

Determinantal point processes: quasi-symmetries and interpolation

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For the sine-process, it is proved that almost every realization with one particle removed is a complete and minimal set for the Paley-Wiener space, whereas if two particles are removed, then one obtains a zero set for the Paley-Wiener space. In joint work with Qiu and Shamov, it is proved that almost every realization of a determinantal point process is a uniqueness set for the underlying Hilbert space.

To every realization of the sine-process we assign a random entire function, an infinite product with zeros at the particles of our configuration, the scaling limit of the ratio of two values of the characteristic polynomial of a random matrix.

In order to study the behaviour of the logarithm of this product, we introduce a Gaussian field under which the variance of the random variable assigned to a given function is proportional to the square of its Sobolev $1/2$ -norm. More specifically now, we consider the Gaussian process, indexed by point in the upper half-plane, of Cauchy transforms of our configuration, as well as the process of anti-derivatives of the Cauchy transforms: precisely, the differences of the logarithms occurring in the logarithm of our random entire function. The key rôle is played by the remark that the Gaussian process formed by the antiderivatives of the Cauchy transforms is invariant under the Lobachevskian isometries of the upper half-plane. The argument then proceeds by taking the scaling limit in the Borodin-Okounkov-Geronimo-Case formula and obtaining an analogue of the Johansson change of variable formula for the sine-process, using the quasi-invariance of the sine-process under the group of diffeomorphisms with compact support.

In joint work with Qiu, the Patterson-Sullivan construction is used to interpolate Bergman functions from a realization of the determinantal point process with the Bergman kernel, in other words, by the Peres-Virág theorem, the zero set of a random series with independent complex Gaussian entries. The invariance of the zero set under the isometries of the Lobachevsky plane again plays a key rôle.

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