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Model order reduction of dynamic skeletal muscle models

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Forward simulations of three-dimensional continuum-mechanical skeletal muscle models are a complex and computationally expensive problem. Considering a fully dynamic modelling framework based on the theory of finite elasticty is challenging as the muscles' mechanical behaviour requires to consider a highly nonlinear, viscoelastic and incompressible material behaviour. The governing equations yield a nonlinear second-order differential algebraic equation (DAE), which represents a challenge to model order reduction techniques.

In detail, the governing equations to be solved in the solution domain are the balance of momentum subject to the incompressibility constraint, i.e.

$$\rho_0(X)\frac{\partial \boldsymbol{V}}{\partial t}(X,t) = \nabla \boldsymbol{P}(X,t) + \boldsymbol{B}(X,t)\,, \qquad \text{s.t. } J(X,t) - 1 = 0\,, \tag{1}$$

and a nonlinear constitutive equation of the form

$$\boldsymbol{P}(X,t) = \boldsymbol{P}^{iso}(X,t) + \boldsymbol{P}^{aniso}(X,t) + \boldsymbol{P}^{active}(X,t) + \boldsymbol{P}^{viscous}(X,t) + p(X,t)\boldsymbol{F}^{-T}(X,t)\,. \tag{2}$$

Herein, ρ_0 is the muscle density, $\boldsymbol{V}$ is the velocity field, $\boldsymbol{P}$ is the first Piola-Kirchhoff stress tensor, $\boldsymbol{B}$ are the body forces, $J := \det \boldsymbol{F}$ is the Jacobian, $\boldsymbol{F}$ is the deformation gradient and p is the pressure. Discretising the governing equations using the finite element method, one obtains the following system

$$\begin{aligned} \boldsymbol{M}\boldsymbol{u}''(t) + \boldsymbol{D}\boldsymbol{u}'(t) + \boldsymbol{K}(\boldsymbol{u}(t),\boldsymbol{w}(t)) &= \boldsymbol{0}\,, \\ \text{s.t. } \boldsymbol{g}(\boldsymbol{u}(t)) &= \boldsymbol{0}\,, \end{aligned} \tag{3}$$

where $\boldsymbol{u}$ is the vector of position coefficients, $\boldsymbol{w}$ contains the pressure coefficients, $\boldsymbol{M}$, $\boldsymbol{D}$, $\boldsymbol{K}$ are the mass, viscous damping and generalised stiffness matrix, respectively, and $\boldsymbol{g}$ is the operator associated with the incompressibility constraint.

For this complex problem, a simple transformation of the system into a first-order system in order to obtain the general form of a parametric nonlinear dynamical sytem is not sufficient. Subsequent reduction using existing MOR methods, such as POD combined with DEIM, see e.g. [3, 1], did not lead to a stable reduced model. Therefore, other reduction methods and solution schemes need to be investigated and modified. Conceivably, one could directly project and solve the second order system by e.g. the Newmark method, see e.g. [2]. Further, to properly treat the constraint, the application of theories for Hessenberg index 2 DAEs and ODEs on constraint manifolds are promising. Here, we will show current results and discuss open questions.

References

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