Optimal snapshot location for POD model order reduction in optimal control

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In this work we present the approximation of an optimal control problem for linear parabolic PDEs. The method is based on a model reduction technique using Proper Orthogonal Decomposition (POD-MOR). POD-MOR is a Galerkin approach where the basis functions are obtained upon information contained in time snapshots of the parabolic PDE related to given input data. In the present work we show that it is important to have knowledge about the controlled system at the right time instances. Several works focus their attention on the choice of the snapshots in order to approximate either dynamical systems or optimal control problems by suitable surrogate models [2, 3, 4].
In our work, we address the question of optimal snapshot location by means of an a-posteriori error control approach proposed in [1], where the optimality system is rewritten as a second order in time and a fourth order in space elliptic equation. This equation then is approximated with a space-time finite element approach whose advantage is the possibility to perform a space-time grid adaptivity based on a-posteriori error estimates. In particular, the time adaptivity will turn out relevant to build the optimal grid which should be used to solve the optimal control problem. Here the contribution for the reduced control problem is twofold: we directly obtain snapshots from the optimal control problem related to an approximation of the optimal control and, at the same time, information about the time grid. Finally, we present numerical tests to illustrate our approach and to show the effectiveness of the method in comparison to existing approaches.

References

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