

Computational reduction strategies for bifurcations and stability analysis in fluid-dynamics: applications to Coanda effect in cardiac flows

G. Pitton¹, A. Quaini², and G. Rozza¹

¹mathLab, Mathematics Area, SISSA, International School for Advanced Studies, Trieste, Italy

²Department of Mathematics, University of Houston, Houston, TX, US

We focus on reduced order modelling for nonlinear parametrized Partial Differential Equations, frequently used in the mathematical modelling of physical systems.

A common issue in this kind of problems is the possible loss of uniqueness of the solution as the parameters are varied and a singular point is encountered [3]. In the present work, the numerical detection of singular points is performed online through a Reduced Basis Method, coupled with a Spectral Element Method [2] for the numerically intensive offline computations.

Numerical results for laminar fluid mechanics problems will be presented, where pitchfork, hysteresis, and Hopf bifurcation points [1] are detected by an inexpensive reduced model.

Some of the presented 2D and 3D flow results [5] deal with the study of instabilities in a simplified model of a mitral regurgitant flow [7] in order to understand the onset of the Coanda effect. The first results are in good agreement with the reference [6, 4].

References

- [1] K. Cliffe, A. Spence, and S. Tavener. The numerical analysis of bifurcation problems with application to fluid mechanics. *Acta Numerica*, 9:39–131, 2000.
- [2] M. Deville, P. Fischer, and E. Mund. *High-Order Methods for Incompressible Fluid Flow*. Cambridge Monographs on Applied and Computational Mathematics. Cambridge University Press, Cambridge, 2002.
- [3] H. Dijkstra, F. Wubs, A. Cliffe, E. Doedel, I. Dragomirescu, B. Eckhardt, A. Gelfgat, A. Hazel, V. Lucarini, A. Salinger, E. Phipps, J. Sanchez-Umbria, H. Schuttelaars, L. Tuckerman, and U. Thiele. Numerical bifurcation methods and their application to fluid dynamics: Analysis beyond simulation. *Communications in Computational Physics*, 15(1):1–45, 2014.
- [4] D. Drikakis. Bifurcation phenomena in incompressible sudden expansion flows. *Physics of Fluids*, (9):76–87, 1997.
- [5] G. Pitton, A. Quaini, and G. Rozza. Computational reduction strategies for bifurcations and stability analysis in fluid-dynamics: applications to coanda effect in cardiac flows. *in preparation*, 2015.
- [6] M. Oliveira, L. Rodd, G. McKinley, and M. Alves. Simulations of extensional flow in microrheometric devices. *Microfluid Nanofluid*, (5):809–826, 2008.
- [7] A. Quaini, R. Glowinski, and S. Canic. Symmetry breaking and Hopf bifurcation for incompressible viscous flow in a contraction-expansion channel. *submitted*, University of Houston, Dept. of Math. Technical Report, 2014, 2014.