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Nonlinear Model Reduction for Complex Systems using Sparse Sensor Locations from Learned Nonlinear Libraries

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We demonstrate the synthesis of sparse sampling and dimensionality-reduction to characterize and model complex, nonlinear dynamical systems over a range of bifurcation parameters. First, we construct modal libraries using the classical proper orthogonal decomposition in order to expose the dominant low-rank coherent structures. Here, libraries of the nonlinear terms are also constructed in order to take advantage of the discrete empirical interpolation method and projection that allows for the approximation of nonlinear terms from a sparse number of grid points. The selected grid points are shown to be effective sensing/measurement locations for characterizing the underlying dynamics, stability, and bifurcations of complex systems. The use of empirical interpolation points and sparse representation facilitates a family of local reduced-order models for each physical regime, rather than a higher-order global model, which has the benefit of physical interpretability of energy transfer between coherent structures. The method advocated also allows for orders-of-magnitude improvement in computational speed and memory requirements.

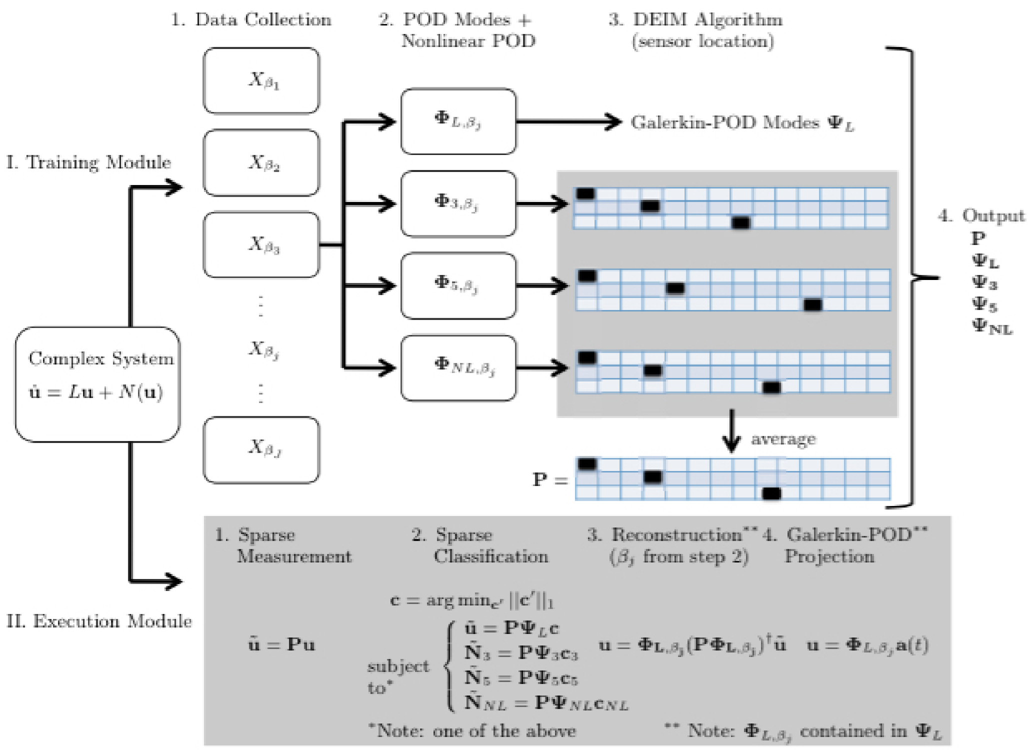

Figure 1: The training module samples the various dynamical regimes $(\beta_1, \beta_2, \cdots, \beta_J)$ through snapshots. For each dynamical regime, low-rank libraries are constructed for the nonlinearities of the complex system $(\boldsymbol{\Phi}_{L,\beta_j}, \boldsymbol{\Phi}_{3,\beta_j}, \boldsymbol{\Phi}_{5,\beta_j}, \boldsymbol{\Phi}_{NL,\beta_j})$. The DEIM algorithm is then used to select sparse sampling locations and construct the projection matrix $\mathbf{P}$. The execution module uses the sampling locations to classify the dynamical regime β_j of the complex system, reconstruct its full state $(\mathbf{u} = \boldsymbol{\Phi}_{\mathbf{L},\beta_\mathbf{j}}(\mathbf{P}\boldsymbol{\Phi}_{\mathbf{L},\beta_\mathbf{j}})^\dagger\tilde{\mathbf{u}})$, and provide a ROM (Galerkin-POD) approximation $(\mathbf{u} = \boldsymbol{\Phi}_{L,\beta_j}\mathbf{a}(t))$.