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Reduced Basis Approximations for Maxwell's Equations in Dispersive Media

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The simulation of the propagation of an electromagnetic pulse through linear, temporally dispersive media (like water) or systems (like a dielectric waveguide) is a typical problem of electromagnetics.

Consider the Maxwell-Debye model formulated in second-order form in the electric field E

$$\frac{1}{\mu_0}\nabla \times \nabla \times E + \epsilon_0\epsilon_\infty \partial_t^2 E = f - \partial_t^2 P,$$
$$\partial_t P + \frac{1}{\tau}P = \frac{\epsilon_0(\epsilon_s - \epsilon_\infty)}{\tau}E,$$

with polarization P, relaxation time τ, relative permittivity at low-frequency limit ϵ_s and relative permittivity at high-frequency limit ϵ_∞, and a broadband input source f, which is modeled as a Gaussian pulse, [2]. The equations are discretized with Nédélec finite elements of first order over a 2D unit square. Dirichlet zero boundary conditions (i.e. PEC, perfectly electric conducting) are imposed on all boundaries.

The model is parametrized by τ and $\Delta_\epsilon = \epsilon_s - \epsilon_\infty$, defining the 2-dimensional parameter domain D. Using a POD-greedy sampling driven by an error indicator, we seek to generate a reduced model which accurately captures the dynamics in the parameter domain D, [3]. Typically, the reduced basis model reduction reduces the model order by a factor of more than 100, while maintaining an approximation error of less than 1%.

Since the POD step in each iteration of the greedy sampling is computationally expensive, we test and compare an interpolatory decomposition, cf. [1], in place of the POD. First numerical results indicate that the interpolative decomposition gives at least comparable or even better results in computational time and approximation quality to the POD for this model.

References

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