POD-Galerkin for finite elements with dynamic mesh adaptivity

S. Ullmann[1,2], J. Lang[1,2], and M. Rotkvic[1]

[1]Department of Mathematics, Technische Universität Darmstadt, Darmstadt, Germany
[2]Graduate School Computational Engineering, Technische Universität Darmstadt, Darmstadt, Germany

Spatial adaptivity has recently gained interest in the context of snapshot-based reduced-order modeling. Typically, one relies on the same spatial mesh for all snapshots, which allows only static adaptivity [3]. Independent snapshot meshes have been investigated so far for wavelet-based [2] and one-dimensional [1] discretization schemes. In this study, we consider POD-Galerkin modeling for two-dimensional unsteady problems with dynamically adapted finite element snapshots.

We concentrate on snapshots obtained with newest vertex bisection based on some fixed initial mesh. For any subset of such snapshots, by refinement one can find a common mesh on which all snapshots in the subset can be represented exactly. Therefore, one way of creating a POD-Galerkin model is to construct the common mesh of all snapshots, interpolate the snapshots onto this mesh, and proceed with standard techniques. Alternatively, for PDEs with polynomial non-linearities one can work with the common grids of all $(N+1)$-tuples of snapshots, where N is the maximum polynomial degree. This approach can be useful if the common mesh of all snapshots contains much more nodes than each individual snapshot mesh.

As an example we study a viscous Burgers equation with smooth initial data. Figure 1 presents adapted meshes at different solution times. Figure 2 shows the error of the projected snapshots (POD) and the error of the POD-Galerkin solution (ROM) with respect to the original snapshots. The solution of the reduced-order model stops converging when the basis functions start reproducing mainly spatial discretization effects. At this point, however, the finite element error of the snapshots (FEM) already suggests snapshot refinement in order to converge to the true solution.

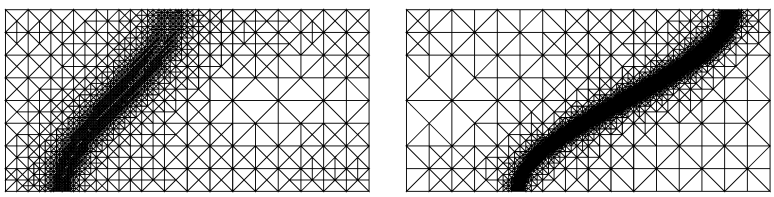

Figure 1: Adaptive meshes at times $t = 0.15$ and $t = 0.3$.

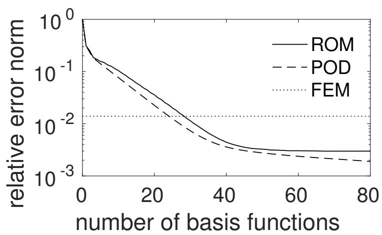

Figure 2: Convergence plot.

A necessary ingredient for our approach to POD-Galerkin modeling with adaptive finite element snapshots is the ability to construct common meshes of sets of snapshots. While this may be difficult to accomplish for arbitrary mesh structures, our work is a proof of concept for nested meshes, which readily generalizes to projection-based methods other than POD.

References

[1] O. Lass. *Reduced order modeling and parameter identification for coupled nonlinear PDE systems.* PhD thesis, 2014.

[2] K. Steih and K. Urban. Reduced basis methods based upon adaptive snapshot computations. http://arxiv.org/abs/1407.1708, submitted.

[3] M. Yano. A minimum-residual mixed reduced basis method: exact residual certification and simultaneous finite-element reduced-basis refinement. *ESAIM: M2AN*, 2015.