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Localization and adaptivity in Reduced Basis methods

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Reduced Basis (RB) methods are a powerful methodology to significantly reduced the computational complexity of solving parameterized partial differential equation problems. They allow for speedups of several orders of magnitude, compared to traditional discretization methods (such as Finite Element methods), by splitting the computational process into an offline and an online part. However, for interesting real-world problems, such as parametric multi-scale flow in porous media, the offline part of the computation can become unbearably costly, thus limiting the usefulness of RB methods in these scenarios. In the past years, several methodologies arose incorporating localization ideas from domain decomposition or numerical multi-scale methods into RB methods, such as the localized reduced basis multi-scale method (LRBMS) [1, 2, 3].

The main idea of the LRBMS is to build several local reduced bases (associated with subdomains of the physical domain) which are coupled in the spirit of discontinuous Galerkin methods. The resulting local reduced bases consist of a locally varying number of local basis functions, representing possible local influences of the parameterization. Since less global solution snapshots are required than with traditional RB methods and all offline work can be executed in parallel on local quantities, the LRBMS allows to balance the computational effort between the offline and the online part of the computation by choosing an appropriate number of subdomains (see [1]).

We present a recent extension of the LRBMS, based on an offline/online decomposable localized a posteriori estimate on the full error (including the discretization as well as the model reduction error, see [2]). This estimate allows to efficiently identify subdomains where the local reduced basis is insufficient during the online part of the computational process. By relaxing the traditional offline/online approach the *online adaptive LRBMS* allows for local high-dimensional computations in order to adaptively enrich these insufficient local reduced bases online, where required (see [3]).

The online adaptive LRBMS is thus suitable for complex real-world problems where we lack sufficient resources to fully prepare a suitable reduced basis offline. We demonstrate the methodology in the context of heterogeneous Darcy flow, compare it to related methods and discuss adaptivity in the context of RB methods in general.

References

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