Integrable systems around the world, September 14-16, 2020

Monday, September 14, 2020 - Wednesday, September 16, 2020

SISSA

Book of Abstracts

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A threefold way to integrable probabilistic models

Author: Thomas Bothner[1]

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This talk is intended for a broad math and physics audience in particular including students. It will focus on the speaker's recent contributions to the analysis of the real Ginibre ensemble consisting of square real matrices whose entries are i.i.d. standard normal random variables. In sharp contrast to the complex and quaternion Ginibre ensemble, real eigenvalues in the real Ginibre ensemble attain positive likelihood. In turn, the spectral radius of a real Ginibre matrix follows a different limiting law for purely real eigenvalues than for non-real ones. We will show that the limiting distribution of the largest real eigenvalue admits a closed form expression in terms of a distinguished solution to an inverse scattering problem for the Zakharov-Shabat system. This system is directly related to several of the most interesting nonlinear evolution equations in 1+1 dimensions which are solvable by the inverse scattering method. The results of this talk are based on our joint work with Jinho Baik (arXiv:1808.02419 and arXiv:2008.01694).

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The "good" Boussinesq equation: long-time asymptotics

Author: Christophe Charlier[1]

[1] *KTH Stockholm*

I will talk about the long-time asymptotics of the solution to the initial-value problem for the "good" Boussinesq equation on the line. The proof is based on a Deift-Zhou steepest descent analysis of a 3x3 RHP.

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Integrable systems in the periodic TASEP

Author: Guilherme Silva[1]

[1] *University of Sao Paolo*

We discuss properties of limiting distribution functions that arise in the periodic totally asymmetric simple exclusion process (pTASEP), and which are believed to be universal limiting distributions in the KPZ universality class for periodic models. For the periodic TASEP with periodic step initial condition, we prove that its one-point limiting distribution interpolates between a Gaussian and the GUE Tracy-Widom distribution, and also find different formulations that are in direct analogy with the latter. One of these representations lead to a purely discrete Riemann-Hilbert problem with infinitely many poles, and using it we connect the pTASEP with a system of coupled mKdV equations, and also with coupled heat equations. In addition, we also find a connection with the KP equation, extending a recent work of Quastel and Remenik. Towards the end of the walk, we also plan to briefly discuss some work in progress, showing how multi-point distributions for both TASEP and pTASEP are connected with matrix versions of the coupled mKdV and heat systems, and also with matrix versions of the Painlev´e II and KP equations. This is based on joint work with Jinho Baik (University of Michigan) and Zhipeng Liu (University of Kansas), and work in progress with Jinho

Baik and
Andrei Prokhorov (University of Michigan).

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Painlevé II tau-function as a Fredholm determinant

Author: Harini Desiraju[1]

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The tau-functions of certain Painlevé equations (III, V, VI) can be expressed as Fredholm determinants of a composition of two suitable Toeplitz operators, called the Widom constant. The key feature of this construction is to reduce the Riemann-Hilbert problem (RHP) associated to the isomonodromic system to a RHP on the circle. In this talk, I will show that the generic Painlevé II tau-function can be expressed as a Fredholm determinant of an integrable (Its-Izergin-Korepin-Slavnov) operator by recasting the RHP of Painlevé II as a RHP on the imaginary axis. This talk is based on the preprint ArXiv:2008.01142v2.

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Complex geometric optics solutions for d-bar problems

Author: Christian Klein[1]

[1] *Institut de Mathématiques de Bourgogne*

Complex geometric optics (CGO) solutions to d-bar problems appear in many
applications, for instance in Electrical Impedance Tomography and in
the scattering theory of integrable PDEs in two dimensions. We discuss various numerical approaches to construct CGO solutions to d-bar systems. Of particular interest is the limit of large values of the spectral parameter where the solutions have an essential singularity.
This is work in collaboration with K. McLaughlin, J. Sjöstrand and N. Stoilov.

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Multiplicative statistics of the Airy process and the Korteweg-de Vries equation

Author: Giulio Ruzza[1]

[1] *Université Catholique de Louvain*

Based on a joint work with Mattia Cafasso and Tom Claeys, we consider a novel class of solutions to the Korteweg-de Vries (KdV) equation, defined for t>0 and blowing up at t=0; they arise in connection with a multiplicative statistics of the Airy point process. Such class can be regarded as a broad generalization of the classical self-similar KdV solution associated with the Ablowitz-Segur Painlevé II (PII) transcendents. In general, these solutions are instead connected with an integro-differential deformation of the PII equation; this deformation has been first found by Amir, Corwin, and Quastel in their study of the KPZ stochastic PDE with narrow wedge initial conditions. We provide a Riemann-Hilbert (RH) approach to these solutions, and thus the study of their initial value problem is amenable by the Deift-Zhou asymptotic analysis of the RH problem.

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Poisson quasi-Nijenhuis manifolds and the Toda system

Author: Marco Pedroni[1]

[1] *Università di Bergamo*

The notion of Poisson quasi-Nijenhuis manifold, introduced by Stiénon and Xu, generalizes that of Poisson-Nijenhuis manifold. The relevance of the latter in the theory of completely integrable systems is well established since the birth of the bi-Hamiltonian approach to integrability. In this talk, we discuss the relevance of the notion of Poisson quasi-Nijenhuis manifold in the context of finite-dimensional integrable systems. Generically, the Poisson quasi-Nijenhuis structure is largely too general to ensure Liouville integrability of a system.
However, we present a general scheme connecting Poisson quasi-Nijenhuis and Poisson-Nijenhuis manifolds, and we give sufficient conditions such that the spectral invariants of the "quasi-Nijenhuis recursion operator" of a Poisson quasi-Nijenhuis manifold (obtained by deforming a Poisson-Nijenhuis structure) are in involution. Then we prove that the closed (or periodic) n-particle Toda lattice, along with its relation with the open (or non periodic) Toda system, can be framed in such a geometrical structure. These results have been obtained in collaboration with Gregorio Falqui, Igor Mencattini, and Giovanni Ortenzi.

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Algorithms for the large scale computation of Maximally-Mutable Laurent Polynomials

Author: Giuseppe Pitton[1]

[1] *Imperial College London*

Fano Polytopes are a family of integral lattice polytopes with important applications in Toric Geometry. Recent results in Mirror Symmetry [1] showed that it is possible to find deformation-equivalent families of Fano varieties by computing some Laurent polynomials, called Maximally-Mutable Laurent Polynomials [2], which are naturally associated to Fano Polytopes.
In this talk, I will illustrate the main challenges and the algorithms involved in the computation of Maximally-Mutable Laurent Polynomials for some families of Fano Polytopes in three dimensions.
I will also discuss the role of Machine Learning algorithms both for tuning the algorithm's parameters and for exploring the database of Maximally-Mutable Laurent Polynomials.
This is a joint work with Tom Coates and Alexander Kasprzyk.
[1] arXiv:1501.05334
[2] arXiv:1212.1785

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Fredholm determinants, exact solutions to the Kardar-Parisi-Zhang equation and integro-differential Painlevé equations.

Author: Alexandre Krajenbrink[1]

[1] *SISSA*

As Fredholm determinants are more and more frequent in the context of stochastic integrability, I discuss in this talk the existence of a common framework in many integrable systems where they appear. This consists in a hierarchy of equations, akin to the Zakharov-Shabat system, connecting

an integro-differential extension of the Painlevé II hierarchy, the finite-time solutions of the Kardar-Parisi-Zhang equation and multi-critical fermions at finite temperature.

The talk is based on the results of the paper arXiv:2008.01509.

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Numerical study of solutions to the Zakharov-Kuznetsov equation in two and three dimensions

Author: Nikola Stoilov[1]

[1] *Institut de Mathématiques de Bourgogne*

In this work we look at the behaviour of solutions to the Zakharov Kuznetsov (ZK) equations, using advanced numerical tools. ZK is a nonlinear dispersive PDE and can be seen as a generalisation of the KdV, however it is not integrable. We demonstrate how the behaviour of its solutions exhibits its dispersive PDE's nature and will look at blow-up, soliton resolution and soliton interaction and discuss how the non-integrability transpires in these cases. We propose several conjectures for the long term behaviour.
Based on joint works with Christian Klein and Svetlana Roudenko.

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Matrix generalization of the PII hierarchy: Lax pair and solutions in terms of Fredholm determinants.

Author: Sofia Tarricone[1]

[1] *Université d'Angers/Concordia University*

This talk is focused on some properties of a matrix generalization of the Painlev´e II hierarchy. In particular, we consider Fredholm determinants of matrix convolution operators associated to matrix versions of the n–th Airy functions. Using the theory of integrable operators, we relate them to a fully noncommutative Painlev´e II hierarchy, defined through a matrix valued version of the Lenard operators. In particular we show how to find a matrix valued Lax pair of this hierarchy and, as by product, how to obtain solutions in terms of the Fredholm determinants above. (https://arxiv.org/abs/2007.05707)

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On the mean Density of States of some matrices related to the beta ensembles and an application to the Toda lattice

Author: Guido Mazzuca[1]

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In this talk we will introduce what we called α ensembles, i.e. some random matrix models related to the classical β ensembles. In particular we will prove the convergence of their empirical spectral distributions to their mean densities of states and we compute them explicitly. As an application we explicitly compute the mean density of states of the Lax matrix of the Toda lattice with periodic boundary conditions with respect to the Gibbs ensemble. The talk is based on https://arxiv.org/abs/2008.04604

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Painlevé equations, anharmonic oscillators and the Shapiro-Tater conjecture

Author: Victor Eduardo Chavez-Heredia[1]

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An intriguing conjecture by Shapiro and Tater (2014) relates symptotically the poles of algebraic solutions of Painlevé 2 and the degenerate spectrum of an eigenvalue problem for a 2nd oder ODE with quartic potential. In this talk we discuss the link between the Shapiro-Tater conjecture and the isomonodromic formulation of P2 and our work towards proving the conjecture using the exact WKB method. This talk is based on ongoing research.

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Sorting networks, staircase Young tableaux and last passage percolation

Author: Fabio Cunden[1]

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I will present new distributional identities relating three random processes: the oriented swap process on n particles, the corner growth process, and the last passage percolation model. The study of these identities leads to interesting combinatorics and involves tools such as the RSK, Burge, and Edelman-Greene correspondences. One of the identity provides precise finite-n and asymptotic predictions on the distribution of the absorbing time of the oriented swap process.
This talk is based on joint works with Elia Bisi, Shane Gibbons and Dan Romik (arXiv:2003.03331, 2005.02043).

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Correlators of classical invariant unitary ensemble and enumerative problems

Author: Massimo Gisonni[1]

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In the early eighties, Bessis, Itzykson and Zuber showed how correlators of the Gaussian Unitary Ensemble (GUE) are related to the counting problem of ribbon graphs. However, their explicit computation was given only thirty years later by Dubrovin and Yang. More recently, it was proven that correlators of the Laguerre Unitary Ensemble (LUE) are likewise related to the counting problem of double weighted monotone Hurwitz numbers.
In this seminar we will show how a generating function for these objects can be obtained starting from the RHP of the associated orthogonal polynomials. We will also discuss how the Laguerre Partition function serves as a generating function for Cubic Hodge Integrals, and further directions of research. This talk is based on joint work with T.Grava and G.Ruzza (https://arxiv.org/abs/1912.00525).

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On the asymptotic analysis of bordered Toeplitz determinants

Author: Alexander Its[1]

[1] *Indiana-Purdue University*

Starting from the seminal works of Szego, Kaufman and Onsager, on the diagonal and row correlations in the Ising model, the Toeplitz determinants have been playing a very important role in many areas of analysis and mathematical physics. A growing interest has been recently developed to the study of certain generalizations of Toeplitz determinants. One of these generalizations, the so called "bordered Toeplitz determinants" is the topic of this talk.
For the first time, the bordered Toeplitz determinants appeared in 1987 work of Au-Yang and Perk where it was shown that the Ising next-to-diagonal correlation functions are described by a bordered Toeplitz determinant. The analysis of this determinant had been continued in 2007 paper of Witte where an important fact - the relation of the bordered Toeplitz determinant of Au-Yang and Perk to the orthogonal polynomials on the circle, had been established. This, in particular, had opened a possibility to use the Riemann-Hilbert method for the analysis of the bordered determinants. However, the asymptotic results similar to the classical Szego and Fisher-Hartwig asymptotic theory of "pure" Toeplitz determinants have not yet been known.
In this talk, a counterpart of the Strong Szeg˝o theorem for a wide class of the bordered Toeplitz determi- nants will be presented and discussed. This is a joint project with Estelle Basor, Torsten

Ehrhardt, Roozbeh Gharakhloo, and Yuqi Li. In our analysis we use the two complementary approaches - the Riemann-Hilbert and the Operators methods. In the talk, the emphasis will be made on the Riemann-HIlbert technique as directly connected to the Inetgrable systems which is the topic of the conference.

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Regularity of the free boundary for the two-phase Bernoulli problem

Author: Guido De Philippis[1]

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I will illustrate a recent result obtained in collaboration with L. Spolaor and B. Velichkov concerning the regularity of the free boundaries in the two phase Bernoulli problems. The new main point is the analysis of the free boundary close to branch points, where we show that it is given by the union of two C^1 graphs. This complete the analysis started by Alt Caffarelli Friedman in the 80's.

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Differential Geometry of Orbit space of Extended Affine Jacobi Group A_1\$

Author: Guilherme Almeida[None]

Dubrovin Frobenius manifolds are the geometric interpretation of a remarkable system of differential equations, called WDVV equations. Since the beginning of the ninethies, there has been a continuous exchange of ideas from fields that are not trivially related to each other, such as: string theory, non-linear waves, singularity theory, reflection groups and its extensions, random matrices theory, integrable systems, and Painleve equations. Dubrovin Frobenius manifolds theory has demonstrated to be thebridge between them.
Orbit space of reflection groups and its extensions are one of the main examples of Dubrovin Frobenius manifolds. In this talk, we define certain extensions of Jacobi groups of A1, prove an analogue of Chevalley Theorem for their invariants, and construct a Dubrovin Frobenius structure on it orbit spaces. This seminar is based on the results of arXiv:1907.01436v3, and arXiv:2004.01780v1.

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On the Dam-Break problem

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The Dam-break problem is the study of the evolution of a fluid partially filling a channel under the effects of the gravity. We present the analysis of the consequences of the initial interface regularity on the expansion process.

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Vacuum states in hydrodynamic models

Author: Roberto Camassa[None]

Vacuum or “dry” states, whereby density-like variables of hydrodynamic models vanish on sets of initial conditions, can cause development of finite time singularities in the evolution of model solutions. This loss of regularity in the ensuing evolution poses challenges from both the analytical and numerical standpoint.
This talk will discuss a selected class of examples where a finite-time singularity can be “unfolded” in appropriately defined variables, and how the local and global evolution of solutions, in either weak or strong form, can thus be recovered and continued past the singularity time.