Control of an elastic structure based on Galerkin approximations

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In this talk, we consider an elastic shell model described by the following variational formulation:

$$\int_D \left\{ \rho \left(\frac{\partial^2 \tilde r}{\partial t^2}, \delta r \right) + \sum_{i,j=1}^{2} \left(\frac{1}{2} \frac{\partial \Phi}{\partial \varepsilon_{ij}} \delta g_{ij} + \frac{\partial \Phi}{\partial \varkappa_{ij}} \delta q_{ij} \right) - (\tilde W, \delta r) \right\} |\mathring{G}|^{1/2} d\xi^1 d\xi^2 - \int_\Gamma (\tilde F, \delta r) ds = 0, \quad (1)$$

for each admissible variation $\delta r(\xi^1, \xi^2, t)$. Here ρ is the surface density, $\tilde r(\xi^1, \xi^2, t)$ is the radius vector describing the median surface of the shell, $(\xi^1, \xi^2) \in D \subset \mathbb{R}^2$ are the Lagrangian coordinates, $\Phi(\varepsilon, \varkappa)$ is the energy density, tensors $\varepsilon = (\varepsilon_{ij})$ and $\varkappa = (\varkappa_{ij})$ are introduced to measure the strain and bending, $\mathring{G}$ is the metric tensor, g_{ij} and q_{ij} are components of the first and the second quadratic forms of the median surface, respectively. We assume that the shell is controlled by the force $\tilde F$ on the boundary Γ of D, and that external disturbances $\tilde W$ are distributed on the shell surface. Variational formulation (1) is obtained by applying Hamilton's principle within the framework of classic shell theory [1].

In order to derive a reduced model, we consider a finite set of basis functions $r_1(\xi^1, \xi^2)$, $r_2(\xi^1, \xi^2)$, ..., $r_N(\xi^1, \xi^2)$ and assume that

$$\tilde r(\xi^1, \xi^2, t) = \sum_{j=1}^{N} q_j(t) r_j(\xi^1, \xi^2) \text{ and } \delta r(\xi^1, \xi^2, t) \in \operatorname{span}\{r_1(\xi^1, \xi^2), r_2(\xi^1, \xi^2), ..., r_N(\xi^1, \xi^2)\}.$$

As a result, we get the following Galerkin system for (1):

$$M\ddot q(t) + Kq(t) = \bar B u + d(t), \quad q(t) = \begin{pmatrix} q_1(t) \\ q_2(t) \\ \vdots \\ q_N(t) \end{pmatrix}, \; u = \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \end{pmatrix}, \quad (2)$$

where $u \in \mathbb{R}^m$ is the control (represented in terms of the integral of $\tilde F$ over Γ) and $d \in \mathbb{R}^N$ is the disturbance vector (represented via the integral of $\tilde W$ over D). The procedure for computing the components of matrices M, K, and $\bar B$ is described in the paper [1].

We address the following problems for the Galerkin system in this talk:

1. for a given disturbance d, find a control u that minimizes the total stress in the shell;
2. identification of the disturbance vector d;
3. observer design problem for system (2) with the output from strain gauges;
4. stabilization of system (2) by a state feedback law and with an observer-based controller.

We also show some simulation results for reduced system (2) with two degrees of freedom where the basis functions $r_1(\xi^1, \xi^2)$ and $r_2(\xi^1, \xi^2)$ are computed by a finite element method.

References

[1] A. Zuyev and O. Sawodny. Modelling and control of a shell structure based on a finite dimensional variational formulation. *Mathematical and Computer Modelling of Dynamical Systems*, to appear, 2015.