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Multifidelity methods for uncertainty quantification

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A classical and common remedy to reduce the runtime of uncertainty quantification is to replace the high-fidelity (full) model by a cheap surrogate model of lower fidelity. Such a surrogate model can be derived in various ways, including data-driven and projection-based model reduction, interpolation and regression approaches from machine learning, and different modeling and physics assumptions. A surrogate model is traditionally constructed with one-time high computational costs in an offline phase and then replaces the high-fidelity model in an online phase to conduct the uncertainty quantification. In this talk, we introduce multifidelity methods for uncertainty quantification that use in the online phase a combination of the high-fidelity model with multiple surrogate models. Our multifidelity methods leverage surrogate models to accelerate the computation, but explicitly allow occasional recourse to the high-fidelity model to establish accuracy guarantees on the overall uncertainty quantification result. These guarantees exist even in the absence of error bounds and error estimators for the surrogate models themselves.

The key component of our multifidelity methods is model management that balances the model evaluations across the high-fidelity and the surrogate models, and that combines the high-fidelity and low-fidelity solutions. We discuss three cases: (1) Model management based on online adaptivity, where a surrogate model (data-driven projection-based reduced model) is corrected online with sparse/partial solutions of the high-fidelity model. (2) Model management based on importance sampling. (3) Model management based on control variates, where we introduce an optimization problem to distribute model evaluations between the high-fidelity model and an arbitrary number of surrogate models of any type, including projection-based reduced models, data-fit interpolation models, and support vector machines.

We show mathematically and demonstrate numerically that combining the high-fidelity model with surrogate models of different approximation quality and costs is often more beneficial than combining the high-fidelity model with accurate surrogate models only. In this sense, surrogate models that inform different aspects of the high-fidelity model are better than surrogate models that are accurate but lack a rich diversity. Numerical results with linear and nonlinear examples show that our multifidelity methods achieve speedups by orders of magnitude compared to methods that invoke a single model only.