Element-based model reduction for parameter dependent parabolic PDE's on networks

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We consider parameter dependent parabolic PDE's on one-dimensional networks. For such networks as well as component-based problems Maday and Rønquist [3] developed the reduced basis element method (RBE). It is based on the idea of constructing a reduced basis for every network edge and afterwards coupling the reduced components by a mortar-like method. This way, large networks can be reduced without calculating a full solution of the given problem.

However, this decomposition procedure leads to a big problem for large parameter dependent networks. Especially on edges far away from the boundary the solution is hardly predictable. Therefore, the required boundary conditions for the edge problem in the POD process are not known. Based on this, the computation of a reduced basis for this edge is not possible or leads to poor basis functions. Initial approaches concerning this problem are given in the context of the static condensation reduced basis element method (SCRBE) [2].

We present a method to calculate basis functions which remedies this problem and provides a good basis representation for the single edges. Our approach uses piecewise linear functions to approximate the unknown boundary conditions. Hereby, a new set of parameters is introduced which is additionally considered in the POD method. Furthermore, we introduce an error analysis which substantiates the good basis representation of the global solution.

We demonstrate our method on an one-dimensional network of heat equations with varying thermal conductivity. The numerical treatment of the basis construction is based on the greedy-POD. Due to the high number of parameters the method of Bui-Thanh et. al. [1] is applied which extends the greedy by an optimization algorithm.

References

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