Simultaneous Empirical Interpolation and Reduced Basis method for non-linear problems

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In this talk, we will focus on the reduced basis methodology in the context of non-linear and non-affinely parametrized partial differential equations in which affine decomposition necessary for the reduced basis methodology are not obtained.

To deal with this issue, it is now standard to apply the Empirical Interpolation Method (EIM) methodology [1, 4] before deploying the Reduced Basis (RB) methodology. The EIM building step can be costly and require many (hundreds) finite element solutions when the terms are non-linear that forbids its application to large non-linear problems.

In this talk, we will introduce a Simultaneous EIM Reduced basis algorithm (SER) [2] based on the use of reduced basis approximations into the EIM building step. Enjoying the efficiency offered by reduced basis approximation, this method provides a huge computational gain and can require as little as $N + 1$ finite element solves where N is the dimension of the RB approximation.

We will start this talk with a brief overview of the EIM and RB methodologies applied to non-linear problems. The identification of the main issue, discussing the changes to be made in the EIM offline step for such problems will then introduce the SER method detailed in the first part of the talk with some of its variants.

The second part of the talk will first illustrate our method with preliminary results obtained on a benchmark introduced in [4]. We will then assess its performances for large scale problems it is designed for, through a 3D non-linear multi-physics reduced model used in high field magnet optimization context introduced in [3].

The SER method is now available in the generic and seamlessly parallel reduced basis framework of the opensource library Feel++ (Finite Element method Embedded Language in C++, `http://www.feelpp.org`).

References

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