Offline Error Bounds for the Reduced Basis Method

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Error quantification is of vital importance in reduced-basis modeling. A reduced model is only useful if it matches the high-dimensional model to within some tolerance. Even before it is time to evaluate the error in a simulation, the reduced-basis method uses a posteriori error bounds to build the model in an intelligent manner.

Reduced-basis models are most often generated iteratively using the greedy algorithm [3]. Each time the basis needs to be expanded, the low-dimensional solution $u_N(\mu)$ and the a posteriori error bound $\Delta_N(\mu)$ are calculated for all parameters μ in some Ξ, which is a finite subset of the parameter domain $\mathcal{D}$. The basis is then enhanced using the high-dimensional solution $u(\mu_{N+1})$ associated with the parameter defined by

$$\mu_{N+1} = \arg\max_{\mu \in \Xi} \frac{\Delta_N(\mu)}{\|u_N(\mu)\|}.$$

The greedy algorithm guarantees that the relative error is small for all $\mu \in \Xi$. In practice, it is expected that if Ξ is sufficently large and properly distributed, then the relative error will also be small everywhere in D. For $\mu \in \mathcal{D} \setminus \Xi$ the only way to verify that a tolerance has been met is to use a posteriori error bounds. The difficulty is in dealing with error bounds that do not meet the tolerance: either precomputed basis elements must be added to the basis [2] or new basis elements need to be computed.

In real-time problems, which have been mentioned by many authors as possible applications of reduced-basis modeling [1, 2], solutions are needed quickly and there is often no time to rerun simulations with additional basis elements.

In this talk we propose a solution to this problem. The idea is to prove that the error will be sufficiently small for all $\mu \in \mathcal{D}$. That cannot be achieved with traditional a posteriori error bounds, but we present new error bounds that make it possible. We also demonstrate these ideas with numerical results for some example problems.

References

[1] C. Prud'homme, D. V. Rovas, K. Veroy, L. Machiels, Y. Maday, A. T. Patera, and G. Turinici. Reliable real-time solution of parametrized partial differential equations: Reduced-basis output bound methods. *J. Fluid. Eng.*, 124(1):70–80, 2002.

[2] G. Rozza, D. Huynh, and A. Patera. Reduced basis approximation and a posteriori error estimation for affinely parametrized elliptic coercive partial differential equations. *Archives of Computational Methods in Engineering*, 15(3):229–275, 2008.

[3] K. Veroy, C. Prud'homme, D. V. Rovas, and A. T. Patera. A posteriori error bounds for reduced-basis approximation of parametrized noncoercive and nonlinear elliptic partial differential equations. In *Proceedings of the 16th AIAA Computational Fluid Dynamics Conference*, 2003. AIAA Paper 2003-3847.