Using sensitivity analysis in the framework of proper orthogonal decomposition with application to cake filtration

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We are considering a problem resulting from the mathematical modeling of an industrial process called cake filtration [3]. It is described by a moving boundary problem involving a system of convection-diffusion-reaction equations and kinetic equations. In our case the moving boundary is evolving in time by a constant rate. The model exhibits various parameters, which influence the solution. The aim of this study is to derive a reduced order model for a reference configuration and to adjust this model under parameter change. Therefore we have to get rid of the time dependence of our computation domain. To this end we apply a pullback operator. This results in a transformed formulation involving a system of PDEs with time-dependent coefficients. As model reduction technique we choose proper orthogonal decomposition (POD). It is well-known that the application of POD involves the solution of an eigenvalue/eigenvector problem (cf. e.g. [5, 6]) using a snapshot matrix. Since we want to account for the parameter sensitivity of our reduced model, a sensitivity analysis is performed. First of all the sensitivity of the solution, i.e. of the snapshot matrix, is needed, in addition also the sensitivity of the eigenvalue/eigenvector problem for computing the reduced basis [2, 4]. To derive an improved approximation of the POD basis we combine POD and sensitivity analysis. Therefore we assume that for at least one reference parameter configuration μ_0 the POD basis and the corresponding sensitivities are available. Then various methods can be used to approximate the POD for an arbitrary parameter value μ, compare [1, 2, 4]. To solve the problem efficiently an offline/online decomposition is used, due to the time dependence of the coefficients.
It is shown that the quality of the methods strongly depend on the underlying problem. In our case the time horizon of the process has a large influence.

References

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