# A subspace projection-based method for stabilization and enhancement of projection-based ROMs of the Navier–Stokes equations

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Computational fluid dynamics (CFD) has become an indispensable tool for many engineering applications. Unfortunately, high-fidelity CFD is often too expensive for parametric, time-critical and many-query applications such as design optimization, control and uncertainty quantification. Reduced order models (ROMs) are a promising tool for bridging the gap between high-fidelity, and real-time, multi-query applications. Despite recent advances in the field, model reduction is still in its infancy, particularly for high-Reynolds number turbulent flows.

For a projection-based ROM to be stable and accurate, the dynamics of the truncated subspace must be taken into account. In fluid flow applications, the traditional approach involves the addition of empirical energy-absorbing eddy-viscosity terms to the ROM. The drawback of this approach is that empirical turbulence models destroy consistency between the Navier-Stokes equations and the ROM. Accurately identifying and matching free coefficients of the turbulence models is another challenge.

In this work, an alternative approach for stabilizing and enhancing ROMs is proposed. Instead of adding an empirical turbulence model term to the ROM, we derive a transformation of the projection subspace that accounts for truncated modes [1]. Because only the projection subspace is modified, consistency between the ROM and the Navier-Stokes equations is retained. The proposed approach can be formulated mathematically as a trace minimization problem on the Stiefel manifold. The reproductive as well as predictive capabilities of the method are evaluated on several compressible flow problems, including a problem involving laminar flow over an airfoil with a high angle of attack (Figure 1), and a channel-driven cavity flow problem.

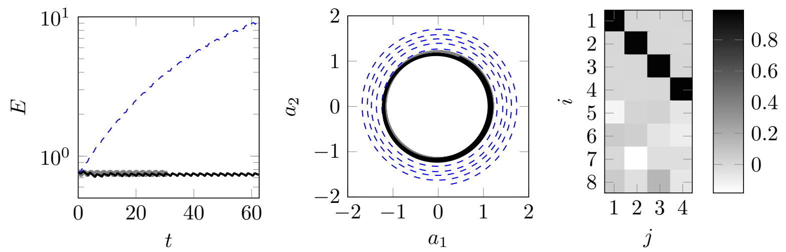


Figure 1: Nonlinear model reduction of the laminar airfoil. Evolution of modal energy (left), and phase plot of the first and second temporal basis, $a_1(t)$ and $a_2(t)$ (middle); DNS (thick gray line), standard 4 DOF ROM (dashed blue line), stabilized 4 DOF ROM (solid black line). Stabilizing transformation matrix (right)

## References

[1] M. Balajewicz, I. Tezaur, and E. Dowell. Minimal subspace rotation on the Stiefel manifold for stabilization and enhancement of projection-based reduced order models for the compressible Navier–Stokes equations. *Under consideration for publication in the journal CMAME.*