

**Integrable Systems in  
Geometry and Mathematical  
Physics, Conference in  
Memory of Boris Dubrovin  
(online, 28 June to 2 July 2021)**

Monday, June 28, 2021 - Friday, July 2, 2021

SISSA



**Book of Abstracts**



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## Variational Bihamiltonian Cohomology and Integrable Hierarchies

**Author:** Youjin Zhang<sup>1</sup>

<sup>1</sup> *Tsinghua University*

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In order to study deformations of Virasoro symmetries of the bihamiltonian integrable hierarchies associated to semisimple Frobenius manifolds, we introduce the notion of variational bihamiltonian cohomology, and compute the cohomology groups that will be used in our study of deformations of Virasoro symmetries. To illustrate its application, we classify the conformal bihamiltonian structures with semisimple hydrodynamic limits.

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## A Riemann-Hilbert approach to q-discrete Painlevé equations

**Author:** Nalini Joshi<sup>1</sup>

<sup>1</sup> *University of Sydney*

**Corresponding Author:** nalini.joshi@sydney.edu.au

The Riemann-Hilbert method provides a powerful framework for describing solutions of the classical Painlevé equations and semi-classical families of orthogonal polynomials. In this talk, I will give an overview and describe some recent results that show how to extend the framework to describe solutions of q-discrete Painlevé equations and q-orthogonal polynomials. (This is based on collaborative work with Tom Lasic Latimer and Pieter Roffelsen; see arXiv:1911.05854 and arXiv:2106.01042.)

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## Numerical Study of Davey-Stewartson systems

**Author:** Christian Klein<sup>1</sup>

<sup>1</sup> *Université de Bourgogne*

We present a detailed numerical study of solutions to Davey-Stewartson (DS) systems, nonlocal non-linear Schrödinger equations in two spatial dimensions. A possible blow-up of solutions is studied, a conjecture for a selfsimilar blow-up is formulated. In the integrable cases, numerical and hybrid approaches for the inverse scattering are presented. We comment on DS II and DS I systems.

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## Logarithmic Painlevé functions and Mathieu stability chart

**Author:** Oleg Lisovyi<sup>1</sup>

<sup>1</sup> LMPT, Tours University

The tau function of Painlevé III<sub>3</sub> equation (parameterless PIII) corresponding to generic monodromy data is known to coincide with the dual Nekrasov-Okounkov partition function and admits explicit combinatorial series representation. I will explain how to derive an analog of this representation for the one-parameter family of non-generic solutions of Painlevé III<sub>3</sub> characterized by the logarithmic asymptotics. I will also discuss a connection between such logarithmic tau functions and the characteristic values of Mathieu equation describing the band structure of the Schroedinger operator with a cosine potential.

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## Generating function of monodromy symplectomorphism, isomonodromic tau-function and its WKB expansion.

**Author:** Dmitry Korotkin<sup>1</sup>

<sup>1</sup> Concordia University

We discuss three closely related problems. First, we consider the  $SL(n)$  character variety of the Riemann surface of genus  $g$  with  $n$  punctures and show how to invert the Goldman Poisson structure on its symplectic leaf in terms of Fock-Goncharov coordinates. A version of this result for the extended character variety proposed by L. Jeffrey in 1994 is presented. Second, we apply this formalism to study the generating function of the monodromy symplectomorphism for the Fuchsian system on the Riemann sphere. In our framework, the symplectic potential on the (extended) character variety is expressed via Fock-Goncharov coordinates. This generating function can be naturally identified with the Jimbo-Miwa tau-function, which allows to fix the dependence of the tau-function on monodromy data. As a by-product, we obtain a new hamiltonian formulation of Schlesinger system which involves quadratic Poisson brackets with dynamical r-matrix. Third, we study the WKB expansion of the generating function; these calculations are based on WKB expansion of Fock-Goncharov coordinates in terms of certain Abelian integrals, known as Voros symbols.

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## On algebraic integrability of the elliptic two-dimensional $CP^n$ sigma model

**Author:** Igor Krichever<sup>1</sup>

<sup>1</sup> Columbia University

Harmonic maps of two-dimensional Riemann surface  $\Sigma$  to a Riemann manifold  $M$  are of interest both in physics and mathematics. They are critical points of the Dirichlet functional, the sigma model action.

In the talk a new approach to the study of these models will be presented. In particular we show that the Dubrovin-Krichever-Novikov hierarchy can be seen as a family of commuting symmetries of the  $CP^n$  sigma model. As a corollary we prove that the spectral curves associated with harmonic maps of two-torus to spheres are algebraic.

The talk is based on a joint work with Nikita Nekrasov

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## Frobenius $k$ -characters, Fricke identities and Markov equation

**Author:** Alexander Veselov<sup>1</sup>

<sup>1</sup> *Loughborough University, UK and Moscow State University, Russia*

**Corresponding Author:** a.p.veselov@lboro.ac.uk

In 1896 Frobenius and Fricke published two seemingly unrelated papers: Frobenius started to develop his theory of  $k$ -characters for finite groups motivated by Dedekind's question about factorisation of the group determinant, while Fricke followed Klein's approach to the uniformization theorem. I will explain that in fact these two works can be naturally linked and both are related to remarkable Markov's paper of 1880 on arithmetic of binary quadratic forms.

The central part of the talk is a brief review and extension of the theory of  $k$ -characters for finite groups initiated by Frobenius, who was motivated by the factorisation problem of the group determinant. We will mainly follow his very deep original work, which was further developed more recently by Johnson, Wiles, Taylor, Buchstaber and Rees.

The talk is based on joint work with V.M. Buchstaber.

Reference:

V.M. Buchstaber and A.P. Veselov Fricke identities, Frobenius  $k$ -characters and Markov equation. In: Integrability, Quantization and Geometry. II : Quantum Theory and Algebraic Geometry. Proc. Symp. Pure Math. 103.2 (2021), 67-78.

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## Integrability Of Integro-Differential Painlevé Equations

**Author:** Mattia Cafasso<sup>1</sup>

<sup>1</sup> *Université d'Angers*

**Corresponding Author:** mattia.cafasso@univ-angers.fr

Motivated by applications in integrable probability, I will discuss the integrability properties of some integro-differential generalisations of Painlevé equations.

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## Hamiltonian aspects of multilayer flows

**Author:** Gregorio Falqui<sup>1</sup>

**Co-authors:** Ortenzi G.<sup>2</sup>; Pedroni M.<sup>3</sup>; R. A. Camassa<sup>4</sup>

<sup>1</sup> *Dipartimento di Matematica e Applicazioni Università di Milano-Bicocca*

<sup>2</sup> *Milano-Bicocca*

<sup>3</sup> *Bergamo*

<sup>4</sup> *UNC - Chapel Hill*

**Corresponding Author:** gregorio.falqui@unimib.it

The theory of Hamiltonian PDEs will be applied to study evolution equations deduced from the Euler equations in the incompressibility regime by means of suitable vertical-averaging and asymptotic

expansion processes. We shall consider two and three layers sharply stratified flows in an infinite 2D channel. A Hamiltonian structure introduced by T.-B. Benjamin will be reviewed and specialised to the models. We shall show how to reduce the full 2D Hamiltonian picture to 1D averaged equations and discuss conservation laws in the long wave dispersionless limit, with a view towards the inclusion of dispersive terms. The Boussinesq approximation of neglecting density differences in the fluids' inertia will be then applied to the leading order equations, showing the equivalence of the two-layer system with the shallow-water Airy system (a/k/a dispersionless NLS). Time permitting, we shall finally discuss time evolutions from a class of suitable initial data.

This is a report of joint ongoing works with R.A. Camassa (UNC - Chapel Hill), G. Ortenzi (Milano-Bicocca), M. Pedroni (Bergamo) and others.

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## Poster - 3 minutes talks

Speakers:

1. Almeida Guilherme - Differential geometry of orbit space of extended Jacobi group  $A_n$
2. Atlasiuk Olena - On generic boundary-value problems for differential system
3. Attarchi Hassan - Why escape is faster than expected
4. Babinet Nicolas - Hirota equation for the supermatrix model
5. Chen Thomas - Constructing Classical Integrable Systems Using Artificial Neural Networks
6. Chouteau Thomas - Hamiltonian structure for higher-order Airy kernel
7. Fairon Maxime - Integrable systems on (multiplicative) quiver varieties
8. Fantini Veronica - On 2d-4d Wall-Crossing Formulas and the extended tropical vertex
9. Gharakhloo Roozbeh - Modulated Bi-orthogonal Polynomials on the Unit Circle: The  $2j-k$  and  $j-2k$  Systems
10. Gubbiotti Giorgio - Recent developments on variational difference equations and their classification
11. Robert Jenkins - Large time asymptotic behavior for the Derivative Nonlinear Schrodinger (DNLS) Equation
12. Kels Andrew -  $\mathbb{C}^8 \times Q(E_8)$  extension of the elliptic Painlevé equation
13. Kramer Reinier - KP for Hurwitz-type cohomological field theories
14. Minakov Alexander - Gap probabilities in the Freud random matrix ensemble
15. Prokhorov Andrei - Large parameter asymptotics of rational solutions of Painlevé III equation near zero.
16. Ruzza Giulio - Jacobi Unitary Ensemble and Hurwitz numbers
17. Tarricone Sofia - Stokes manifolds and cluster algebras
18. Vergallo Pierandrea - Classification of bi-Hamiltonian pairs extended by isometries
19. Vermeeren Mats - A Lagrangian perspective on integrability
20. Zhang Ziruo - Black Hole Entropy from the Superconformal Index
21. Zhou Jing - An exponential Fermi accelerator, a Rectangular Billiard with Moving Slits

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## Isomonodromy aspects of the $tt^*$ equations of Cecotti and Vafa. Iwasawa factorization and asymptotics.

**Author:** Alexander Its<sup>1</sup>

<sup>1</sup> *Indiana University - Purdue University Indianapolis*

**Corresponding Author:** aits@iupui.edu

In this talk the results concerning the global asymptotic analysis of the  $tt^*$  - Toda equation,

$$2(w_i)_{t\bar{t}} = -e^{2(w_{i+1}-w_i)} + e^{2(w_i-w_{i+1})},$$



where, for all  $i$ ,  $w_i = w_{i+4}$  (periodicity),  $w_i = w_i(|t|)$  (radial condition), and  $w_i + w_{-i-1} = 0$  (“anti-symmetry”), will be presented.

The problem has been intensively studied since the early 90s work of Cecotti and Vafa. In these work a prominent role of the  $tt^*$  equations in the classification of supersymmetric field theories had been revealed and a series of important conjectures about their solutions has been formulated. We study the question using a combination of methods from PDE, isomonodromic deformations (Riemann-Hilbert method), and loop groups (Iwasawa factorization). We place these global solutions into the broader context of solutions which are smooth near 0. For such solutions, we compute explicitly the Stokes data and connection matrix of the associated meromorphic system, in the resonant cases as well as in the non-resonant case. This allows us to give a complete picture of the monodromy data, holomorphic data, and asymptotic data of the global solutions.

This is a joint work with Martin Guest and Chang-Shou Lin.

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## On K-theory of Deligne-Mumford spaces

**Author:** Alexander Givental<sup>None</sup>

I will discuss the state of affairs in the theory of K-theoretic Gromov-Witten invariants of the point target space.

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## From Frobenius manifolds to hyperKähler geometry via Donaldson-Thomas invariants

**Author:** Ian Strachan<sup>1</sup>

**Co-author:** Tom Bridgeland

<sup>1</sup> *University of Glasgow*

**Corresponding Author:** [ian.strachan@glasgow.ac.uk](mailto:ian.strachan@glasgow.ac.uk)

In the theory of Frobenius manifolds a connection with a regular and an irregular singularity, with associated Stokes phenomena, plays a fundamental role. In this talk the link between Donaldson-Thomas (DT) invariants and such isomonodromy problems - with an infinite dimensional Lie algebra - is studied. The DT-invariants control the Stokes factors between sectors, and the various objects can be combined to form what is called a Joyce structure, and this in turn defines a (complex) hyperKähler structure on a certain tangent bundle  $TM$ . Finally, borrowing ideas from the deformation quantisation programme, the relationship between quantum DT-invariants and Moyal-deformations of hyperKähler structures is studied.

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## The universal unfolding is an atlas of Stokes data for the simple and the simple elliptic singularities

**Author:** Claus Hertling<sup>1</sup>

<sup>1</sup> *Universität Mannheim*

A holomorphic vector bundle on  $\mathbb{C}$  with a meromorphic connection with an order 2 pole at 0 can be encoded by its monodromy data. In many cases from algebraic geometry, the pole part is semisimple with pairwise different eigenvalues  $u_1, \dots, u_n$ , and the monodromy data boil down to these eigenvalues and an upper triangular matrix with integer entries, a Stokes matrix. Isomonodromic deformations lead to a braid group orbit of Stokes matrices and base spaces, where the eigenvalues  $u_1, \dots, u_n$  are locally canonical coordinates. The talk discusses this for the simple and the simple elliptic singularities. Here distinguished bases of the Milnor lattice are marked monodromy data. For the simple singularities (Looijenga and Deligne 73/74) and the simple elliptic singularities (Hertling and Roucairol 07/18), this leads to an understanding of the base spaces of certain global versal unfoldings as atlases of marked monodromy data.

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## Second-order integrable Lagrangians and WDVV equations

**Author:** Evgeny Ferapontov<sup>1</sup>

**Co-authors:** Lingling Xue ; M.V. Pavlov

<sup>1</sup> *University of Loughborough*

**Corresponding Author:** e.v.ferapontov@lboro.ac.uk

We investigate the integrability of Euler-Lagrange equations associated with 2D second-order Lagrangians.

By deriving integrability conditions for the Lagrangian density, examples of integrable Lagrangians expressible via elementary functions,

Jacobi theta functions and dilogarithms are constructed. A link of second-order integrable Lagrangians to WDVV equations is established.

Generalisations to 3D second-order integrable Lagrangians are also discussed.

Based on:

E.V. Ferapontov, M.V. Pavlov, Lingling Xue,

Second-order integrable Lagrangians and WDVV equations, Lett. Math. Phys. (2021); doi:10.1007/s11005-021-01403-3; arXiv:2007.03768.

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## Initial data for Airy kernel determinant solutions of the Korteweg-de Vries equation

**Author:** Tom Claeys<sup>None</sup>

**Co-authors:** Mattia Cafasso ; Giulio Ruzza <sup>1</sup>; Christophe Charlier

<sup>1</sup> *SISSA*

**Corresponding Authors:** gruzza@sissa.it, mattia.cafasso@univ-angers.fr

Fredholm determinants associated to deformations of the Airy kernel characterize a specific solution of the Kardar-Parisi-Zhang equation, and the same determinants appear in models for finite temperature free fermions. These determinants belong to a class of solutions to the Korteweg-de Vries equation with singular initial data. I will discuss the initial data in detail, and I will explain the implications for the Kardar-Parisi-Zhang equation.

The talk will be based on joint work (in progress) with Mattia Cafasso, Giulio Ruzza and Christophe Charlier.

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## The two-periodic Aztec diamond and matrix valued orthogonality

**Author:** Arno Kuijlaars<sup>1</sup>

**Co-author:** Maurice Duits<sup>2</sup>

<sup>1</sup> *Katholieke Universiteit Leuven*

<sup>2</sup> *KTH Stockholm*

**Corresponding Author:** arno.kuijlaars@kuleuven.be

I will discuss how polynomials with a non-hermitian orthogonality on a contour in the complex plane arise in certain random tiling problems. In the case of periodic weightings the orthogonality is matrix valued. In work with Maurice Duits (KTH Stockholm) the Riemann-Hilbert problem for matrix valued orthogonal polynomials was used to obtain asymptotics for domino tilings of the two-periodic Aztec diamond. This model is remarkable since it gives rise to a gaseous phase, in addition to the more common solid and liquid phases.

Reference:

M. Duits and A.B.J. Kuijlaars,  
The two periodic Aztec diamond and matrix valued orthogonal polynomials,  
J. Eur. Math. Soc. 23 (2021), 1075-1131.

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## Soliton versus the gas

**Author:** Ken McLaughlin<sup>1</sup>

<sup>1</sup> *Colorado State University*

We study the dynamics of a single (tracer) soliton as it interacts with a continuum limit of solitons. Detailed asymptotics of the soliton's position will be enjoyed.

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## TALK CANCELLED: The focusing NLS equation with oscillating backgrounds: the shock problem

**Author:** Anne Boutet de Monvel<sup>1</sup>

<sup>1</sup> *Université de Paris, IMJ-PRG*

This lecture is cancelled, the talks will resume as usual at 10:20.

I will consider a solution  $q(x, t)$  for the focusing nonlinear Schrödinger equation  $iq_t + q_{xx} + 2|q|^2q = 0$  with initial values  $q(x, 0) \approx A_1 e^{i\phi_1} e^{-2iB_1 x}$  as  $x \rightarrow -\infty$  and  $q(x, 0) \approx A_2 e^{i\phi_2} e^{-2iB_2 x}$  as  $x \rightarrow +\infty$ .

I'm interested in its long-time asymptotics. It is qualitatively different in sectors  $\xi_{i+1} < \xi := \frac{x}{t} < \xi_i$  of the  $(x, t)$  half-plane and the goal is to determine these sectors and the asymptotics of  $q$  in each of them.

I will concentrate on the shock case ( $B_1 < B_2$ ). The case  $B_1 = B_2$  has already been studied by Biondini and Mantzavinos (CPAM 2017) and the rarefaction case ( $B_2 < B_1$ ) is close to the case  $A_1 = 0$  studied in a paper with Kotlyarov and Shepelsky (IMRN 2011).

The shock case has already been considered by Buckingham and Venakides (CPAM 2007). I will show it is actually rich in asymptotic scenarios. I will present these different scenarios. They depend on the relative values of the parameters  $A_j, B_j$ . (This is joint work with Jonatan Lenells and Dmitry Shepelsky.)

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## Poncelet property and quasi-periodicity of the integrable Boltzmann system

**Author:** Giovanni Felder<sup>1</sup>

<sup>1</sup> *ETH Zurich*

**Corresponding Author:** felder@math.ethz.ch

The integrable Boltzmann system describes the motion of a particle in a plane subject to an attractive central force with inverse-square law on one side of a wall at which the particle is reflected elastically. This model is a special case of a class of systems, involving also an inverse-cube law centrifugal force, considered by L. Boltzmann to illustrate his ergodic hypothesis. The system without centrifugal force was recently shown by G. Gallavotti and I. Jauslin to admit a second integral of motion additionally to the energy. By recording the subsequent positions and momenta of the particle as it hits the wall, we obtain a three-dimensional discrete-time dynamical system. We show that this system has the Poncelet property: if for given generic values of the integrals one orbit is periodic, then all orbits for these values are periodic and have the same period. We also prove a conjecture of Gallavotti and Jauslin on the quasi-periodicity of the integrable Boltzmann system, implying the applicability of Kolmogorov-Arnold-Moser perturbation theory to the Boltzmann system with weak centrifugal force. As in the analogous case of the Poncelet porism, which I will review, the results rely on the theory of elliptic curves.

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## Extended nonlinear Schrodinger hierarchy and higher genera Catalan numbers

**Author:** Guido Carlet<sup>1</sup>

<sup>1</sup> *Université de Bourgogne*

**Corresponding Author:** guido.carlet@u-bourgogne.fr

In the framework of the Givental construction of total descendent potential of the two-dimensional charge  $d=-1$  Dubrovin-Frobenius manifold, we derive Hirota equations and study the Lax representation of the associated extended Schrodinger hierarchy. The total descendent potential is related to higher general Catalan numbers via CEO topological recursion.

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## Geometry and topological recursion

**Author:** Gaetan Borot<sup>1</sup>

<sup>1</sup> *Humboldt-Universität zu Berlin*

**Corresponding Author:** gaetan.borot@hu-berlin.de

I will describe various ramifications of the theory of topological recursion in intersection theory of the moduli space, integrable systems, Gromov-Witten theory and gauge theories, together with a few open problems.

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## **A noncommutative generalization of Witten's conjecture**

**Author:** Alexandr Buryak<sup>1</sup>

<sup>1</sup> *ETH Zurich*

**Corresponding Author:** buryaksh@gmail.com

The classical Witten conjecture says that the generating series of integrals of monomials in the psi-classes over the moduli spaces of curves is a solution to the KdV hierarchy. Together with Paolo Rossi, we present the following generalization of Witten's conjecture, which remarkably involves a noncommutative integrable system. On one side, let us deform Witten's generating series by inserting in the integrals certain naturally defined cohomology classes, the so-called double ramification cycles. It turns out that the resulting generating series is conjecturally a solution of a noncommutative KdV hierarchy, where one spatial variable is replaced by two spatial variables and the usual multiplication of functions is replaced by the noncommutative Moyal multiplication in the space of functions of two variables.

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## **Interpolation, integrals, and indices**

**Author:** Andrei Okounkov<sup>1</sup>

<sup>1</sup> *Columbia University*

There is an interesting topology behind such classical questions as interpolation and solving linear  $q$ -difference equations by integrals. It has to do with counting algebraic curves in some very specific geometries, which can be also phrased as computing indices in certain  $(2+1)$  dimensional supersymmetric QFTs. In particular, the  $q$ -difference equations appear as  $q$ -analogs of the Dubrovin connection. The talk will be an introduction to this circle of ideas.

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## **From SFT to Integrable Systems: my conversations with Boris Dubrovin**

**Author:** Yakov Eliashberg<sup>None</sup>

The formalism of Symplectic Field Theory proposed by A. Givental, H. Hofer and the speaker naturally leads to quantum integrable systems. This link was first observed in our discussions with Boris Dubrovin. In the talk I recall our conversations.

## Geometry and Arithmetic of Integrable Hierarchies of KdV-type (Part 1)

**Author:** Don Zagier<sup>None</sup>

This is a joint talk about certain rational numbers (special FJRW invariants)  $\tau_{\mathfrak{g}}(g)$  indexed by a non-negative integer  $g$  (genus) and a simply-laced Lie algebra  $\mathfrak{g}$  (for us always  $A_l, D_l$  or  $E_6$ ), with the case of  $A_{r-1}$  being Witten's 1-point  $r$ -spin intersection numbers. These numbers can be defined geometrically as integrals over compactified moduli spaces of curves of products of psi-classes or else in the language of integrable systems as the Taylor expansion coefficients of the logarithms of tau-functions for certain integrable hierarchies, but they also have various elementary descriptions in terms of differential equations, recursions, explicit formulas or generating functions.

Some of these explicit formulas will be given in Part I, in which the emphasis is on the algebraic and arithmetic properties of the numbers, especially in the  $A_4$  case (5-spin intersection numbers). Here we show that there are three different ways to make the numbers  $\tau_g = \tau_{A_4}(g)$  integral by multiplying by suitable elementary denominators (products of Pochhammer symbols). One of these three sequences of integers is the Taylor expansion of an algebraic function, and this works in all cases and will be discussed in Part II. The other two, which are proved by  $p$ -adic arguments, are in some ways more interesting since they give sequences that according to a famous conjecture of Y. André about " $G$ -functions" should be the Taylor coefficients of period functions (solutions of Picard-Fuchs differential equations). This conjecture could be verified, and indeed these two other generating functions also turned out to be algebraic, but in a quite unexpected way related to Klein's formulas for the icosahedron. For other  $A_l$  cases, we find an exact formula for the part of the intersection numbers made up of small prime numbers, giving "best possible" denominators which, however, no longer fit into the framework of  $G$ -functions. This seems to be an interesting new phenomenon in the arithmetic theory of algebraic differential equations, even apart from this application.

The second part of the talk will explain more about the geometric and integrable system backgrounds for the FJRW invariants and will contain a sketch of the proof of the above-mentioned algebraicity by using the method of wave functions. We also give several other different types of formulas, including a closed formula via residues of pseudo-differential operators, another closed formula (obtained also by Liu-Vakil-Xu) based on Brézin-Hikami's approach from matrix models, and asymptotic formulas in the large-genus limit. Moreover, we find that the all-genera one-point FJRW invariants for the  $A_l, D_l$  and  $E_6$  cases coincide with the coefficients of the calibration of the underlying Frobenius manifold evaluated at a special point.

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## Geometry and Arithmetic of Integrable Hierarchies of KdV-type (Part 2)

**Author:** Di Yang<sup>1</sup>

<sup>1</sup> *Max Planck Institute for Mathematics*

**Corresponding Author:** diyang@mpim-bonn.mpg.de

This is a joint talk about certain rational numbers (special FJRW invariants)  $\tau_{\mathfrak{g}}(g)$  indexed by a non-negative integer  $g$  (genus) and a simply-laced Lie algebra  $\mathfrak{g}$  (for us always  $A_l, D_l$  or  $E_6$ ), with the case of  $A_{r-1}$  being Witten's 1-point  $r$ -spin intersection numbers. These numbers can be defined geometrically as integrals over compactified moduli spaces of curves of products of psi-classes or else in the language of integrable systems as the Taylor expansion coefficients of the logarithms of tau-functions for certain integrable hierarchies, but they also have various elementary descriptions in terms of differential equations, recursions, explicit formulas or generating functions.

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The second part of the talk will explain more about the geometric and integrable system backgrounds for the FJRW invariants and will contain a sketch of the proof of the above-mentioned algebraicity by using the method of wave functions. We also give several other different types of formulas, including a closed formula via residues of pseudo-differential operators, another closed formula (obtained also by Liu-Vakil-Xu) based on Brézin-Hikami’s approach from matrix models, and asymptotic formulas in the large-genus limit. Moreover, we find that the all-genera one-point FJRW invariants for the  $A_l$ ,  $D_l$  and  $E_6$  cases coincide with the coefficients of the calibration of the underlying Frobenius manifold evaluated at a special point.

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## Ricci curvature and quantum geometry

**Author:** Mauro Carfora<sup>None</sup>

We describe a few elementary aspects of the circle of ideas associated with a quantum field theory (QFT) approach to Riemannian Geometry, a theme related to how Riemannian structures are generated out of the spectrum of (random or quantum) fluctuations around a background fiducial geometry. In such a scenario, Ricci curvature with its subtle connections to diffusion, optimal transport, Wasserstein geometry, and renormalization group features prominently.

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## Isomonodromic deformations: Confluence, Reduction and Quantization

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In this talk I will discuss the isomonodromic deformations of systems of differential equations with poles of any order on the Riemann sphere as Hamiltonian flows on the product of co-adjoint orbits of the Takiff algebra (i.e. truncated current algebra). This is based on work in collaboration with Ilia Gaiur and Volodya Rubtsov. Our motivation is to produce confluent versions of the celebrated Knizhnik–Zamolodchikov equations and explain how their quasiclassical solution can be expressed via the isomonodromic  $\tau$ -function.

In order to achieve this, we study the confluence cascade of  $r + 1$  simple poles to give rise to a singularity of arbitrary Poincaré rank  $r$  as a Poisson morphism and explicitly compute the isomonodromic Hamiltonians.

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## Constrained Schlesinger system

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We propose a modification of the classical Schlesinger system where some of the independent variables become functions of the other variables. This construction is motivated by considering a special variation of a hyperelliptic curve related to generalized Chebyshev polynomials. We also construct an algebro-geometric solution to the constrained Schlesinger system in terms of such a family of hyperelliptic curves.

This is a joint work with Vladimir Dragovic (UTD).

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## Geometry and dynamics of isorotational and iso-harmonic deformations

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The talk is based on a strong interrelation between integrable billiards and Poncelet polygons, extremal polynomials, Riemann surfaces, potential theory, and isomonodromic deformations. We discuss injectivity properties of rotation and winding numbers. We construct and describe isorotational families of Poncelet polygons inscribed in a given circle and subscribed about conics from a confocal family. After introducing a new notion of iso-harmonic deformations, we study their isomonodromic properties in the first nontrivial examples and indicate the genesis of a new class of the so-called constrained isomonodromic deformations. The talk is based on the work in progress with Vasilisa Shramchenko and:

1. V. Dragovic, M. Radnovic, Periodic ellipsoidal billiard trajectories and extremal polynomials, *Communications. Mathematical Physics*, 2019, Vol. 372, p. 183-211.
2. V. Dragovic, V. Shramchenko, Algebro-geometric solutions of the Schlesinger systems and the Poncelet-type polygons in higher dimensions, *International Math. Research Notices*, 2018, Vol. 2018, No 13, p. 4229-4259.
3. V. Dragovic, V. Shramchenko, Algebro-geometric approach to an Okamoto transformation, the Painleve VI and Schlesinger equations, *Annales Henri Poincare*, 2019, Vol. 20, No. 4, 1121–1148.
4. V. Dragovic, V. Shramchenko, Deformation of the Zolotarev polynomials and Painleve VI equations, *Letters Mathematical Physics*, 111, 75 (2021). <https://doi.org/10.1007/s11005-021-01415-z>.
5. V. Dragovic, M. Radnovic, Poncelet polygons and monotonicity of rotation numbers: iso-periodic confocal pencils of conics, hidden traps, and marvels, arXiv: 2103.01215.
6. G. Andrews, V. Dragovic, M. Radnovic, Combinatorics of the periodic billiards within quadrics, arXiv: 1908.01026, *The Ramanujan Journal*, DOI: 10.1007/s11139-020-00346-y.