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Reduced Basis Approximation for Parametric Large Scale Algebraic Riccati Equations

A. Schmidt[1] and B. Haasdonk[1]

[1]Institute for Applied Analysis and Numerical Simulation, University of Stuttgart, Stuttgart, Germany

The Algebraic Riccati Equation (ARE) is a quadratic matrix valued equation with a variety of important applications in the field of dynamical systems, such as optimal feedback control, filtering and state estimation, see e.g. [1]. As the modelling of many complex technical phenomena is usually performed by means of partial differential equations, which after semidiscretization in space yield very large time dependent ODE systems, the question arises how the resulting AREs can be solved efficiently. Although many very efficient solution algorithms exist, real-time applications or many-query scenarios in a parametric case can lead to infeasible calculation times. For that reason we are interested in extending the reduced basis (RB) method (see e.g. [2]) for obtaining approximate solutions to the ARE rapidly and with reliable error statements. RB methods aim at splitting the calculation in two parts: A potentially expensive offline phase, where a suitable problem adapted basis is constructed, followed by a projection of the original problem onto the low dimensional subspace. This expensive procedure then pays off in the online phase, where approximate solutions can be calculated rapidly for varying parameters, along with error bounds that quantify the quality of the approximation.

In order to apply the RB method for parametric AREs, we formulate a new procedure called "Low Rank Factor Greedy" algorithm, which exploits a low rank structure in the solution matrices to build a suitable low dimensional basis, cf. [4, 3]. We show how a new residual based error estimator can be derived and how a computable version can be formulated by stating upper bounds for the constants occuring in the error estimator. Furthermore, parameter separability of the data matrices will allow a calculation completely independent of the original (high) dimension.

The solution to the ARE directly determines a so called gain matrix which solves an optimal feedback control problem for linear systems (linear quadratic regulator, LQR). As this is certainly among the most interesting applications of the ARE, we state error bounds on the suboptimality imposed by using the approximation from the proposed RB method as a surrogate for the true (expensive) solution of the ARE, including bounds on the cost functional, the state and the output of the control systems.

References

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