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Geometry and Arithmetic of Integrable Hierarchies of KdV-type (Part 1)

Friday, July 2, 2021 9:30 AM (40 minutes)

This is a joint talk about certain rational numbers (special FJRW invariants) $\tau_{\mathfrak{g}}(g)$ indexed by a non-negative integer g (genus) and a simply-laced Lie algebra $\mathfrak{g}$ (for us always A_l, D_l or E_6), with the case of A_{r-1} being Witten's 1-point r-spin intersection numbers. These numbers can be defined geometrically as integrals over compactified moduli spaces of curves of products of psi-classes or else in the language of integrable systems as the Taylor expansion coefficients of the logarithms of tau-functions for certain integrable hierarchies, but they also have various elementary descriptions in terms of differential equations, recursions, explicit formulas or generating functions.

Some of these explicit formulas will be given in Part I, in which the emphasis is on the algebraic and arithmetic properties of the numbers, especially in the A_4 case (5-spin intersection numbers). Here we show that there are three different ways to make the numbers $\tau_g = \tau_{A_4}(g)$ integral by multiplying by suitable elementary denominators (products of Pochhammer symbols). One of these three sequences of integers is the Taylor expansion of an algebraic function, and this works in all cases and will be discussed in Part II. The other two, which are proved by p-adic arguments, are in some ways more interesting since they give sequences that according to a famous conjecture of Y. André about "G-functions" should be the Taylor coefficients of period functions (solutions of Picard-Fuchs differential equations). This conjecture could be verified, and indeed these two other generating functions also turned out to be algebraic, but in a quite unexpected way related to Klein's formulas for the icosahedron. For other A_l cases, we find an exact formula for the part of the intersection numbers made up of small prime numbers, giving "best possible" denominators which, however, no longer fit into the framework of G-functions. This seems to be an interesting new phenomenon in the arithmetic theory of algebraic differential equations, even apart from this application.

The second part of the talk will explain more about the geometric and integrable system backgrounds for the FJRW invariants and will contain a sketch of the proof of the above-mentioned algebraicity by using the method of wave functions. We also give several other different types of formulas, including a closed formula via residues of pseudo-differential operators, another closed formula (obtained also by Liu-Vakil-Xu) based on Brézin-Hikami's approach from matrix models, and asymptotic formulas in the large-genus limit. Moreover, we find that the all-genera one-point FJRW invariants for the A_l, D_l and E_6 cases coincide with the coefficients of the calibration of the underlying Frobenius manifold evaluated at a special point.

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