MoRePaS 2015

Sparse-Grid, Reduced-Basis Bayesian Inversion

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September 20, 2015

We report on reduced basis accelerated Bayesian inversion methods for affine-parametric, linear operator equations, from joint work with Peng Chen [2, 1]. We allow a general class of non-affine parametric, nonlinear operator equations. We present an analysis of sparsity of parametric forward solution maps and of Bayesian inversion in to the fully discrete setting. The analysis includes Petrov-Galerkin high-fidelity (HiFi) discretization of the forward maps. We develop adaptive, stochastic collocation based reduction methods for the efficient computation of reduced bases on the parametric solution manifold. The nonlinearity with respect to the distributed, uncertain parameters and the unknown solution is collocated; specifically, by the so-called Generalized Empiricial Interpolation Method (GEIM). For the corresponding Bayesian inversion problems, computational efficiency is enhanced in two ways: first, expectations with respect to the posterior are computed by adaptive quadratures with dimension-independent convergence rates, following [4, 3]. Our analysis accounts for the impact of the PG discretization in the forward maps on the expectation of the Quantities of Interest (QoI). Second, we perform the Bayesian estimation only with respect to a parsimonious, reduced basis approximation of the posterior density. In [5], under general conditions on the forward map, the infinite-dimensional parametric, deterministic Bayesian posterior was shown to admit N-term approximations which converge at rates which depend only on the sparsity of the parametric forward map. We present dimension-adaptive collocation algorithms to build finite-dimensional parametric surrogates which realize these rates. In several numerical experiments, the proposed algorithms exhibit parameter dimension-independent convergence rates which equal, at least, the currently known rate estimates for N-term approximation. We propose to accelerate Bayesian estimation by offline computation of reduced basis surrogates of the Bayesian posterior density. The parsimonious surrogates can be employed for online data assimilation and for Bayesian estimation. They also open a perspective for optimal experimental design.

Supported in part by Swiss National Science Foundation (SNF) and by the European Research Council (ERC) under AdG 247277.

References

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