Space-Time RBM for parabolic PDEs with Parameter Functions

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We consider reduced basis methods for parabolic partial differential equations (PDEs) in space-time variational formulation. The space-time approach avoids the (costly) time-stepping scheme in the RB-online phase and provides a posteriori error bounds (cf. [2]). The PDE solution is a function in space and time in an appropriate Bochner space.

We do not only allow parameters in the coefficients (for e.g. model calibration) but choose the initial condition as a parameter (function) as well. A reduced basis method is introduced that handels the parameter function and the corresponding infinite dimensional parameter space in a two-step greedy procedure (cf. [1]). The space-time variational formulation is splitted. A reduced basis for the initial condition is constructed first using a greedy procedure or proper orthogonal decompostion. In the second step a greedy procedure leads to a basis for the evolutionary part of the solution. Here, the reduced basis for the initial condition is involved. A posteriori error estimates are available. Online we solve two small linear equation systems.

In option pricing, the method offers the possibility to use the same reduced basis for different types of options as they differ only by their initial value (assuming the same model approach). Using the reduced basis does not only reduce the computational effort in pricing but can also be used in the PDE-constrained optimisation of the model calibration process.

References

[1] A. Mayerhofer and K. Urban. A reduced basis method for parabolic partial differential equations with parameter functions and application to option pricing. *arXiv preprint arXiv:1408.2709*, 2014.

[2] K. Urban and A. T. Patera. An improved error bound for reduced basis approximation of linear parabolic problems. *Mathematics of Computation*, 83(288):1599–1615, 2014.