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POD model reduction for constrained optimal control problems

M. Gubisch[1] and S. Volkwein[1]

[1]Universität Konstanz, Konstanz, Germany

We consider an optimal control problem for a parabolic partial differential equation (PDE) subject to control and state constraints. Since the optimality conditions are nonlinear, model order reduction provided by proper orthogonal decomposition (POD) is applied [2] to limit the numerical effort. Classical POD basis constructions usually contain *heuristic* information of the expexted optimal state solution to the PDE and therefore lack of a-priori error estimators; these are available only if the *optimal* state dynamics are included into the basis elements.

One way to compensate this drawback is to augment the reduced optimal control problem by interpreting the POD basis functions as optimization variables as well, introducing the information about the optimal state trajectory by postulating the state equation as a side condition: The optimality system proper orthogonal decomposition strategy (OSPOD). We will present a-priori [3] and a-posteriori [4] error estimations for this augmented reduced-order optimal control problem, propose an efficient solution algorithm [1] and illustrate our results by numerical tests.

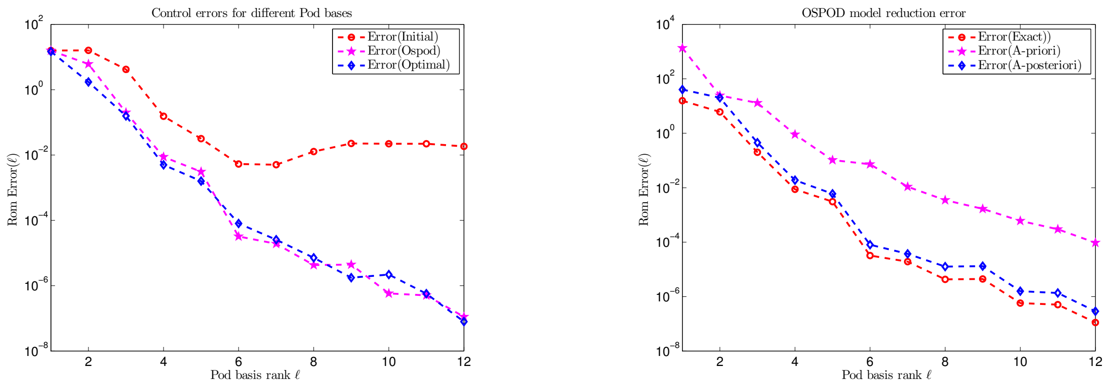

Figure 1: On the left, we see that the initial reference trajectory does not capture enough dynamics of the optimal state to build up a sufficiently accurate reduced-order model ∘ while the quality of the OSPOD basis ⋆ matches the accuracy of the optimal one ⋄. On the right, we estimate the OSPOD model error ∘ by a-priori bounds ⋆ based on eigenvalue estimations and by a-posteriori bounds ⋄ derived by a perturbation argument.

References

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