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MOR-based Uncertainty Quantification in Transcranial Magnetic Stimulation

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Electromagnetic computations rely on the perfect knowledge of material parameters. However, for a wide range of examples in electrical and biomedical engineering some uncertainty should be associated in that knowledge in the modeling process. In order to quantify the uncertainty of the output quantities of interest coming from the lack of knowledge of the input material parameters, the spectral stochastic finite element method based on Polynomial Chaos Expansion (PCE) can be applied. This method, in both the intrusive and non-intrusive forms [2], allows to dramatically reduce computational time with respect to Monte Carlo (MC) methods. Nevertheless, in many situations, computational complexity can still be prohibitively large. This is the case of Transcranial Magnetic Stimulation (TMS), a non-invasive technique to stimulate cortical regions of the human brain by the principle of electromagnetic induction. In TMS there is an essential need for more effective techniques of uncertainty quantification due to the increasing model complexity [1]. For such problems, a novel approach based on Model Order Reduction (MOR) is proposed here.

The electromagnetic problem at hand is simplified due to the low electrical conductivities not exceeding 10 S/m and moderate excitation frequencies which are in the range of $2-3$ kHz so that the secondary magnetic field from the induced eddy currents are neglected. We use a realistic head model which contains five different tissues, namely scalp, skull, cerebrospinal fluid (CSF), grey matter (GM) and white matter (WM). The electrical conductivities of scalp and skull are modeled as deterministic, while the conductivities of CSF, GM, and WM, which show a wide spread across individuals and measurements, are modeled as uniform distributed random variables.

The starting point of the new MOR-based approach is a non-intrusive PCE method, in which the PCEs of variables are estimated from the solutions of the deterministic problems for all values of the random tissue parameters belonging to a sparse grid. The main idea of the proposed algorithm is that of reducing the number of solutions of such deterministic problems by constructing a parametric reduced order model, which is used to approximate the solution of the deterministic problems. Such parametric reduced order model is tailored to approximating with chosen accuracy the deterministic problems for the values of the random tissue parameters in the chosen sparse grid. The parametric reduced order model is generated in an efficient way by solving a reduced number of deterministic problems with respect to the non-intrusive PCE approach. The computational cost to construct the parametric reduced order model is optimized by exploiting the relatedness among the solutions to the deterministic problems for different values of the random material parameters.

References

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