebraic equations which arise from transient gas network modeling. We present an approach based on a linearization around parametrized static states, linear time-invariant reduction and interpolation. The approximation quality is crucially determined by the choice of the representative network states and the interpolation strategy, but also the underlying spatial discretization and the index of the resulting temporal differential algebraic system play a non-negligible role.

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An algebraic approach to deal with nonlinearities in reduced basis methods: the matrix discrete empirical interpolation

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The low cost associated with the solution of reduced order models (ROMs) has in turn allowed their use to accelerate real-time analysis, PDE-constrained optimization [2, 4, 5] and uncertainty quantification [3] problems. In all these cases a suitable offline/online stratagem becomes mandatory to gain a strong computational speedup. However, the complex parametric dependence of the discretized PDE operators, as well as the nonlinear and (possibly, unsteady) nature of the equation, have a major impact on the computational efficiency.

In this talk we show how to apply a Matrix version of the so-called Discrete Empirical Interpolation (MDEIM) for the efficient reduction of nonlinear and nonaffine systems arising from the discretization of parametrized PDEs [1]. Dealing with affinely parametrized operators is crucial in order to enhance the online solution of ROMs such as the reduced basis method [6]. However, in many cases such an affine decomposition is not readily available, and must be recovered through (often) intrusive procedures, such as the empirical interpolation method (EIM) and its discrete variant DEIM. The MDEIM approach presented in this talk allows instead to deal with nonlinearities, as well as with non affinities arising from complex physical and geometrical parametrizations, in a non-intrusive, efficient and purely algebraic way. We propose different strategies to combine MDEIM with a state approximation resulting either from a greedy algorithm or Proper Orthogonal Decomposition. The capability of MDEIM to generate accurate and efficient ROMs is demonstrated on the solution of some computationally-intensive classes of problems occurring in engineering contexts, namely parametrized coupled problems, PDE-constrained optimization and uncertainty quantification problems.

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Model order reduction of dynamic skeletal muscle models

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Forward simulations of three-dimensional continuum-mechanical skeletal muscle models are a complex and computationally expensive problem. Considering a fully dynamic modelling framework based on the theory of finite elasticty is challenging as the muscles' mechanical behaviour requires to consider a highly nonlinear, viscoelastic and incompressible material behaviour. The governing equations yield a nonlinear second-order differential algebraic equation (DAE), which represents a challenge to model order reduction techniques.
In detail, the governing equations to be solved in the solution domain are the balance of momentum subject to the incompressibility constraint, i.e.

$$\rho_0(X)\frac{\partial \boldsymbol{V}}{\partial t}(X,t) = \nabla \boldsymbol{P}(X,t) + \boldsymbol{B}(X,t)\,, \qquad \text{s.t. } J(X,t) - 1 = 0\,, \tag{1}$$

and a nonlinear constitutive equation of the form

$$\boldsymbol{P}(X,t) = \boldsymbol{P}^{iso}(X,t) + \boldsymbol{P}^{aniso}(X,t) + \boldsymbol{P}^{active}(X,t) + \boldsymbol{P}^{viscous}(X,t) + p(X,t)\boldsymbol{F}^{-T}(X,t)\,. \tag{2}$$

Herein, ρ_0 is the muscle density, $\boldsymbol{V}$ is the velocity field, $\boldsymbol{P}$ is the first Piola-Kirchhoff stress tensor, $\boldsymbol{B}$ are the body forces, $J := \det\boldsymbol{F}$ is the Jacobian, $\boldsymbol{F}$ is the deformation gradient and p is the pressure. Discretising the governing equations using the finite element method, one obtains the following system

$$\begin{aligned} \boldsymbol{M}\boldsymbol{u}''(t) + \boldsymbol{D}\boldsymbol{u}'(t) + \boldsymbol{K}(\boldsymbol{u}(t),\boldsymbol{w}(t)) &= \boldsymbol{0}\,, \\ \text{s.t. } \boldsymbol{g}(\boldsymbol{u}(t)) &= \boldsymbol{0}\,, \end{aligned} \tag{3}$$

where $\boldsymbol{u}$ is the vector of position coefficients, $\boldsymbol{w}$ contains the pressure coefficients, $\boldsymbol{M}$, $\boldsymbol{D}$, $\boldsymbol{K}$ are the mass, viscous damping and generalised stiffness matrix, respectively, and $\boldsymbol{g}$ is the operator associated with the incompressibility constraint.
For this complex problem, a simple transformation of the system into a first-order system in order to obtain the general form of a parametric nonlinear dynamical sytem is not sufficient. Subsequent reduction using existing MOR methods, such as POD combined with DEIM, see e.g. [3, 1], did not lead to a stable reduced model. Therefore, other reduction methods and solution schemes need to be investigated and modified. Conceivably, one could directly project and solve the second order system by e.g. the Newmark method, see e.g. [2]. Further, to properly treat the constraint, the application of theories for Hessenberg index 2 DAEs and ODEs on constraint manifolds are promising. Here, we will show current results and discuss open questions.

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Time-Galerkin integrators for the dynamic simulation of local reduced order models.

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Over the past decades, model order reduction (MOR) techniques have aided in the efficient description of many highly computationally demanding models. One of the basic requirements for most developed approaches, is that the response of a model can be well captured by a smaller (fewer variables, less nonlinear equation evaluations, ...) model. However, some systems exhibit such strong nonlinearities or are excited over such a wide range, that one such model cannot be constructed. In these cases regular MOR methods cannot create a sufficiently small model to obtain the required size reduction.

Local reduced order models (LROM) can offer a solution when traditional MOR techniques are not sufficient [1, 2]. In this approach a series of LROMs are constructed where each one provides good accuracy only over a subregion of the system behavior. During the online evaluation the most suitable LROM for a certain subregion is selected and evaluated. However, in a dynamic context, many jumps from one LROM to the other can occur. These changes are generally nontrivial and have to be handled properly in order to avoid parasitic dynamic effects. This is especially problematic for systems which naturally exhibit a strongly energy conserving behavior, like (nonlinear) elastodynamic systems.

In this work we propose a time-Galerkin integrator which enables a highly flexible choice in how the LROMs of (nonlinear) elastodynamic problems are handled while providing a guarantee of energy conserving behavior, even over sudden model changes. Due to the common time-points between two timeslabs, the time-Galerkin is perfectly suitable to perform model changes online. The proposed approach is demonstrated on both linear and nonlinear examples. Through these examples, good convergence and stability are demonstrated.

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Model Order Reduction of Nonlinear Magnetodynamics with Manifold Interpolation

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In the model order reduction community, linear systems have been widely studied and reduced thanks to various techniques [6, 7]. Proper Orthogonal Decomposition (POD) in particular has been very successful, and has recently been gaining popularity in computational electromagnetics [4]. However, the efficiency of POD degrades considerably for nonlinear problems, in particular for nonlinear magnetodynamic models—necessary for designing most of today's electrical machines and drives.

We propose to investigate an algorithm which first applies the POD to construct reduced order models of nonlinear magnetodynamic problems for discrete sets of values of the input parameters. Then, for a new set of values of the input parameters, a nonlinear interpolation on manifolds is performed to determine the reduced basis. This interpolation method is based on the theory previously studied for aerodynamic problems [1, 2].

The goal here is not to speed up single shot calculations as in [5], but to be able to determine efficiently reduced models for nonlinear problems based on previous offline computations. As a simple application, we apply the procedure to a nonlinear inductor-core system, solved using a classical finite element method [3]. In order to gauge the interest of manifold interpolation we compare the proposed approach to the direct use of a precomputed reduced basis, as well as with the use of standard Lagrange interpolation.

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Computational reduction strategies for bifurcations and stability analysis in fluid-dynamics: applications to Coanda effect in cardiac flows

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We focus on reduced order modelling for nonlinear parametrized Partial Differential Equations, frequently used in the mathematical modelling of physical systems.

A common issue in this kind of problems is the possible loss of uniqueness of the solution as the parameters are varied and a singular point is encountered [3]. In the present work, the numerical detection of singular points is performed online through a Reduced Basis Method, coupled with a Spectral Element Method [2] for the numerically intensive offline computations.

Numerical results for laminar fluid mechanics problems will be presented, where pitchfork, hysteresis, and Hopf bifurcation points [1] are detected by an inexpensive reduced model.

Some of the presented 2D and 3D flow results [5] deal with the study of instabilities in a simplified model of a mitral regurgitant flow [7] in order to understand the onset of the Coanda effect. The first results are in good agreement with the reference [6, 4].

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Reduced Basis Approximation of Coupled Problems

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The coupling of two variational problems has many farreaching applications in several fields, such as fluid flow through porous media [1] or chemical reactions in a catalyst or a fuel cell. We study a coupled (time-dependent) convection-dominated problem on a domain Ω_1 and a diffusion-reaction problem on a domain Ω_2, whereby Ω_1, Ω_2 are only connected through an interface. On this interface, the interchange and interference between these two processes is modeled and therefore meaningful boundary conditions have to be established. Different inflow conditions and reaction coefficients serve as parameters.

Firstly, we can deduce a saddle-point formulation. However, existence and uniqueness of a solution is not straightforward to prove. To overcome this issue, we will present an alternative approach partly following [2]. Additionally, we apply the Reduced Basis Method on our coupled problem aiming a reduction of computational time. To achieve this goal, we use an offline/online-decomposition and efficient bounds for the arising error. Our approach is a space-time variational formulation of the coupled problem and we utilize amongst others the error estimator developed in [3]. In this context, we want to discuss the difficulties originating from the interferences on the coupling boundary. We particularly consider the effect on the numerical determination of accurate bounds for the inf-sup and continuity constants, which are crucial for the fast computation of efficient error estimators.

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Reduced Order Models for Distributed Adaptive Monitoring of Atmospheric Dispersion Processes

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Atmospheric dispersion of hazardous materials due to chemical leaks can highly affect human health and well-being. For this reason, online state and parameter estimation of these processes is an important step for disaster response to enable the assessment of future impacts. The estimation procedure relies on a combination of the forecasts of a process PDE-model and on measurements obtained by multiple mobile sensor platforms, which are adaptively guided to locations where additional measurements are most useful. The latter challenge can be solved by a cooperative vehicle controller maximizing the quality of the estimates based on the current error covariance matrix [1, 2].

The described approach can be made more flexible and less prone to error if the required calculations (model forecast, estimation procedure, vehicle control) are performed locally on-board of the sensor vehicles instead of using a central supercomputer. While the on-board computing power is limited and results have to be obtained in real-time, complex PDE-models are required to describe the dynamics and to compute accurate forecasts. This highly motivates the use of model order reduction in this context and demonstrates at the same time that the described problem scenario is a paradigmatic application area for reduced order models.

A reduced joint state parameter estimation approach is developed for the advection-diffusion equation. The initial condition as well as possible source functions, shapes and locations are unknown. However, it is assumed that the initial condition can be approximated by several radial basis functions with height and shape parameter to be determined. Furthermore, source effects can be represented by the convolution of the same radial basis functions with their height. In the offline phase, multiple simulations with the different radial basis function as initial conditions are performed and snapshots are taken. With the aid of Proper Orthogonal Decomposition, the reduced order model is constructed out of the snapshot matrix and reduced model forecasts are performed locally to repeatedly and jointly estimate process state and parameters of the radial basis functions with the Kalman Filter.

As a first step, a basic two-dimensional test-case, in which the true state is simulated along with the estimation, is set up and promising results are obtained.

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Isogeometric analysis based reduced order modelling for incompressible viscous flows in parametrized shapes: applications to underwater shape design

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We provide a new concept "tool" from CAD-like geometry to final simulation with the aim of dealing with parametrized shapes managed by efficient free-form deformation techniques into an isogeometric analysis setting. This tool is totally integrated into model order reduction techniques, based on POD, developed for stable incompressible viscous flows (velocity and pressure) in parametrized shapes. This computational environment has been created in the framework of the project UBE – Underwater Blue Efficiency – for the optimization of the shapes of immersed parts of motor yachts, including exhausting flows devices. The study is benefitting of several properties of reduced order modeling approaches such as offline-online calculations for parametric design, as well as sinergies with isogeometric analysis properties. Convergence, stability and consistency properties are verified as well in test case and then applicative results are introduced.

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Physical and constraint DOF reduction of redundant multibody systems using the Proper Orthogonal Decomposition

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Multibody simulation software has entered the industrial user's market several years ago and is nowadays part of a CAE engineer's everyday life. Due to usability and time advantages in the modeling process, such a simulation software is typically based on modeling strategies using a redundant set of coordinates. Although such a redundant set simplifies the modeling process, modeling without a minimal set of coordinates forces the multibody dynamics into differential algebraic equations (DAEs). Unnecessary degrees of freedom (DOFs) are introduced easily through kinematic relations and especially when talking about planar problems, modeled in a 3D space, the number of unnecessary DOFs increases rapidly. Further, unnecessary constraint equations which address such kinematic chains or unstressed DOFs are typically introduced. The d-index three DAE systems of second order considered in this contribution are modeled in the freeware multibody simulation software*FreeDyn*[1] using Euler parameters and solved in the open source software *Scilab* [3] applying a HHT solver algorithm. Due to the characteristic nonlinear system matrices, model reduction is not possible a-priori but needs at least one full system forward simulation to rest on. Inspired by repetitive simulations necessary for e.g. parameter identification [2], we focus on reducing such a highly redundant set of coordinates close to a minimal set of coordinates representation by eliminating both, physical and constraint DOFs. This contribution first gives a brief introduction to the underlying DAE system. In the second part model reduction of redundant physical DOFs using a modified linear Galerkin projection based on a velocity level Proper Orthogonal Decomposition (POD) is proposed. The subspace found by POD is further applied to identify unneeded constraint equations related to a.) - constraints addressing unneeded physical DOFs and b.) constraints which are orthogonal to the found subspace and, hence, satisfied by the projection matrix. The method is illustrated using a simple analytical example and further applied to a rigid V8 crankdrive consisting of 119 DOFs as well as 118 constraint equations. In the conclusion the proposed methods are discussed and an outlook to parameter identification, using a reduced adjoint system, and therewith arising special needs to the underlying subspace is given.

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Stochastic optimal control problem for Markov jump process: asymptotic analysis and algorithms

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We consider certain stochastic optimal control problems for Markov jump process in the large number regime. For both open loop and feedback control problem, based on Kurtz's limiting theorems, we prove the convergence of the value functions for the optimal control problem of Markov jump process as the "species" number goes to infinity. In the case of finite time horizon, A hybrid control policy is proposed to overcome the difficulties due to the large state space as the "species" number increases. Numerical examples are studied to demonstrate the analysis and algorithms.

This is a joint work [1] with Prof. Carsten Hartmann and Max von Kleist at Freie Universität Berlin.

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Websites:
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Cover: Trieste, Piazza Unità d'Italia from Molo Audace.

MoRePaS 2015

Model Reduction of Parametrized Systems III

October 13–16, 2015

SISSA, International School for Advanced Studies, Trieste, Italy

EU-MORNET
European Network for Model Reduction