Reduced Basis Landweber method for nonlinear ill-posed inverse problems

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The numerical solution of nonlinear inverse problems such as the identification of a parameter in a partial differential equation (PDE) from a noisy solution of the PDE via iterative regularization methods, e.g. the Landweber method or Newton-type methods, usually requires numerous amounts of forward solutions of the respective PDE. One way to speed up the solution process therefore is to reduce the computational time of the forward solution, e.g. via the reduced basis method.

The reduced basis method is a model order reduction technique which constructs a low-dimensional subspace of the solution space. Galerkin projection onto that space allows for an approximative solution. An efficient offline/online decomposition enables the rapid computation of the approximative solution for many different parameters.

The simple and intuitive approach of weaving reduced basis methods into the solution process of inverse problems is the replacement of the forward solution by a global reduced basis approximation in a given regularization algorithm. The limitations of this approach in the context of imaging (very high-dimensional parameter spaces) will be shortly discussed in this talk.

The main topic is the new Reduced Basis Landweber method. It combines the key concepts of the reduced basis method with the Landweber method and is inspired by [1]. The general idea is to adaptively construct a small, problem-oriented reduced basis space instead of constructing a global reduced basis space like it is normally the case in reduced basis methods. This will be done in an iterative procedure: the inverse problem will be solved up to a certain accuracy with a Landweber method that is projected onto the current reduced basis space. The resulting parameter then is utilized to enrich the reduced basis space and therefore fit it to the given problem. This iteration is performed until an iterate is accepted as the solution of the inverse problem. Numerical results will show a significant speed-up and the possibility to reconstruct very high-dimensional parameters.

References

[1] V. Druskin and M. Zaslavsky. On combining model reduction and Gauß-Newton algorithms for inverse partial differential equation problems. *Inverse Problems*, 23(4):1599, 2007.