

PMOR for Nanoelectronic Coupled Problems

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Designs in nanoelectronics often lead to problems that are very large and that include strong feedback couplings. Industry demands to include variability to guarantee quality and yield. These parameter variations maybe due to material properties, system configurations, etc. It also requests to incorporate higher abstraction levels to allow for system simulation in order to shorten design cycles, while preserving accuracy. The transient circuit modelling and space discretized full Maxwell equations yield parameterized differential-algebraic systems (DAEs). In this work, we consider those DAEs that are quadratic in state space $\mathbf{x} = \mathbf{x}(\mu, t)$ and parameterized by a vector $\mu \in \mathbb{R}^d$,

$$\begin{aligned} \mathbf{E}(\mu)\mathbf{x}' &= \mathbf{A}(\mu)\mathbf{x}(\mu) + \mathbf{x}^T\mathbf{F}(\mu)\mathbf{x} + \mathbf{B}(\mu)\mathbf{u}(t), \\ \mathbf{y} &= \mathbf{C}(\mu)\mathbf{x} + \mathbf{D}(\mu)\mathbf{u}(t), \end{aligned} \quad (1)$$

where d is the number of parameters, $\mathbf{x} \in \mathbb{R}^n$ is the state vector, the matrix $\mathbf{E}(\mu) \in \mathbb{R}^{n \times n}$ is singular for every parameter μ , $\mathbf{A}(\mu) \in \mathbb{R}^{n \times n}$, $\mathbf{B}(\mu) \in \mathbb{R}^{n \times m}$, $\mathbf{C}(\mu) \in \mathbb{R}^{\ell \times n}$, $\mathbf{D}(\mu) \in \mathbb{R}^{\ell \times m}$. The tensor $\mathbf{F}(\mu) \in \mathbb{R}^{n \times n \times n}$ is a 3-D array of n matrices $\mathbf{F}_i(\mu) \in \mathbb{R}^{n \times n}$, $i = 1, \dots, n$, and $\mathbf{x}^T\mathbf{F}(\mu)\mathbf{x} := [\mathbf{x}^T\mathbf{F}_1(\mu)\mathbf{x}, \dots, \mathbf{x}^T\mathbf{F}_n(\mu)\mathbf{x}]^T \in \mathbb{R}^n$. We assume that the matrices $(\mathbf{E}(\mu), \mathbf{A}(\mu), \mathbf{B}(\mu), \mathbf{C}(\mu), \mathbf{D}(\mu))$ and the tensor $\mathbf{F}(\mu)$ can be written as an affine parameter dependence, that is $\mathbf{M}(\mu) = \mathbf{M}_0 + \sum_{i=1}^m f_i(\mu)\mathbf{M}_i$, where the scalar functions f_i determine the parametric dependency, which can be nonlinear function of μ , and \mathbf{M}_i can either be a constant matrix or a constant tensor. $\mathbf{u} = \mathbf{u}(t) \in \mathbb{R}^m$ and $\mathbf{y} = \mathbf{y}(t, \mu) \in \mathbb{R}^\ell$ are the inputs (excitations) and the desired outputs (observations), respectively. We note that system (1) includes electro-thermal couplings. The scientific challenges are to develop efficient and robust techniques for fast simulation of strongly coupled systems, that exploit the different dynamics of subsystems, and that can deal with signals that differ strongly in the frequency range and parameter variations. This calls for the application of parameterized model order reduction (PMOR) techniques. However, there exist no such PMOR techniques for quadratic coupled DAEs. In [1], a PMOR method for Electro-Thermal Package Models is proposed. This PMOR method involves first decoupling the parameterized quadratic DAE (1) into quadratic differential (thermal part) and algebraic (electrical) parts. Then the standard PMOR techniques such as PMOR based on implicit moment-matching [2] can be used to reduce both parts which leads to reliable and accurate PROM. We intend to extend these ideas to other coupled problems from nanoelectronics such as Power-MOS devices.

References

- [1] N. Banagaaya, L. Feng, P. Meuris, W. Schoenmaker, and P. Benner. Model order reduction of an electro-thermal package model. In *8th Vienna International Conference on Mathematical Modelling (MathMod Vienna)*, Feb 17-20, volume 8, pages 934–935. Elsevier, 2015.
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