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An adaptive sampling approach for nonlinear dimensionality reduction based on manifold learning

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In [2], the authors and colleagues had proposed a non-intrusive dimensionality reduction method for nonlinear parametric flow problems governed by the Navier-Stokes equations. The approach is based on the manifold learning method of Isomap [3] combined with an interpolation scheme and will be referred to hereafter as as Isomap+I. Via this method, a low-dimensional *embedding space* is constructed that is approximately *isometric* to the manifold that is assumed to be formed by the high-fidelity Navier-Stokes flow solutions under smooth variations of the inflow conditions.

As with almost all model reduction methods, the offline stage for the Isomap+I approach requires a suitable *design of experiment*, i. e., a well-chosen sampling of high-fidelity flow solutions, the so-called *snapshots*. The online stage, however, might be considered as an adaptive way for choosing for each low-order prediction the most suitable local snapshot neighborhood rather than using all available snapshot information in a brute-force way. The notion of locality is based on the Isomap metric.

This talk will focus on an adaptive construction and refinement of the underlying design of experiment. Since Isomap comes with a natural non-Euclidean metric for measuring snapshot distances, we make use of this metric to detect gaps in the embedding space. By the (approximate) isometry between the embedding space and the manifold of flow solutions, we obtain in this way a *manifold filling* design of experiment. In contrast, standard approaches like the Latin Hypercube method [1] aim at a parameter-space filling design of experiment. The performance of the proposed manifold filling method will be illustrated by numerical experiment, where we consider nonlinear parameter-dependent steady-state Navier-Stokes flows in the transonic regime.

References

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