

The Versatile Cross Gramian

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The cross gramian matrix [1] is a tool for model order reduction and system identification of linear control systems with special focus on symmetric systems. We illustrate the cross gramian's relation to the controllability and observability gramian, the Hankel operator and balanced truncation, which can be used for model reduction. For system identification, the cross gramian can be utilized for sensitivity analysis [7] and decentralized control [6]. Furthermore, the cross gramian can boast with relations to the associated transfer function such as the Cauchy-index and system gain.

Beyond linear systems the cross gramian has been extended to nonlinear systems by a generalization to gradient systems [5]. Alternatively, an empirical cross gramian [7, 2] has been developed, which can also be used for parametrized systems [3]. Alongside, an extension of the cross gramian for parameter identification and thus combined state and parameter reduction [2] is available. More recently the cross gramian has been extended to non-symmetric systems [4] with a surprising result.

We give a tour through the results and applications of this system gramian uniting controllability and observability as well as recent developments and the parallel empirical computation.

References

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