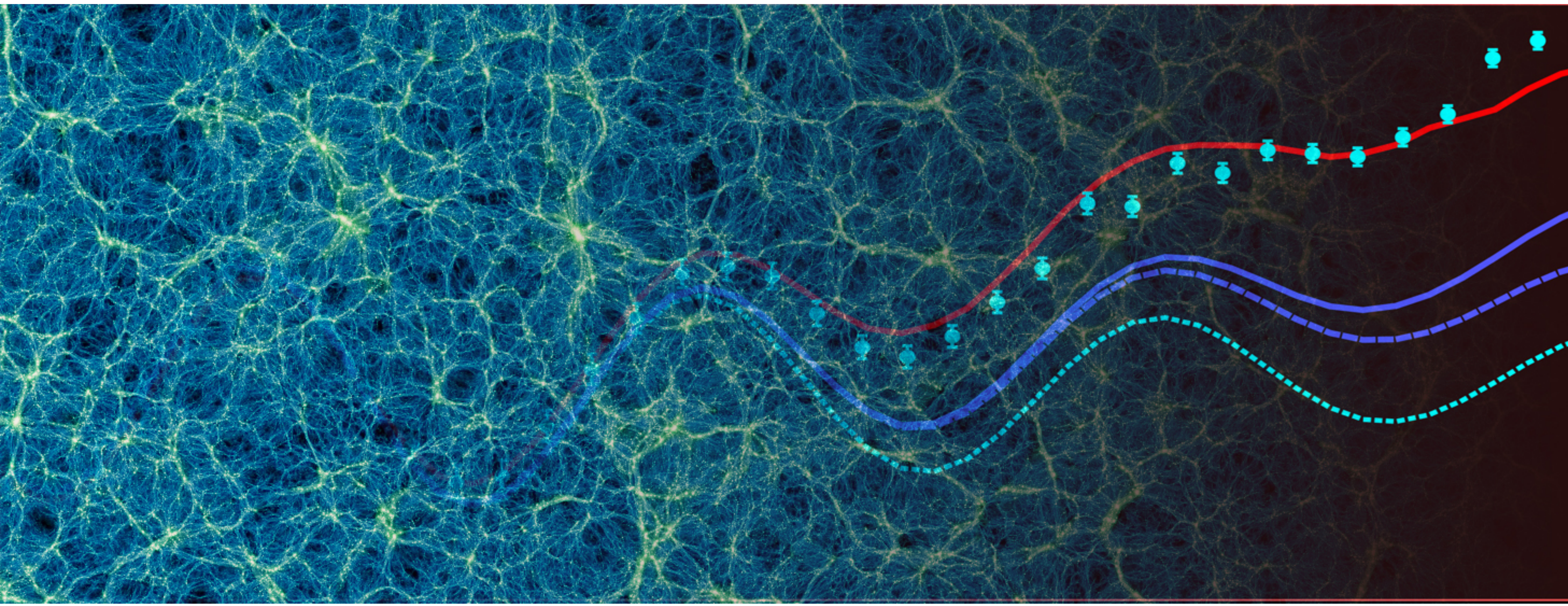
Cosmological Constraints from Spectroscopic Surveys

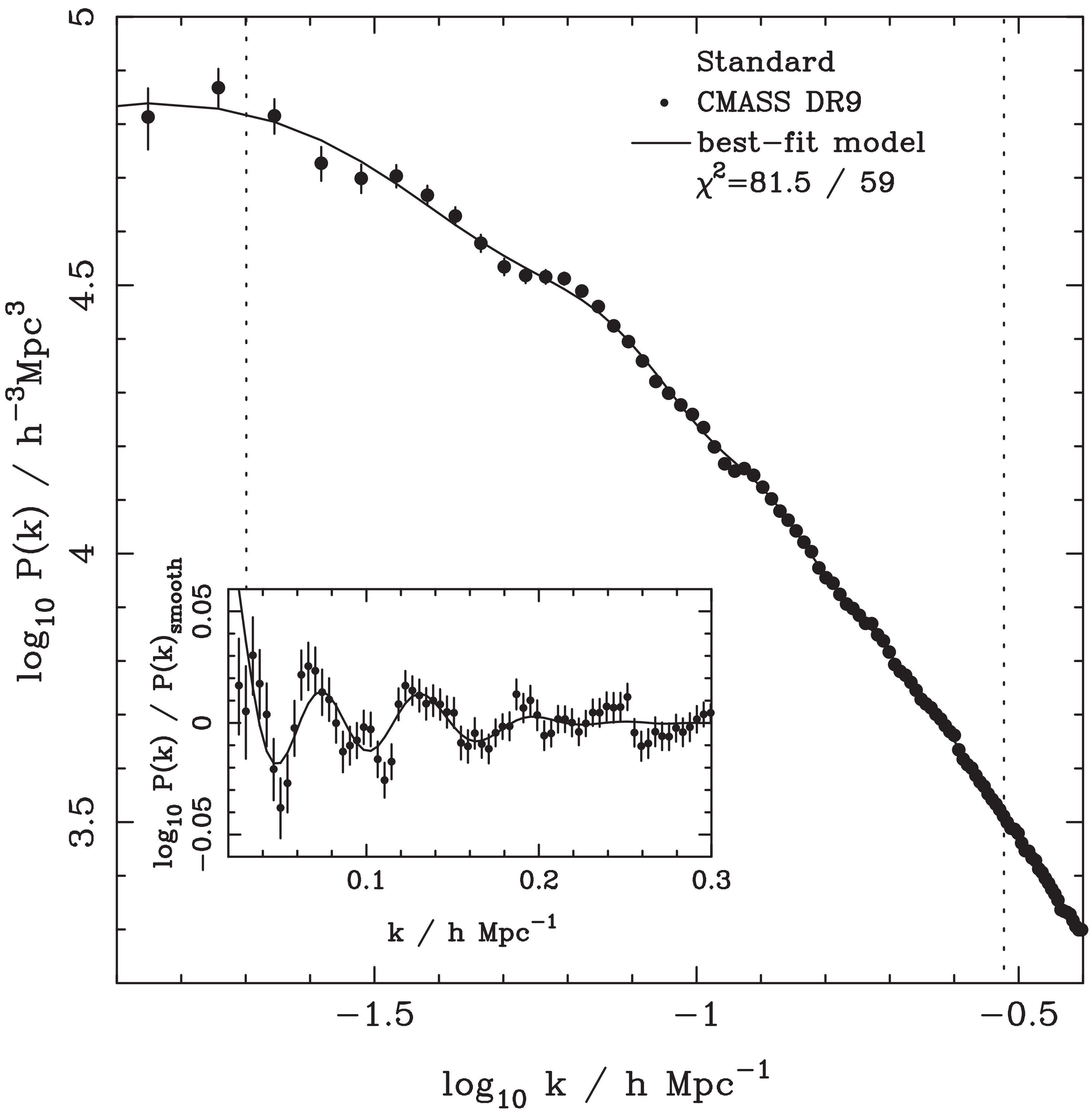
(Intro to Large-Scale Structure in 1 hour or so)



Emiliano Sefusatti
Astronomical Observatory of Trieste

The goal

The galaxy
power spectrum:
what is it and
why it matters



Outlook

Homogeneous cosmology

Density perturbations

The growth of matter perturbations

The power spectrum

Baryonic Acoustic Oscillations,
Redshift-space Distortions &
Full-Shape Analysis

The Homogeneous Universe

A metric for the Universe

We *assume* **homogeneity** and **isotropy**:
the **Friedmann-Lemaître-Robertson-Walker metric**

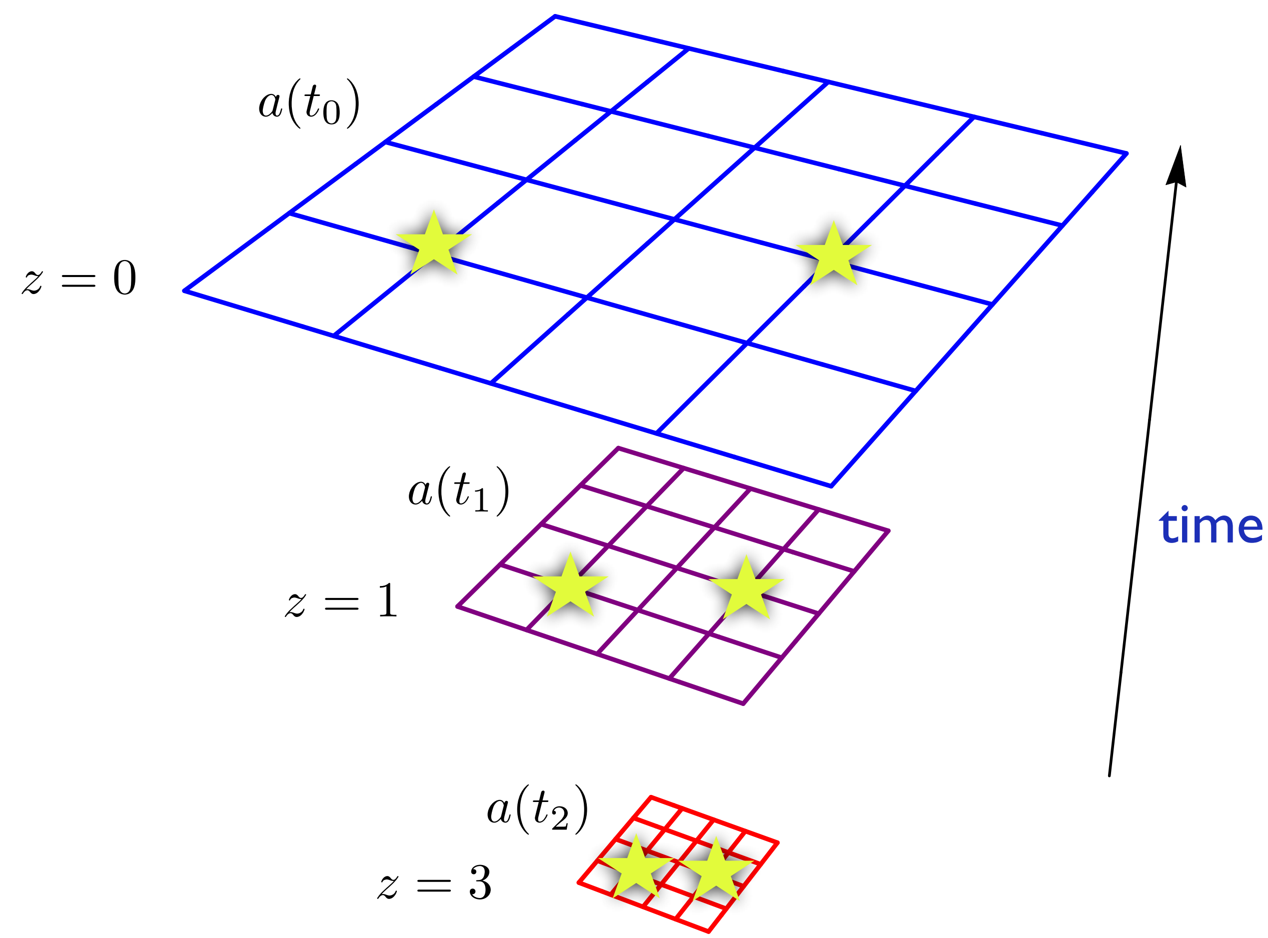
$$ds^2 = c^2 dt^2 - a^2(t) \left[\frac{dx^2}{1 - kx^2} + x^2 \left(d\theta^2 + \sin^2\theta \, d\phi^2 \right) \right]$$

scale factor

coordinates:

$$\vec{r} = a(t)\, \vec{x}$$

physical comoving



If the two points have constant *comoving* coordinates

$$\frac{dr}{dt} = \dot{a}\, x = \frac{\dot{a}}{a}\, a\, x \equiv H(t)\, r$$

Hubble's law

A metric for the Universe

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scale factor

the Universe is not static!

Einstein's equations reduce to **Friedmann's equations** for the scale factor

$$H^2(a) \equiv \left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

Given the **fluid equation of state**

$$\rho = w\, p$$

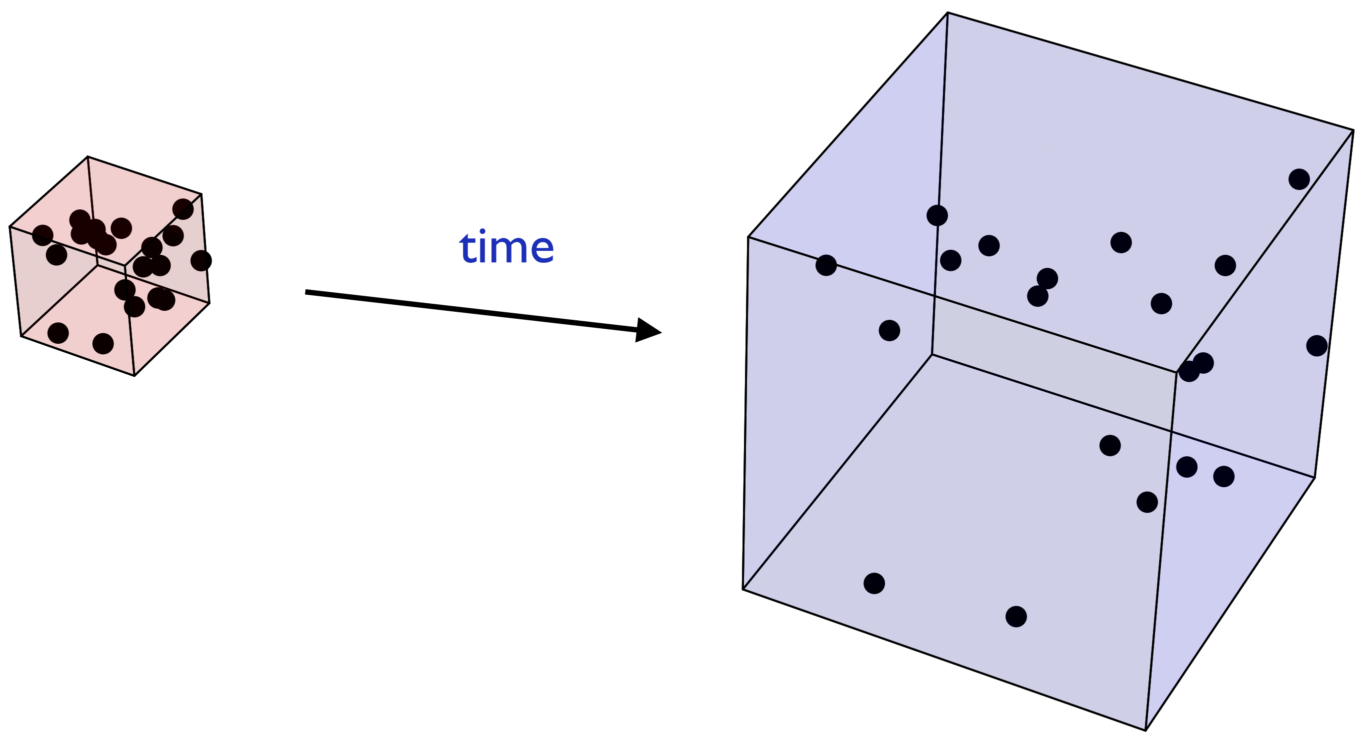
we can find the dependence of the density on the scale

$$\rho(a) \sim a^{-3(1+w)}$$

and finally solve for the evolution of the scale factor …

What fluid?

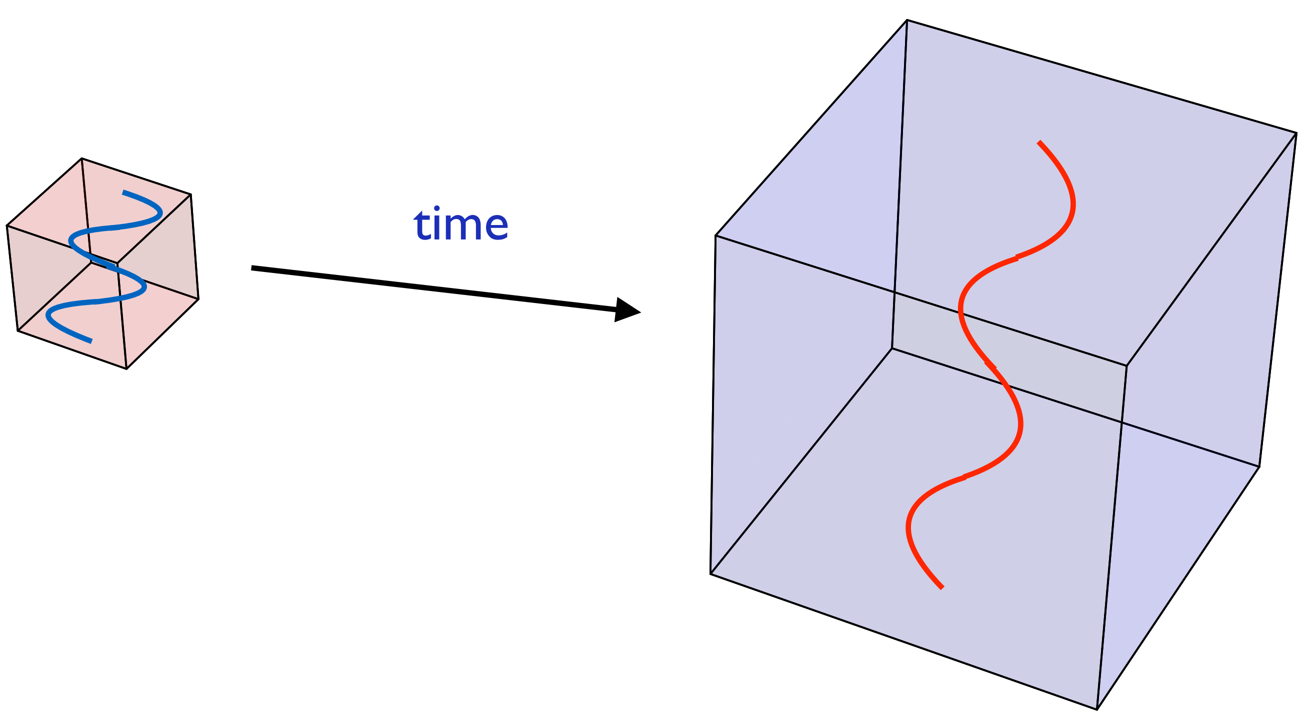
Matter: baryons and *dark matter*



$$\boxed{\rho_m \sim \frac{1}{a^3}}$$

(conservation of matter)

Radiation



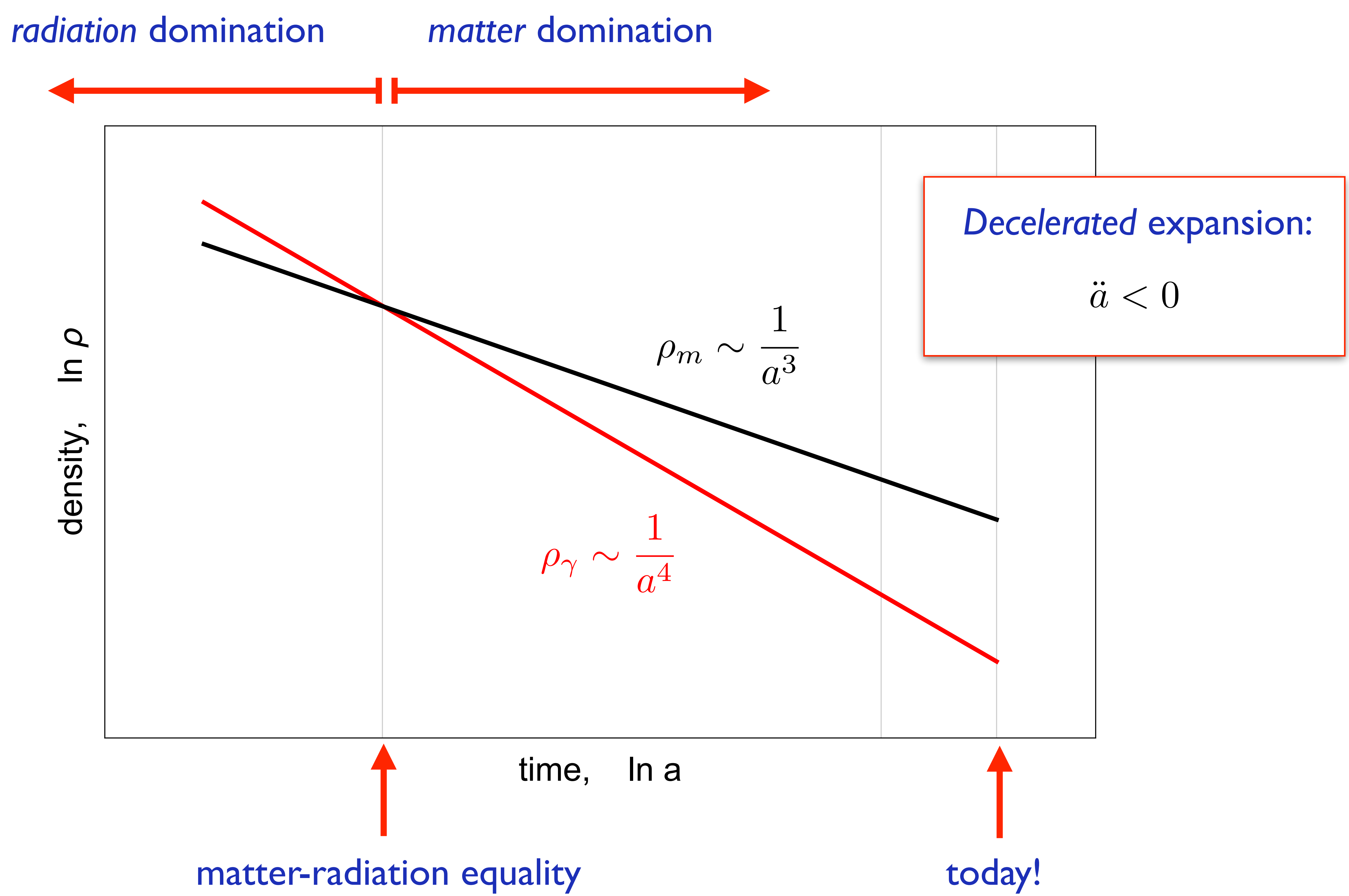
$$\boxed{\rho_\gamma \sim \frac{1}{a^4}}$$

In addition to the volume expansion, the energy of each photon decreases as

$$\epsilon_\gamma = \frac{h}{\lambda} \sim \frac{1}{a}$$

The energy density of the Universe

Extrapolating back in time …

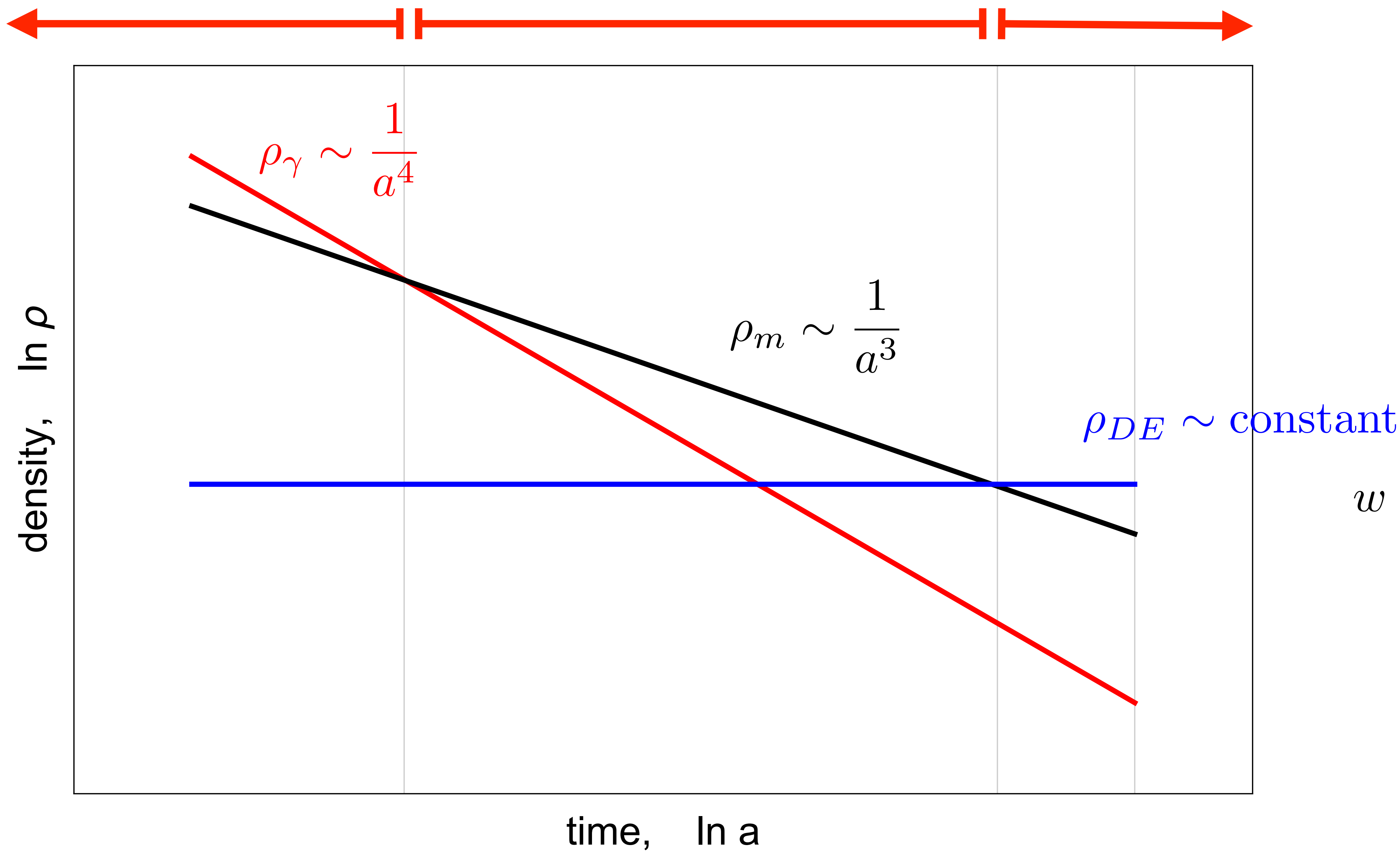


The energy density of the Universe

An accelerated expansion?

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) > 0 \qquad p = w\,\rho \qquad w < -\frac{1}{3}$$

radiation domination   *matter* domination   *dark energy* domination

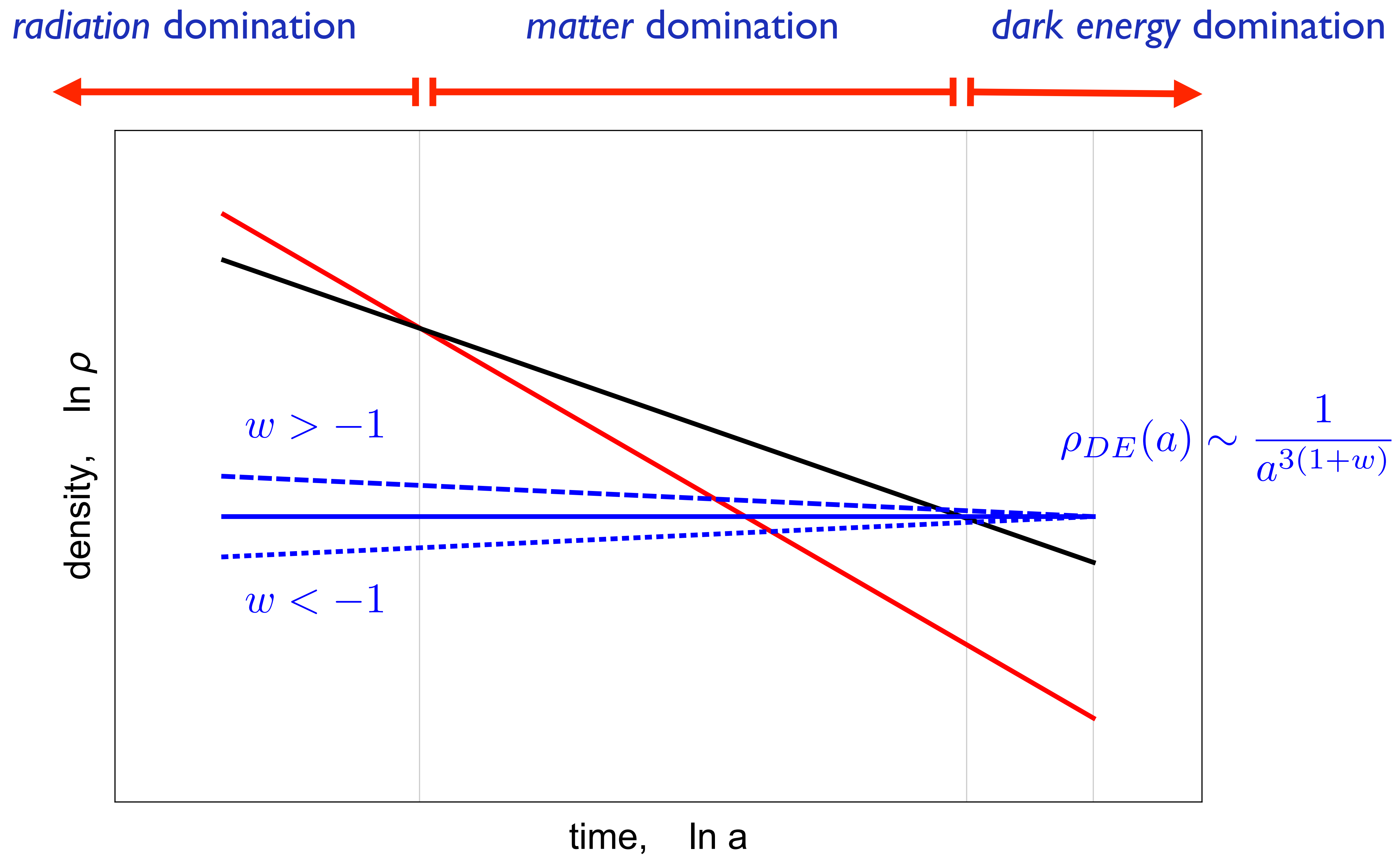


$w \simeq -1$

Cosmological constant?

The energy density of the Universe

An accelerated expansion?



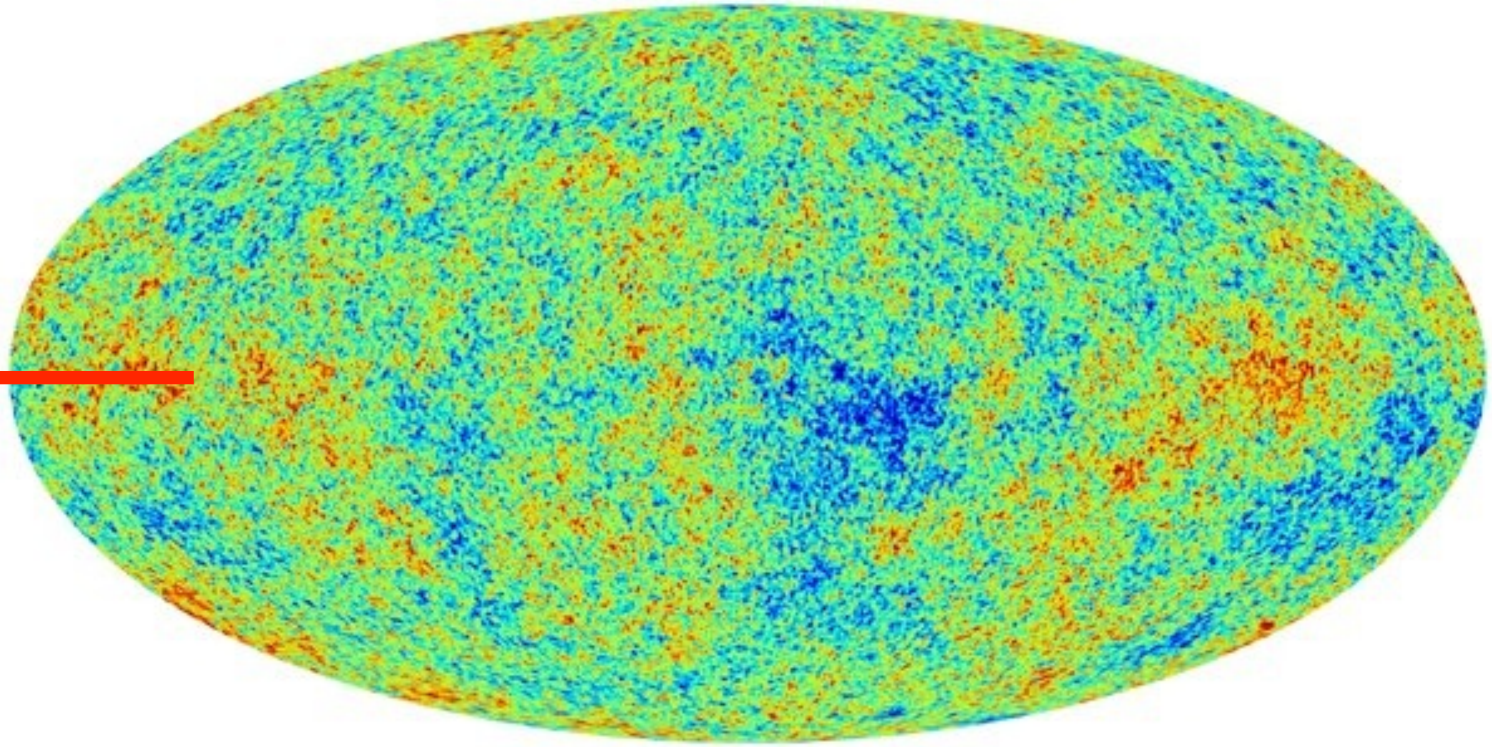
Quintessence? Modified gravity? …

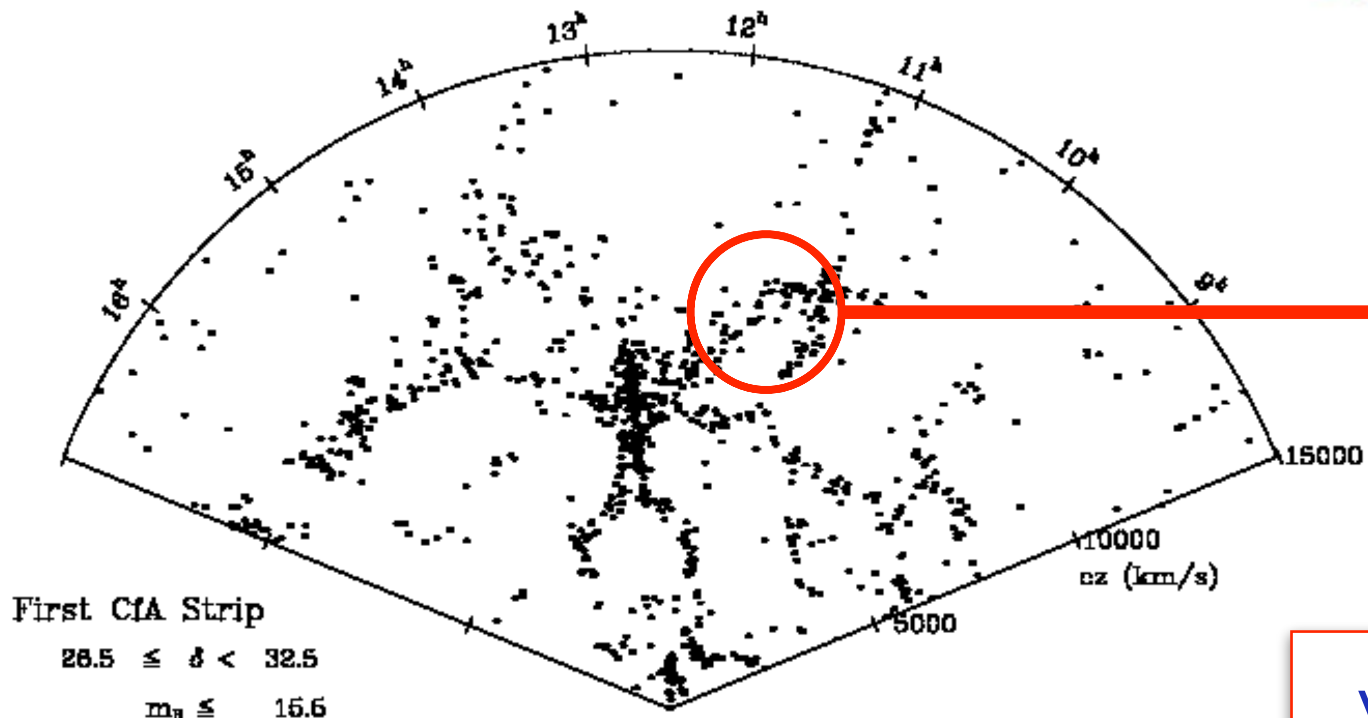
The Perturbed Universe

Cosmological perturbations

CMB temperature fluctuations

$T(\hat{n})$





$n_g(\vec{x})$

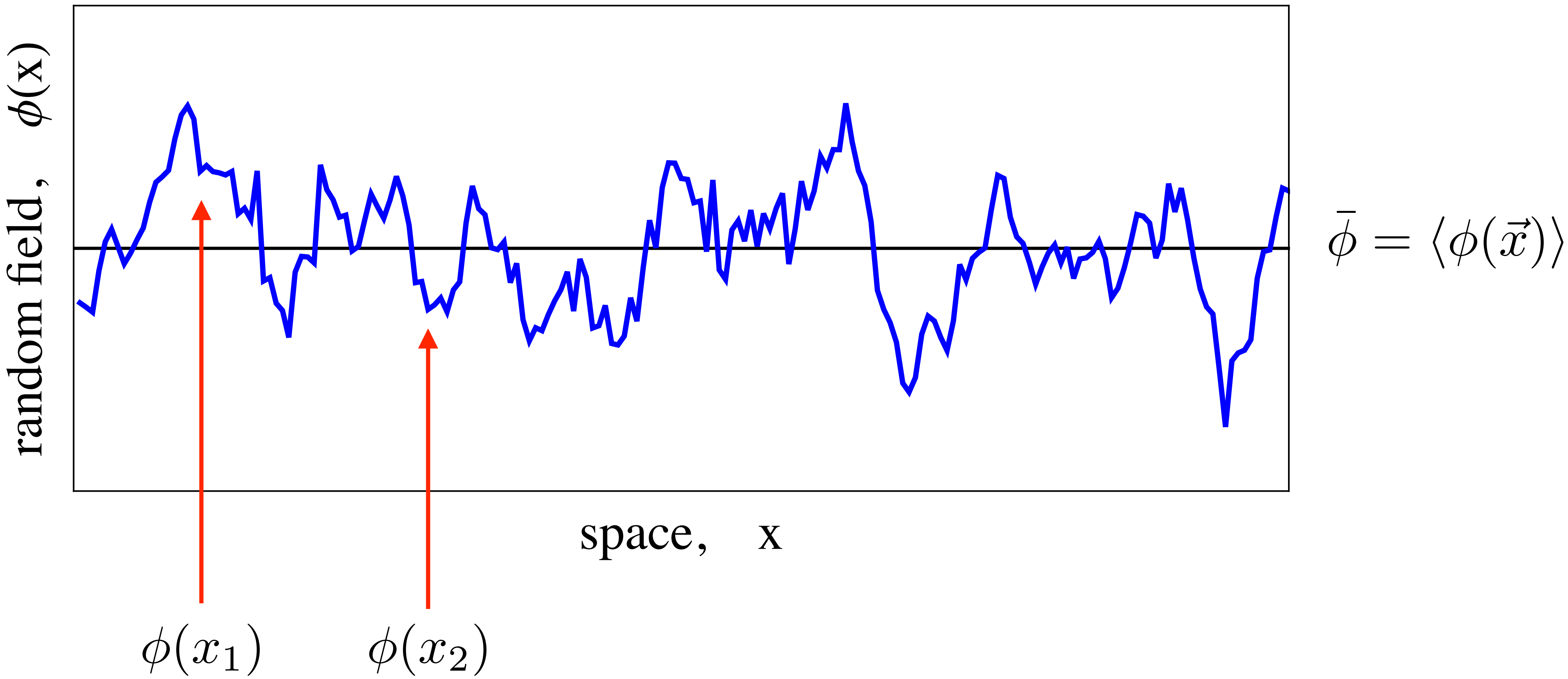
number density of galaxies

We study the **statistical properties** of cosmological perturbations

Mathematically, these are **random fields**

Random fields

If $\phi(\vec{x})$ is a **random *field*** we can compute **correlation functions**



two-point function $\quad \langle \phi(x_1)\phi(x_2) \rangle = \langle \phi(x_1) \rangle \langle \phi(x_2) \rangle + \langle \phi(x_1)\phi(x_2) \rangle_c$

three-point function $\quad \langle \phi(x_1)\phi(x_2)\phi(x_3) \rangle = \langle \phi(x_1) \rangle \langle \phi(x_2) \rangle \langle \phi(x_3) \rangle +$

$$+\langle \phi(x_1)\phi(x_2) \rangle_c \langle \phi(x_3) \rangle + \text{perm.} +$$

$$+\langle \phi(x_1)\phi(x_2)\phi(x_3) \rangle_c$$

...

n-point function $\quad \langle \phi(x_1)\phi(x_2)\ldots\phi(x_n) \rangle$

The distribution of galaxies in the Universe

The **galaxy number density** and its

$$n_g(\vec{x}) \equiv \bar{n}_g \left[1 + \delta_g(\vec{x})\right] \qquad \delta_g(\vec{x}) \equiv \frac{n_g(\vec{x}) - \bar{n}_g}{\bar{n}_g}$$

mean galaxy number density

galaxy **overdensity**
or density contrast

N.B. $\langle \delta_g(\vec{x}) \rangle \equiv 0$

$\delta_g(\vec{x}) \geq -1$

Similarly, for the **matter density** we have

$$\rho(\vec{x}, t) = \bar{\rho}(t) \left[1 + \delta(\vec{x}, t)\right] \qquad \delta(\vec{x}, t) \equiv \frac{\rho(\vec{x}, t) - \bar{\rho}(t)}{\bar{\rho}(t)}$$

mean matter density

matter **overdensity**

The galaxy two-point correlation function

What is the probability of finding two galaxies in the volume elements dV_1 and dV_2 ?

$$dP = dV_1\, dV_2\, \langle\, n_g(\vec{x}_1)\, n_g(\vec{x}_2)\, \rangle$$

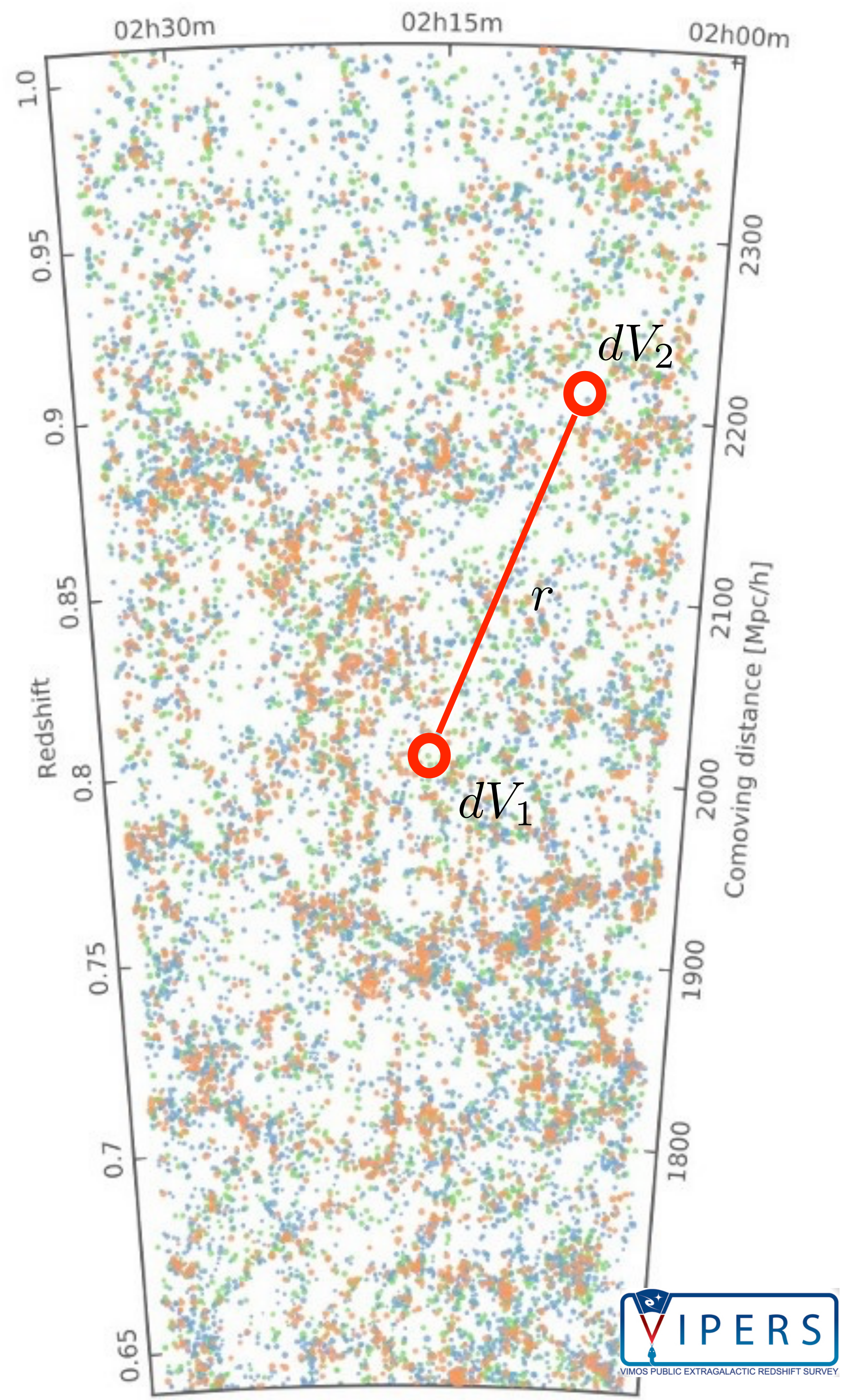
$$= dV_1\, dV_2\, \bar{n}_g^2\, [1 + \langle\, \delta_g(\vec{x}_1)\, \delta_g(\vec{x}_2)\, \rangle]$$

excess probability

We now make the assumption of ***statistical* homogeneity and isotropy**

$$\xi(|\vec{x}_1 - \vec{x}_2|) \;\equiv\; \langle\, \delta_g(\vec{x}_1)\, \delta_g(\vec{x}_2)\, \rangle$$

the two-point correlation function $\xi(r)$ *only depends on the distance* $r = |\vec{x}_1 - \vec{x}_2|$ *between the two points*



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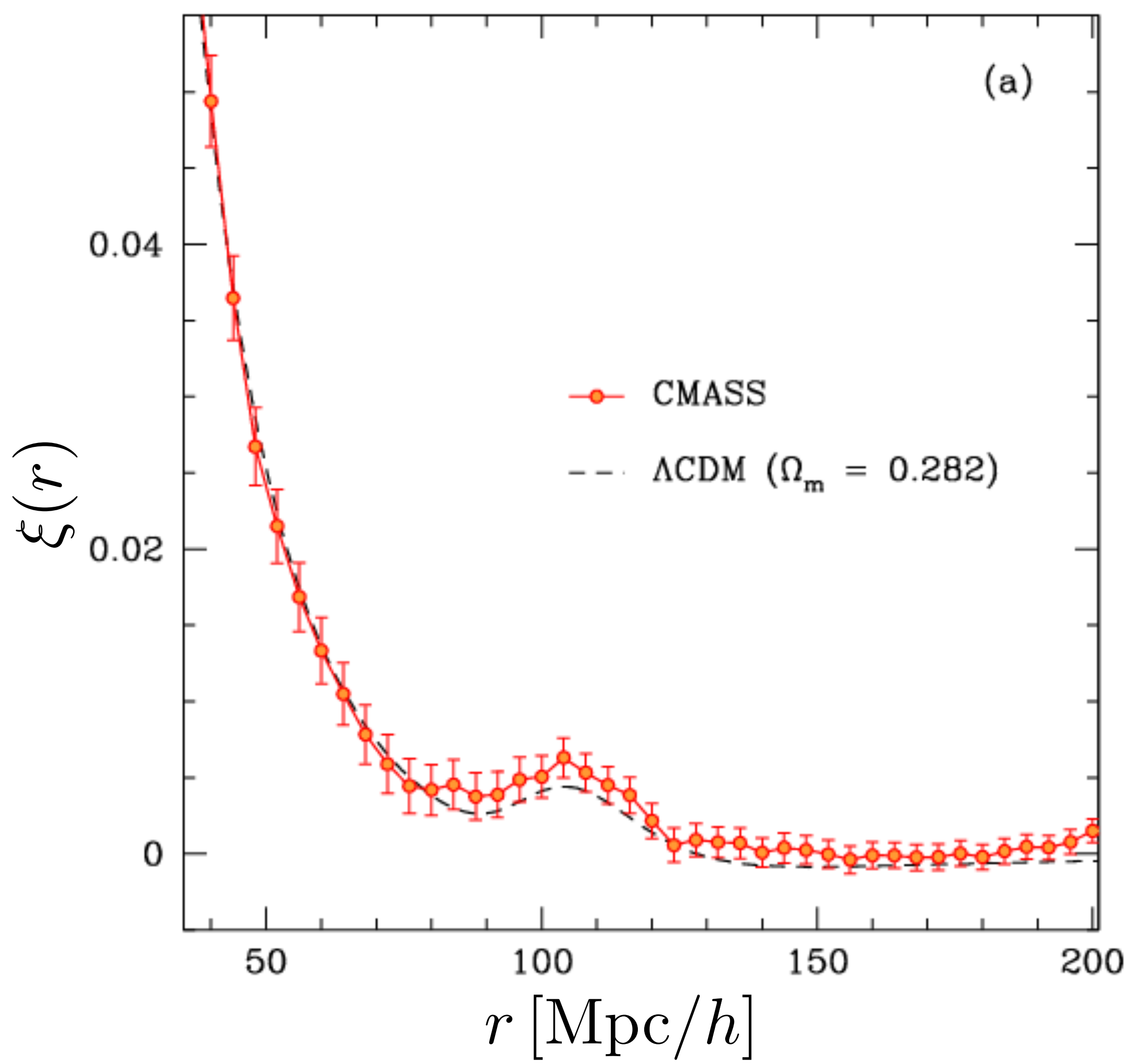
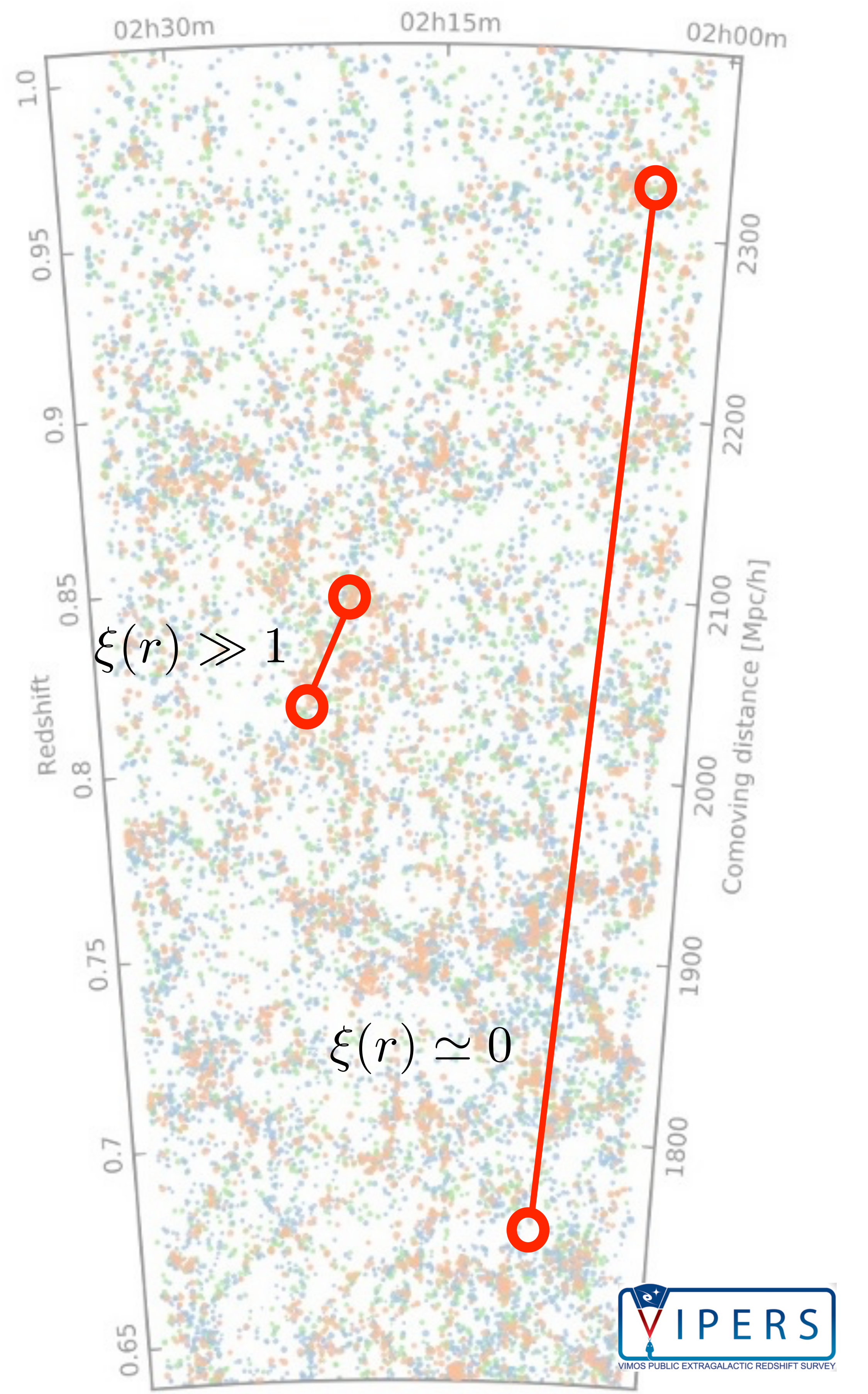
excess probability





The galaxy three-point correlation function

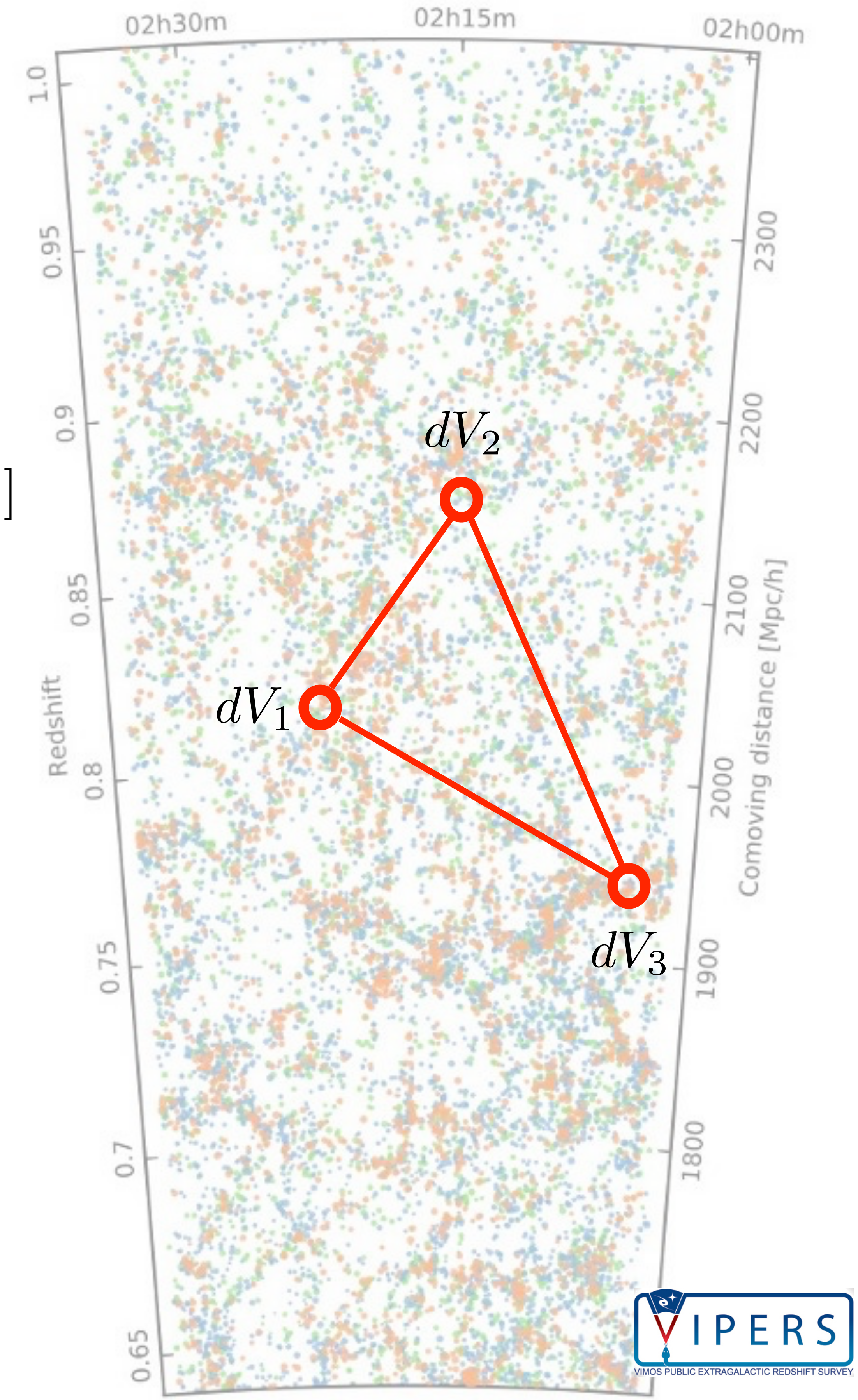
Similarly I can ask the probability of finding three galaxies in the volume elements dV_1 , dV_2 and dV_3

$$dP = dV_1 dV_2 dV_3 \langle n_g(\vec{x}_1) n_g(\vec{x}_2) n_g(\vec{x}_3) \rangle$$
$$= dV_1\, dV_2\, dV_3\, \bar{n}_g^3\, [1 +$$
$$+ \xi(r_{12}) + \xi(r_{13}) + \xi(r_{23}) + \zeta(r_{12}, r_{13}, r_{23})]$$

excess probability

$$\zeta(r_{12}, r_{13}, r_{23}) \equiv \langle \delta_g(\vec{x}_1) \delta_g(\vec{x}_2) \delta_g(\vec{x}_3) \rangle$$

the 3-point correlation function represents the (excess) probability to find 3 galaxies forming a triangle of a given shape and size

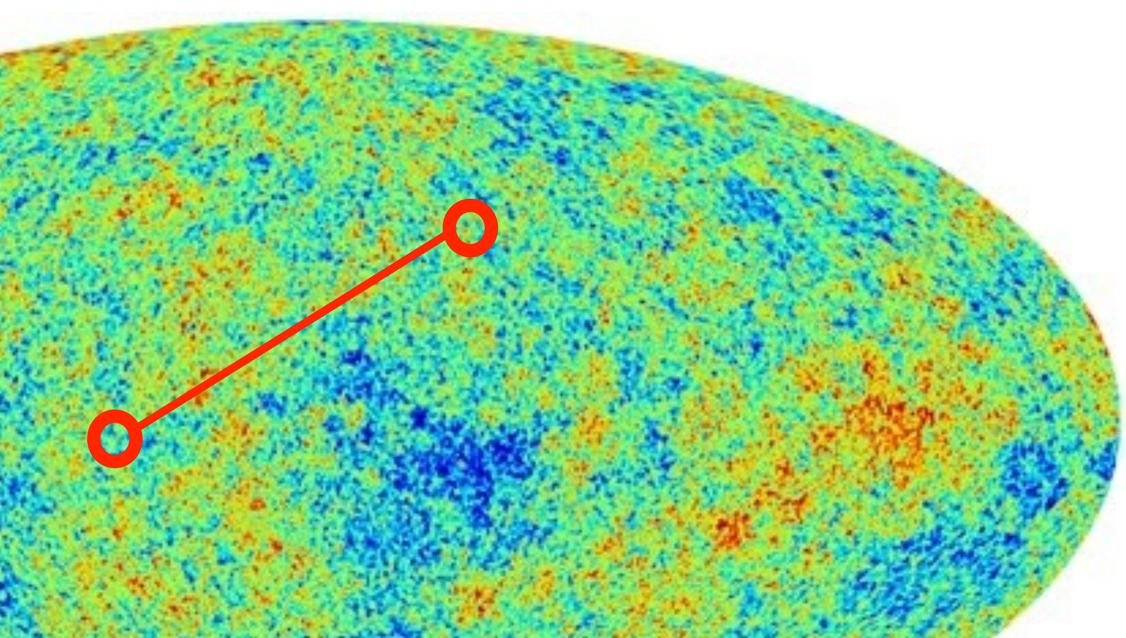


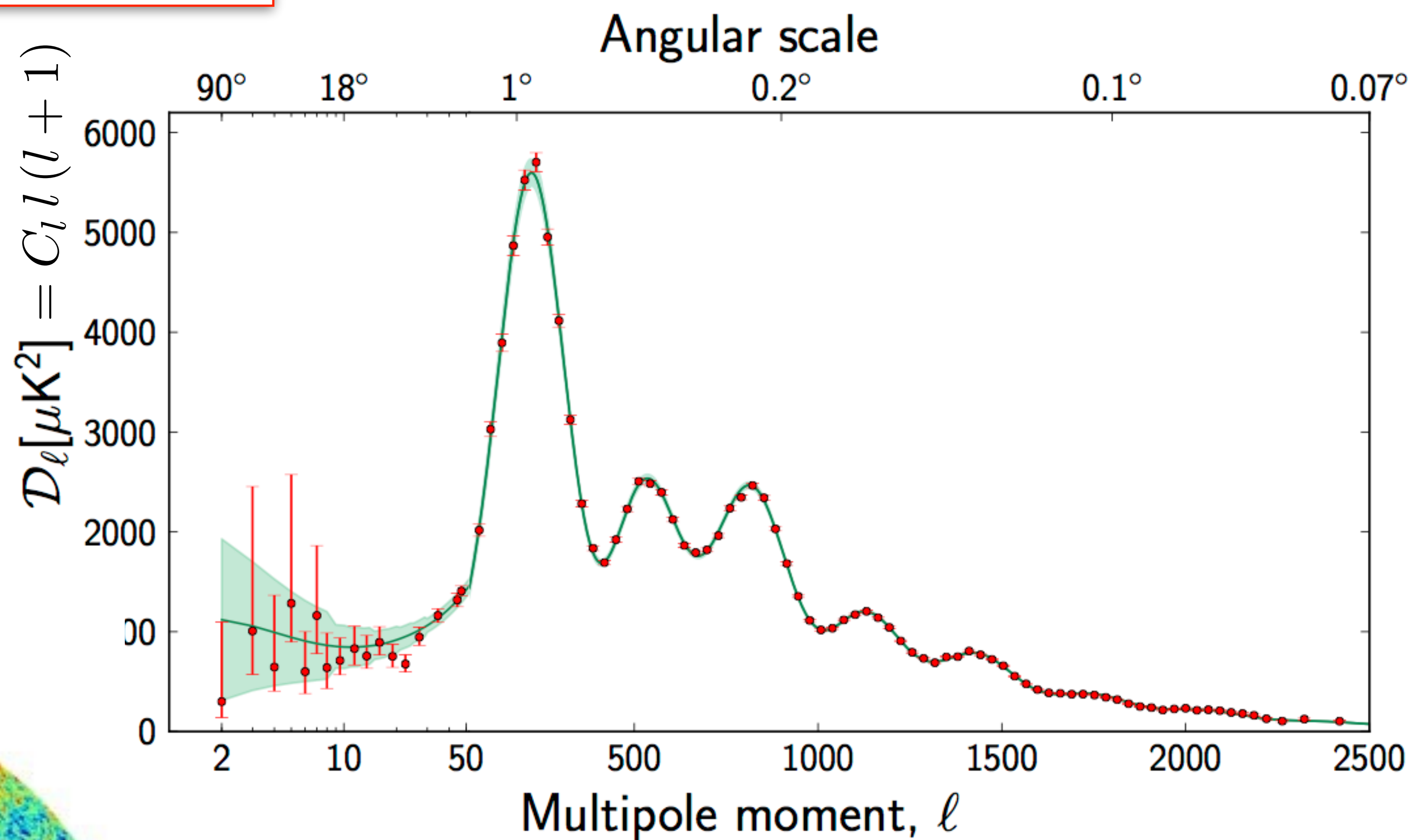
Gaussian and non-Gaussian random fields

The statistical properties of a **Gaussian random field** are completely characterised by its 2-point correlation function. All higher-order, *connected* correlation functions are vanishing

$$\delta_T(\hat{n}) \equiv \frac{T(\hat{n}) - \bar{T}}{\bar{T}}$$

$$\mathcal{P}[\delta_T(\hat{n})] = \frac{1}{\sqrt{2\pi\sigma_T^2}} e^{-\frac{1}{2}\frac{\delta_T^2}{\sigma_T^2}}$$



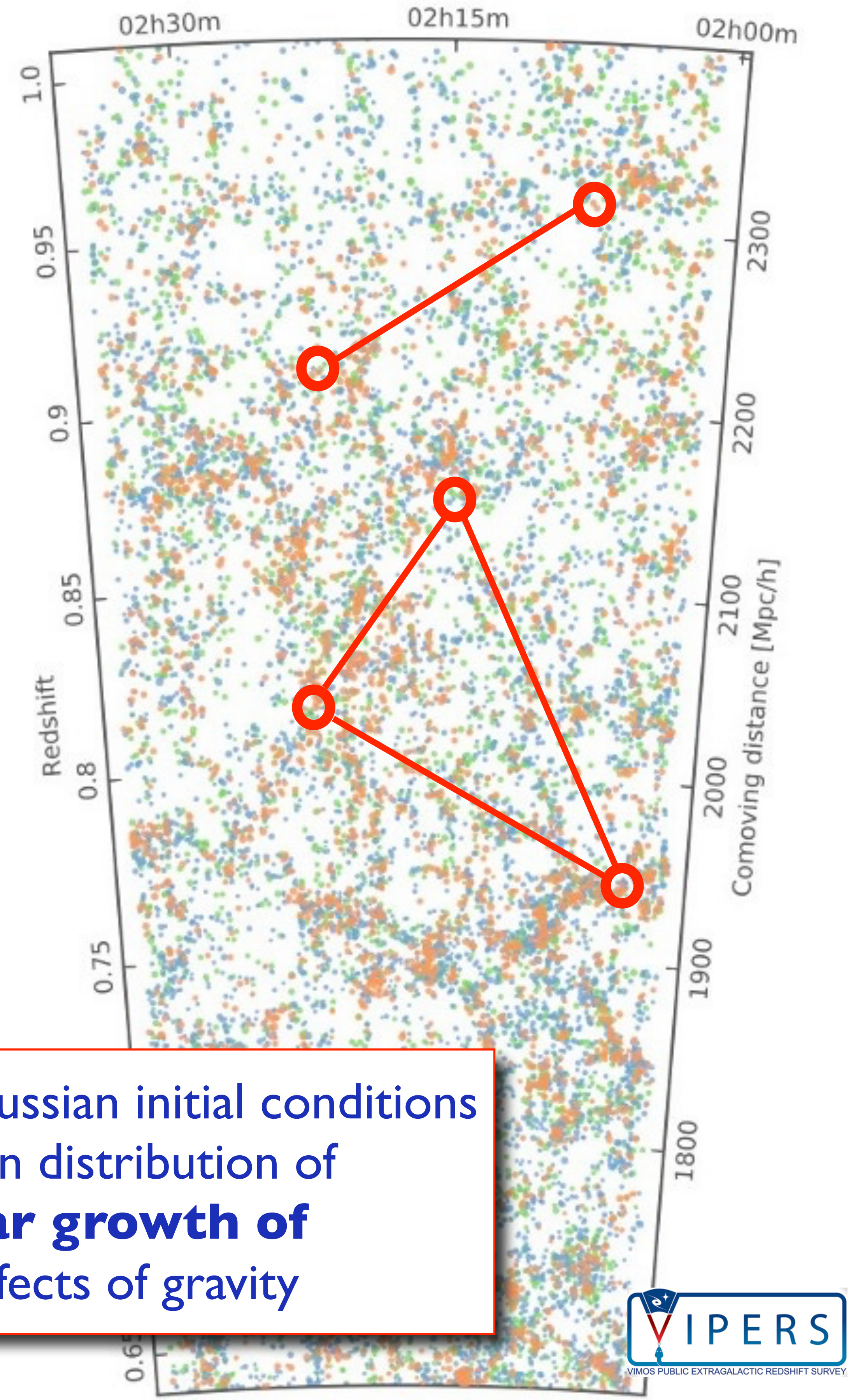


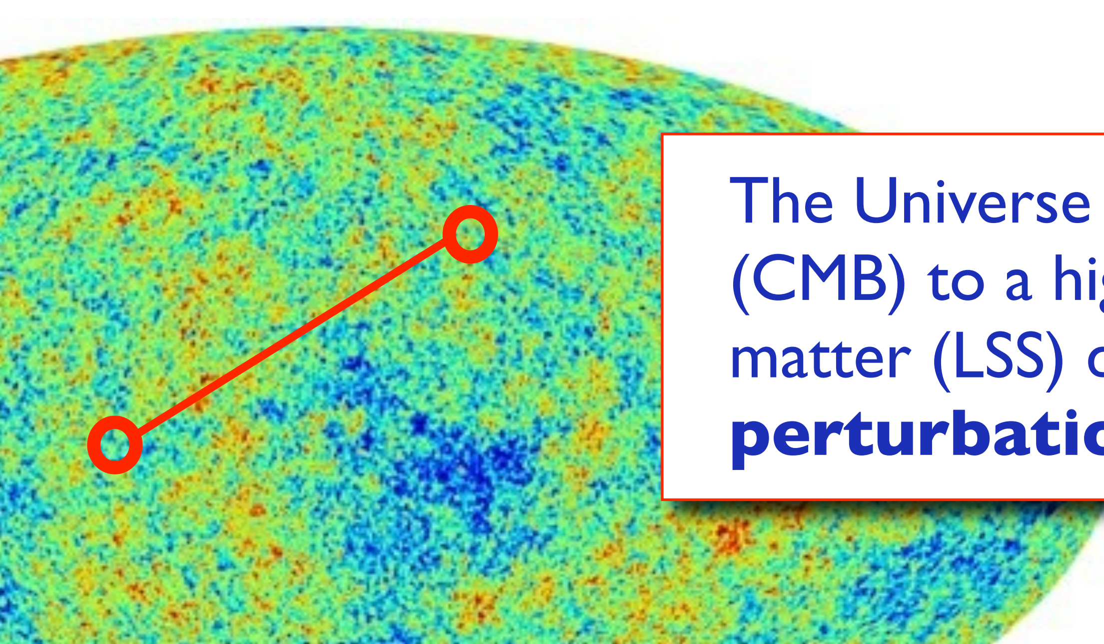
Perturbations in the CMB are a very good example of Gaussian random field!

Gaussian and non-Gaussian random fields

The statistical properties of a **Gaussian random field** are completely characterised by its 2-point correlation function. All higher-order, *connected* correlation functions are vanishing

random fields describing perturbations at low redshift, however, are typically *non-Gaussian*





The Universe evolves from Gaussian initial conditions (CMB) to a highly non-Gaussian distribution of matter (LSS) due to **nonlinear growth of perturbations** under the effects of gravity

Ergodic hypothesis and observations

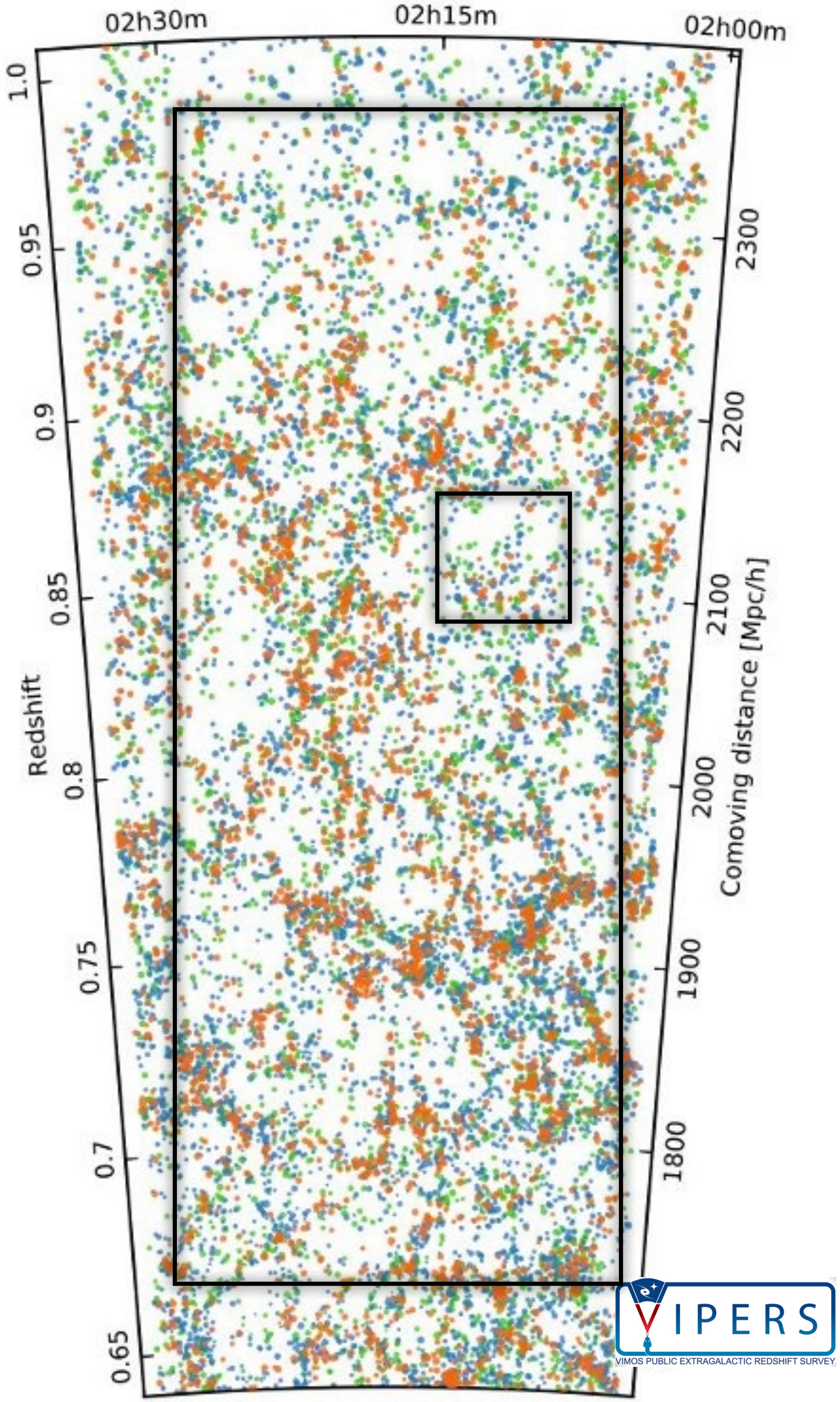
Expectation values, in principle, are to be intended as *ensemble averages*, i.e. averages over many "realisations of the Universe" …

… but we only have one Universe!

We have to assume the **ergodic hypothesis: ensemble averages are equal to spatial averages**

$$\langle \phi(\vec{x}) \rangle \equiv \int d\phi \, \phi \, \mathcal{P}(\phi) = \frac{1}{V} \int_V d^3x \, \phi(\vec{x})$$

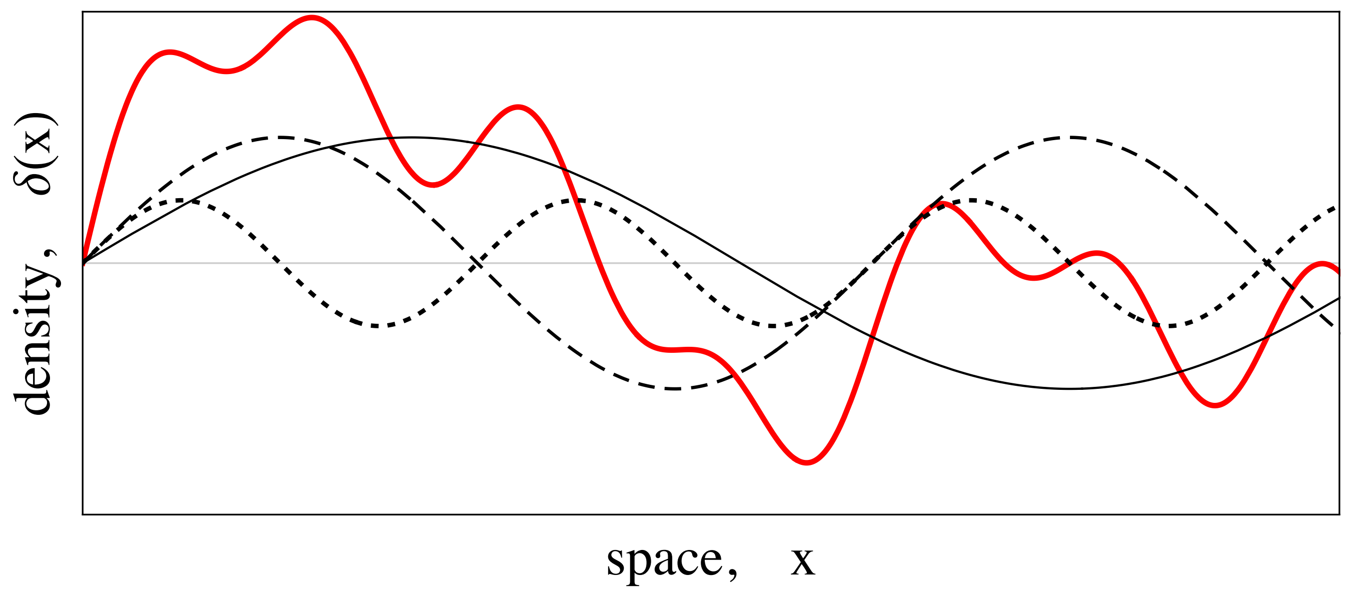
We should make sure, however, that the observed volume correspond to a "fair sample" of the Universe



Fourier space

Theoretical predictions for the matter correlation functions are performed in **Fourier space**

Fourier analysis naturally separates perturbations at different scales:



$$\delta_{\vec{k}} = \int \frac{d^3x}{(2\pi)^3} e^{-i\vec{k}\cdot\vec{x}} \delta(\vec{x})$$

$$\delta(\vec{x}) = \int d^3k e^{i\vec{k}\cdot\vec{x}} \delta_{\vec{k}}$$

- Since $\delta(\vec{x})$ is a random field $\delta_{\vec{k}}$ is also a random field
- Since $\delta(\vec{x})$ is real $\delta^*_{\vec{k}} = \delta_{-\vec{k}}$

The power spectrum

The **power spectrum** is the 2-point function in Fourier space

$$\langle\, \delta_{\vec{k}_1}\, \delta_{\vec{k}_2} \rangle = \delta_D(\vec{k}_1 + \vec{k}_2)\, P(k_1)$$

homogeneity & isotropy

The power spectrum is in fact the *Fourier Transform* of the 2-point correlation function

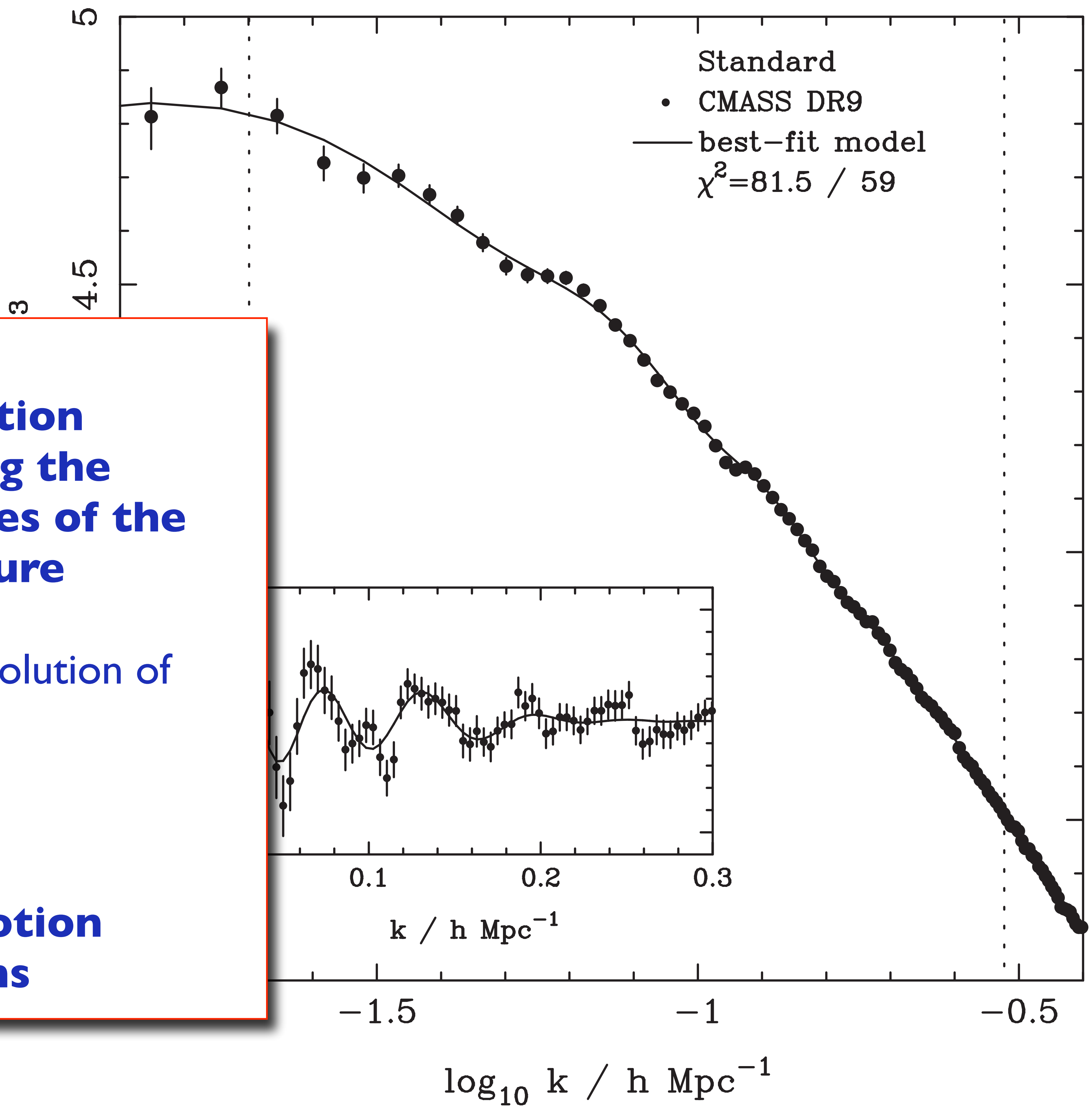
$$P(k) = \int \frac{d^3x}{(2\pi)^3}\, e^{i\,\vec{k}\cdot\vec{x}} \xi(x) \quad \longleftrightarrow \quad \xi(x) = \int d^3k\, e^{-i\vec{k}\cdot\vec{x}} P(k)$$

The power spectrum is a measure of the amplitude of perturbations *as a function of scale*

$$\sigma^2 \equiv \langle \delta^2 \rangle = 4\pi \int dk\, k^2 P(k) = \int \frac{dk}{k} \Delta(k)$$

$$\Delta(k) \equiv 4\pi k^3\, P(k)$$

The goal



Goal:
predict the correlation functions describing the statistical properties of the Large-Scale Structure

for this we study the evolution of

$$\delta_{\vec{k}}(t)$$

We need:
1. **Equations of motion**
2. **Initial conditions**

Evolution of matter perturbation: Initial Conditions

The "initial" matter power spectrum

Inflation predicts a (nearly) *scale-invariant initial power spectrum* for perturbations in the gravitational potential

$$\Delta_\Phi(k) \equiv 4\pi k^3 P_\Phi(k) \simeq \text{constant}$$

$$P_\Phi(k) \simeq \frac{C}{k^3}$$

$$\sigma_\Phi^2 = \int \frac{dk}{k} \Delta_\Phi(k)$$

The variance of perturbations in the gravitational potential receives equal contributions from perturbations at all scales
(Harrison-Zeldovich power spectrum)

Departures from this simple prediction provide *constraints on inflation*

The "initial" matter power spectrum

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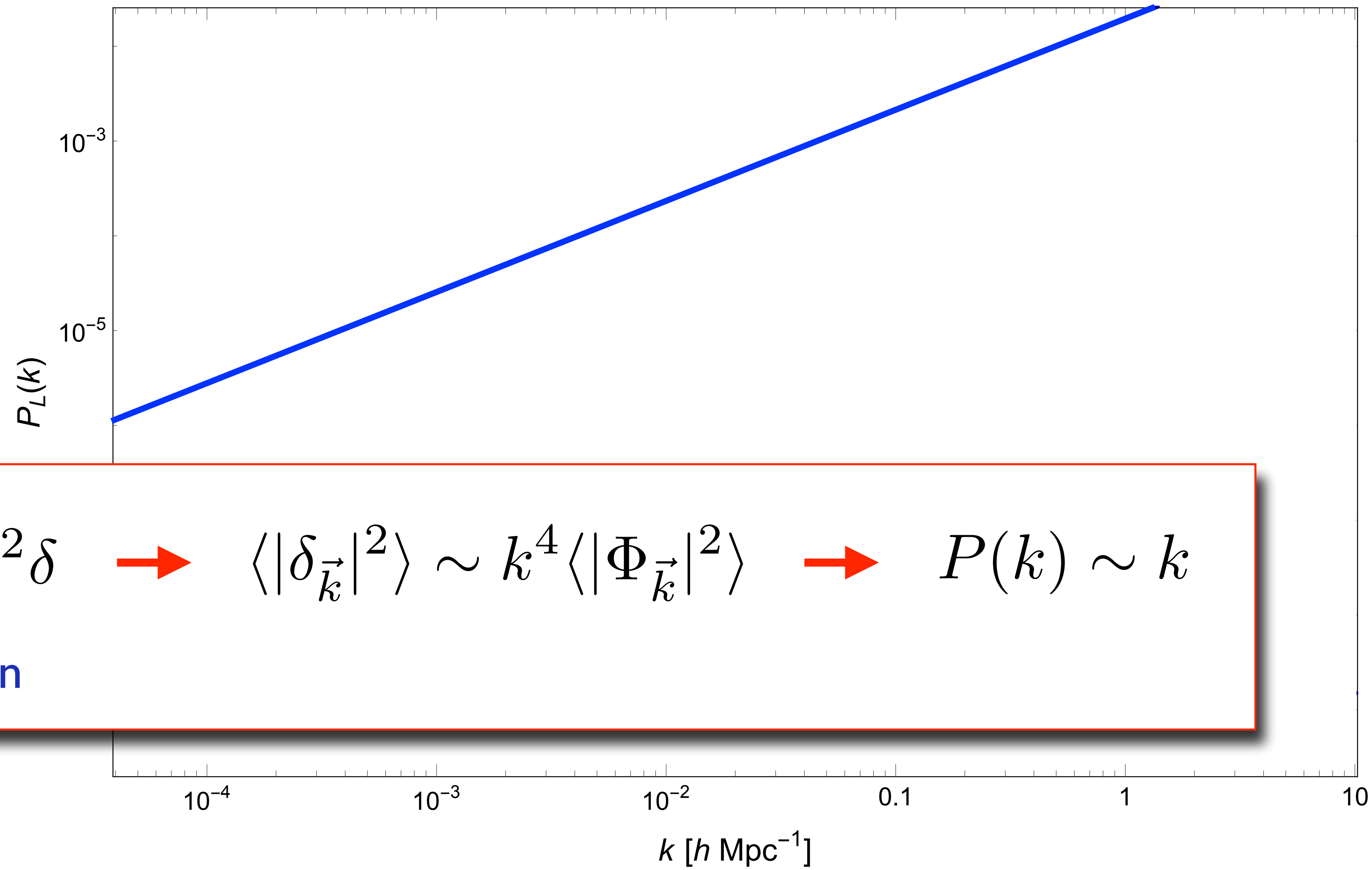
$$\Delta_\Phi(k) \equiv 4\pi k^3 P_\Phi(k) \simeq \text{constant} \qquad P_\Phi(k) \simeq \frac{C}{k^3}$$

In terms of matter perturbations:



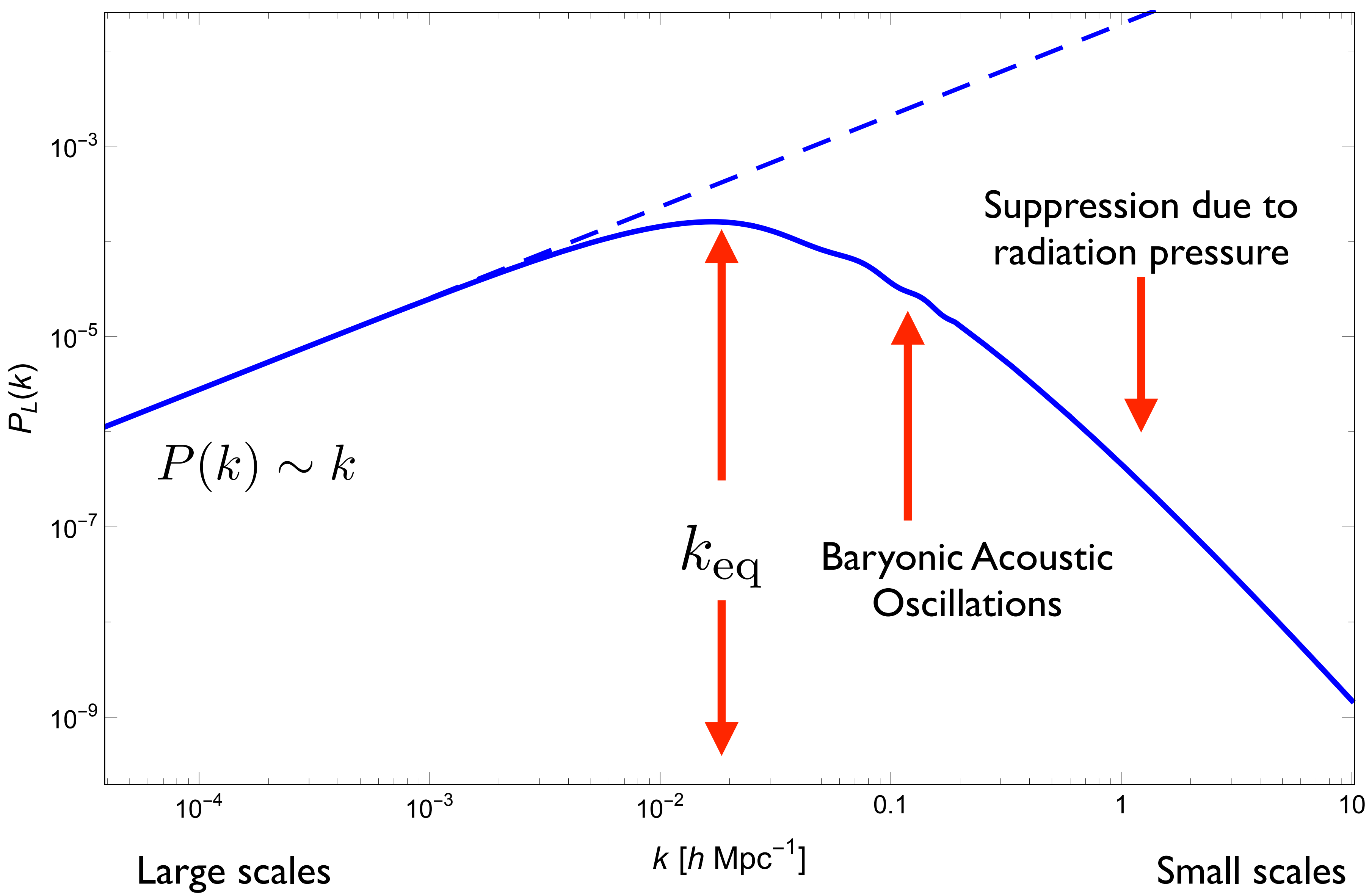
$$\nabla^2\Phi = \frac{3}{2}\mathcal{H}^2\delta \rightarrow \langle|\delta_{\vec{k}}|^2\rangle \sim k^4\langle|\Phi_{\vec{k}}|^2\rangle \rightarrow P(k) \sim k$$

Poisson equation

The “initial” matter power spectrum

Linear evolution during radiation and matter domination changes the *shape* of the initial matter power spectrum

At $z \simeq 1000$



Evolution of matter perturbations:
Equations of motion

Evolution of matter perturbations

In first approximation we can study the evolution of matter perturbations assuming:

1. **All matter is cold** (ignore the effects of baryons & neutrinos)

2. **Newtonian approximation:**

   $k \gg a\,H(a)$ scales much smaller than the horizon

   $v \ll c$ velocities much smaller than the speed of light

3. **Matter domination** (ignore effects of dark energy at late times)

Fluid equations

Assuming **CDM as ideal fluid** we need the following equations:

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}_r \cdot (\rho\, \vec{v}) = 0$$

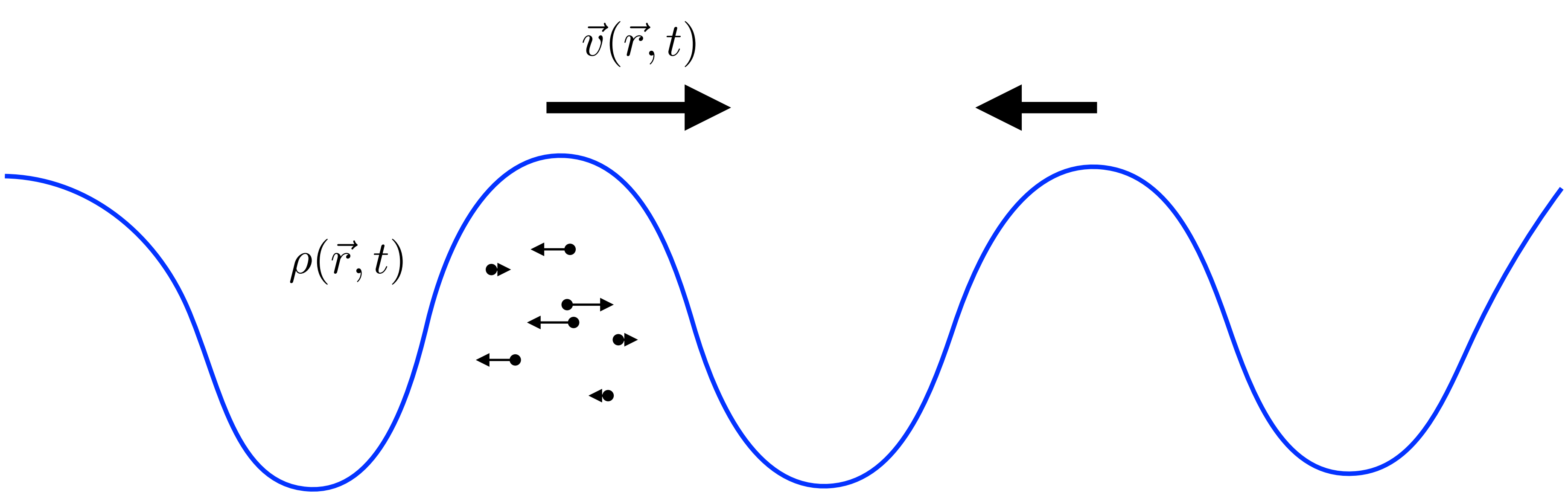
continuity equation
(conservation of mass)

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \vec{\nabla}_r)\, \vec{v} = -\cancel{\frac{\vec{\nabla}\, p}{\rho}} - \vec{\nabla}_r \Phi_{tot}$$

pressure term
(vanishing for CDM)

force

Euler's equation
(conservation of momentum)



Single-stream approximation

Fluid equations for perturbations

Assuming **CDM as ideal fluid** we need the following equations:

$$\boxed{\frac{\partial \delta}{\partial \tau} + \vec{\nabla} \cdot \left[(1+\delta)\, \vec{u} \right] = 0}$$

continuity equation
(conservation of mass)

$$\boxed{\frac{\partial \vec{u}}{\partial \tau} + \mathcal{H}\vec{u} + (\vec{u} \cdot \vec{\nabla})\, \vec{u} = -\vec{\nabla}\Phi}$$

Euler's equation
(conservation of momentum)

Approximated!

$$\boxed{\nabla^2 \Phi = \frac{3}{2}\mathcal{H}^2 \delta}$$

Poisson's equation

Now written for the perturbations:

$$\rho(\vec{x},\tau) = \bar{\rho}(\tau)[1+\delta(\vec{x},\tau)]$$

$\delta(\vec{x},\tau)$ matter perturbations

$$\vec{v}(\vec{x},\tau) = \underbrace{\mathcal{H}(\tau)\, \vec{x}(\tau)}_{\text{Hubble flow}} + \vec{u}(\vec{x},\tau)$$

$\vec{u}(\vec{x},\tau)$ peculiar velocities

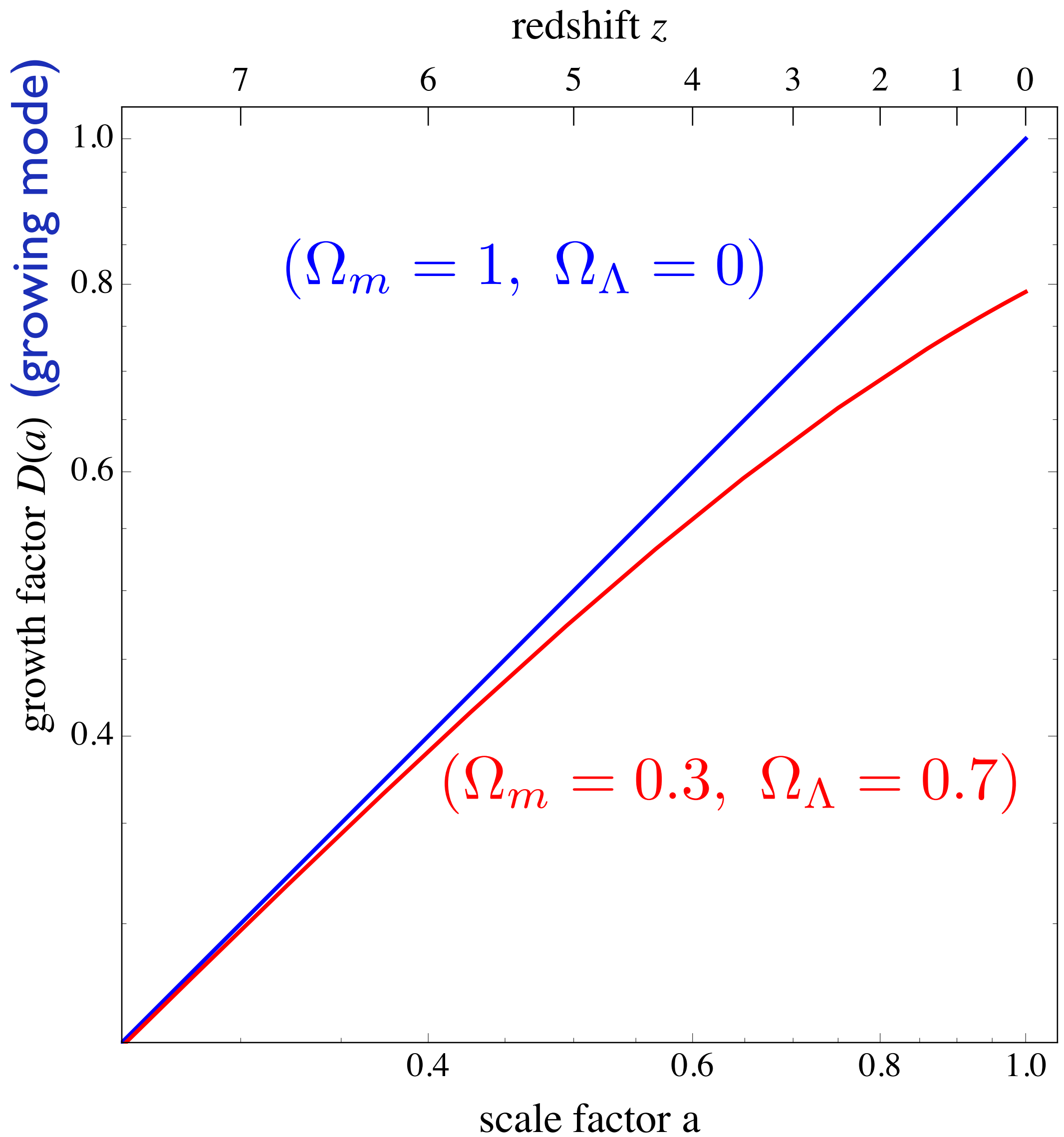
Linear solution

Linearising and combining the equations we obtain

$$\boxed{\frac{\partial^2 \delta}{\partial \tau^2} + \underbrace{\mathcal{H}\frac{\partial \delta}{\partial \tau}}_{\text{friction}} - \underbrace{\frac{3}{2}\mathcal{H}^2\,\delta}_{\text{gravity}} = 0}$$

2nd order equation

We find a separable solution like $\delta_{\vec{k}}(\tau) = D(\tau)\, A_{\vec{k}}$ $D(\tau)$ **growth factor**



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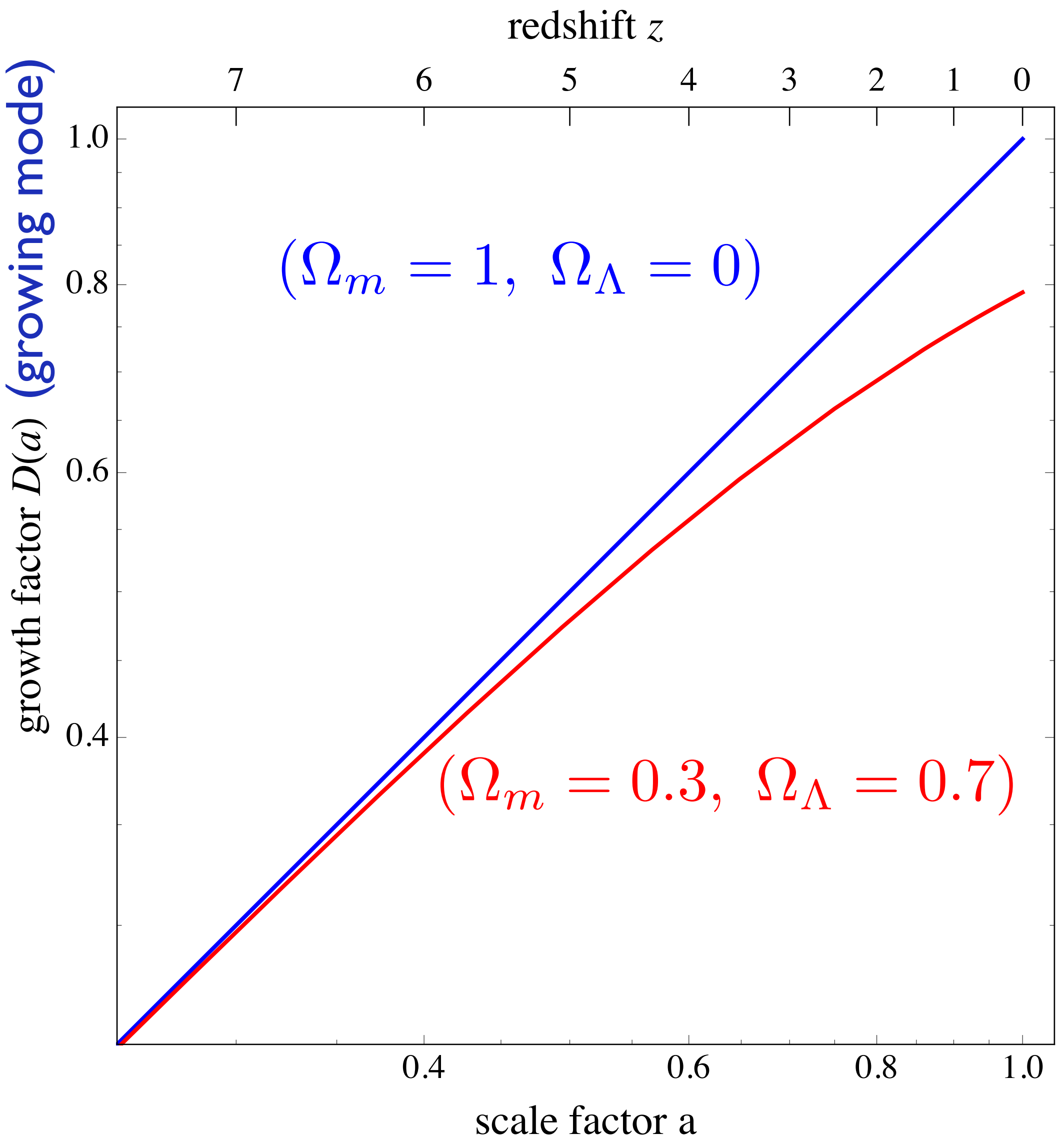
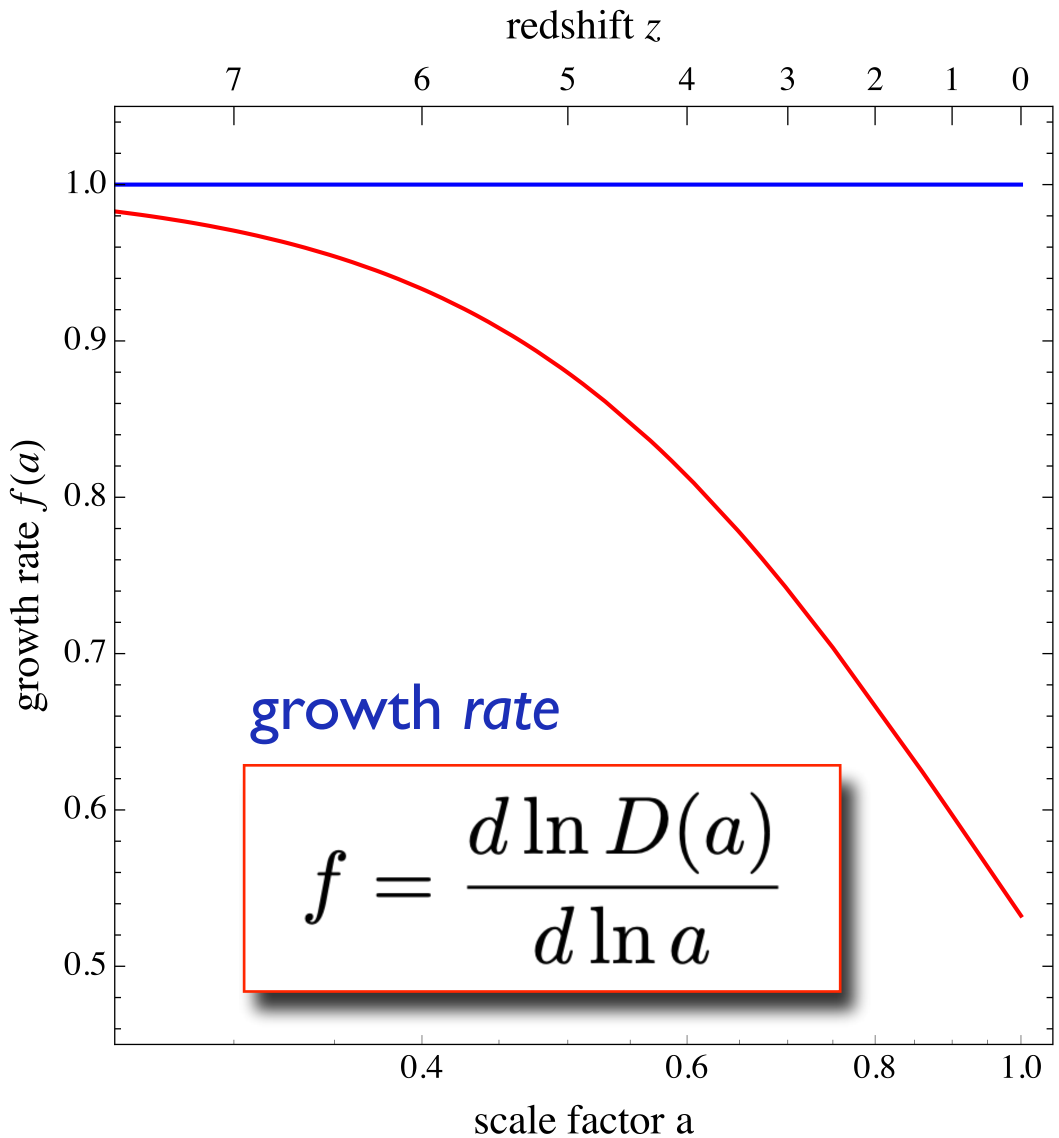
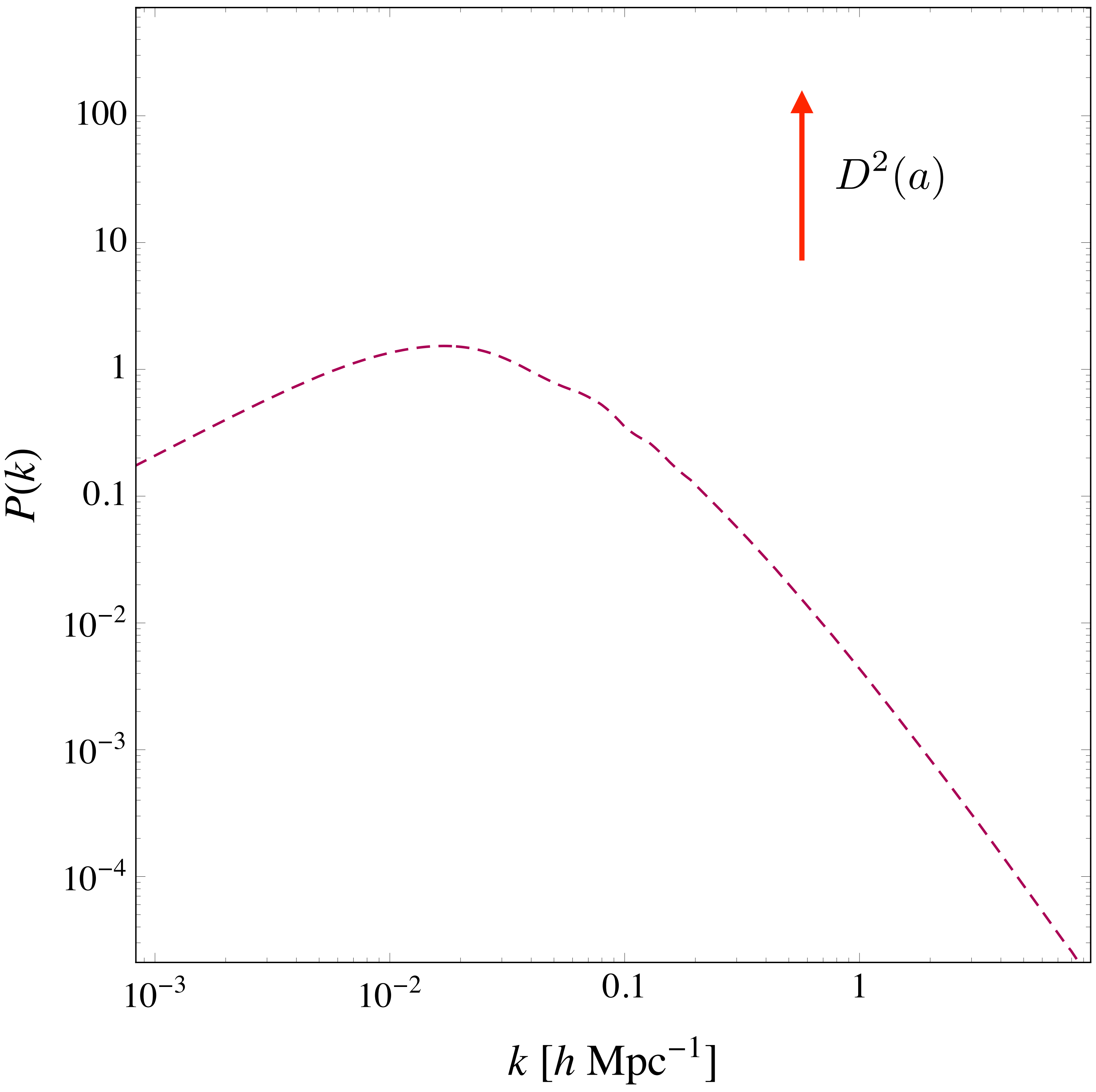




Linear vs Nonlinear evolution

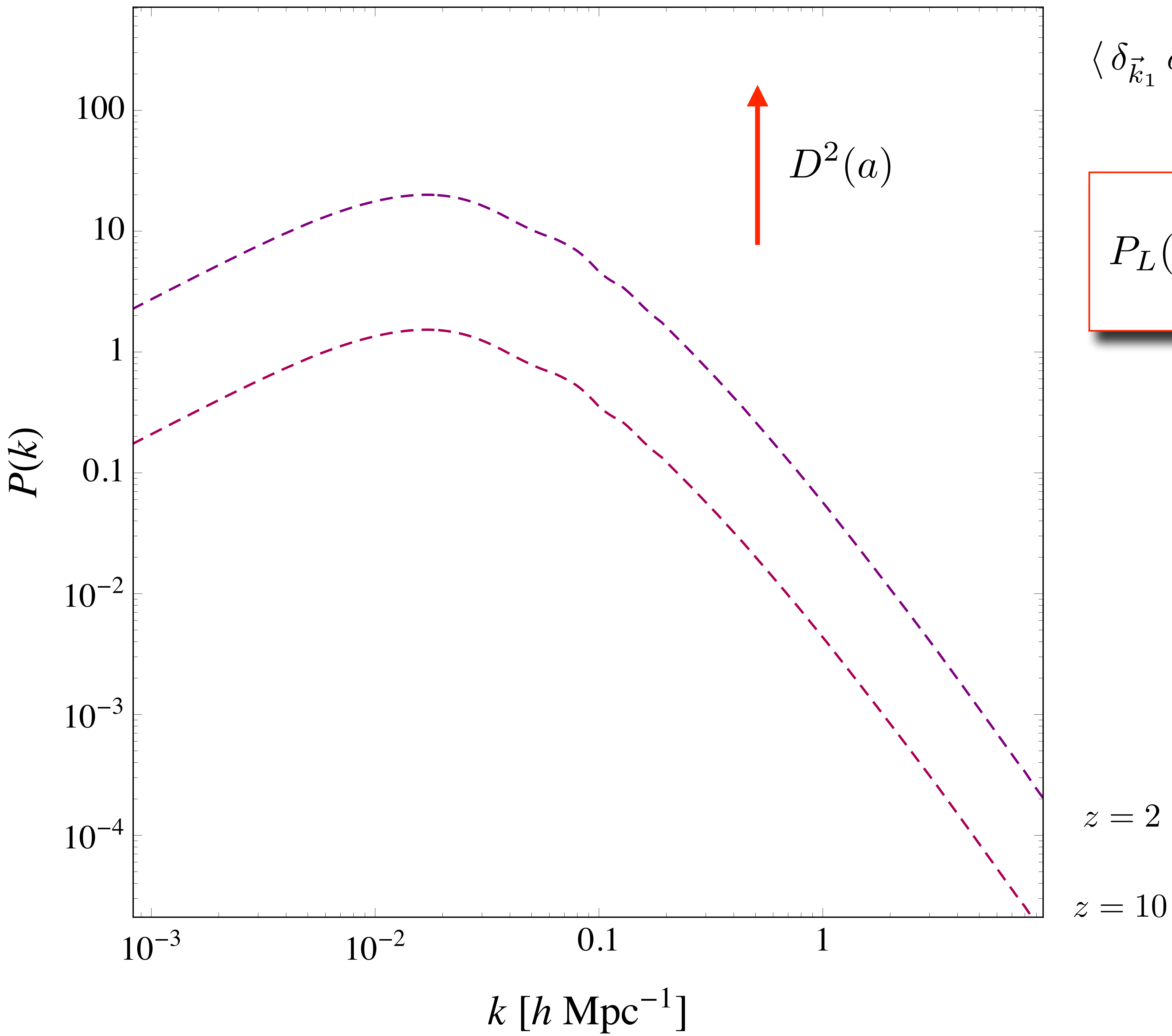


$$\langle\, \delta_{\vec{k}_1}\, \delta_{\vec{k}_2} \rangle = \delta_D(\vec{k}_1 + \vec{k}_2)\, P(k_1)$$

$$\boxed{P_L(k,a) = D^2(a)\, P_0(k)}$$

$z = 10$

Linear vs Nonlinear evolution



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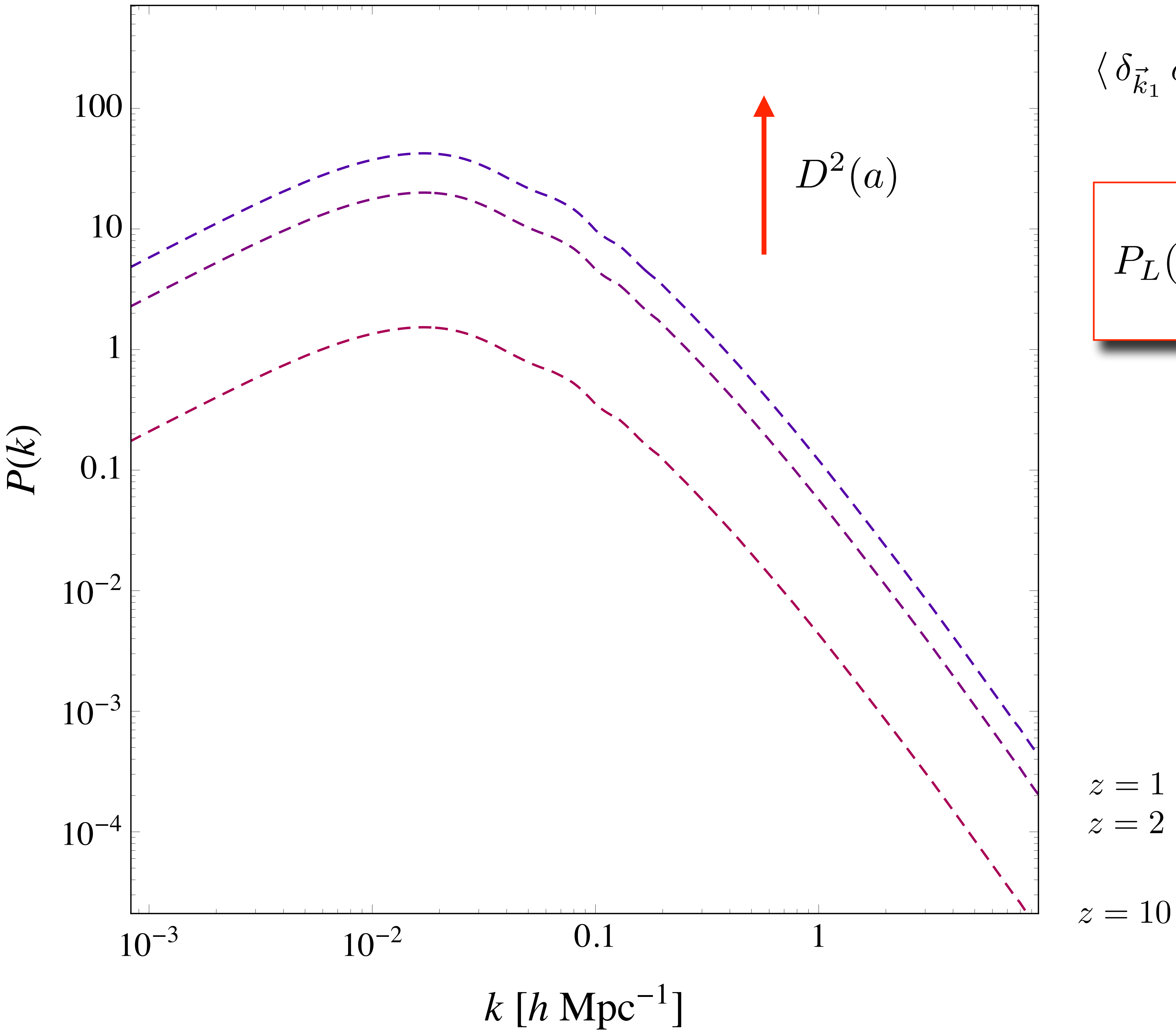
Linear vs Nonlinear evolution



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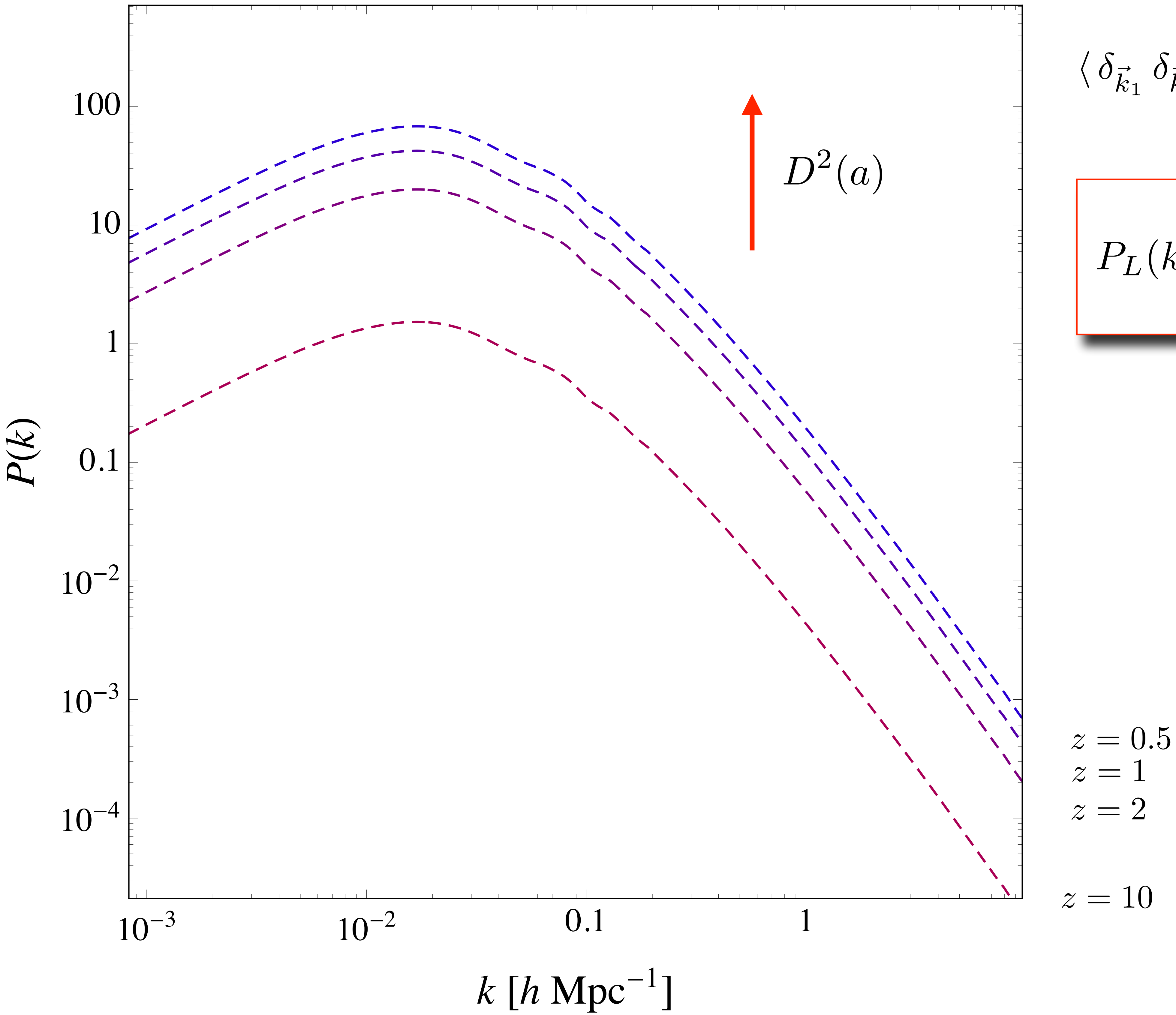
Linear vs Nonlinear evolution



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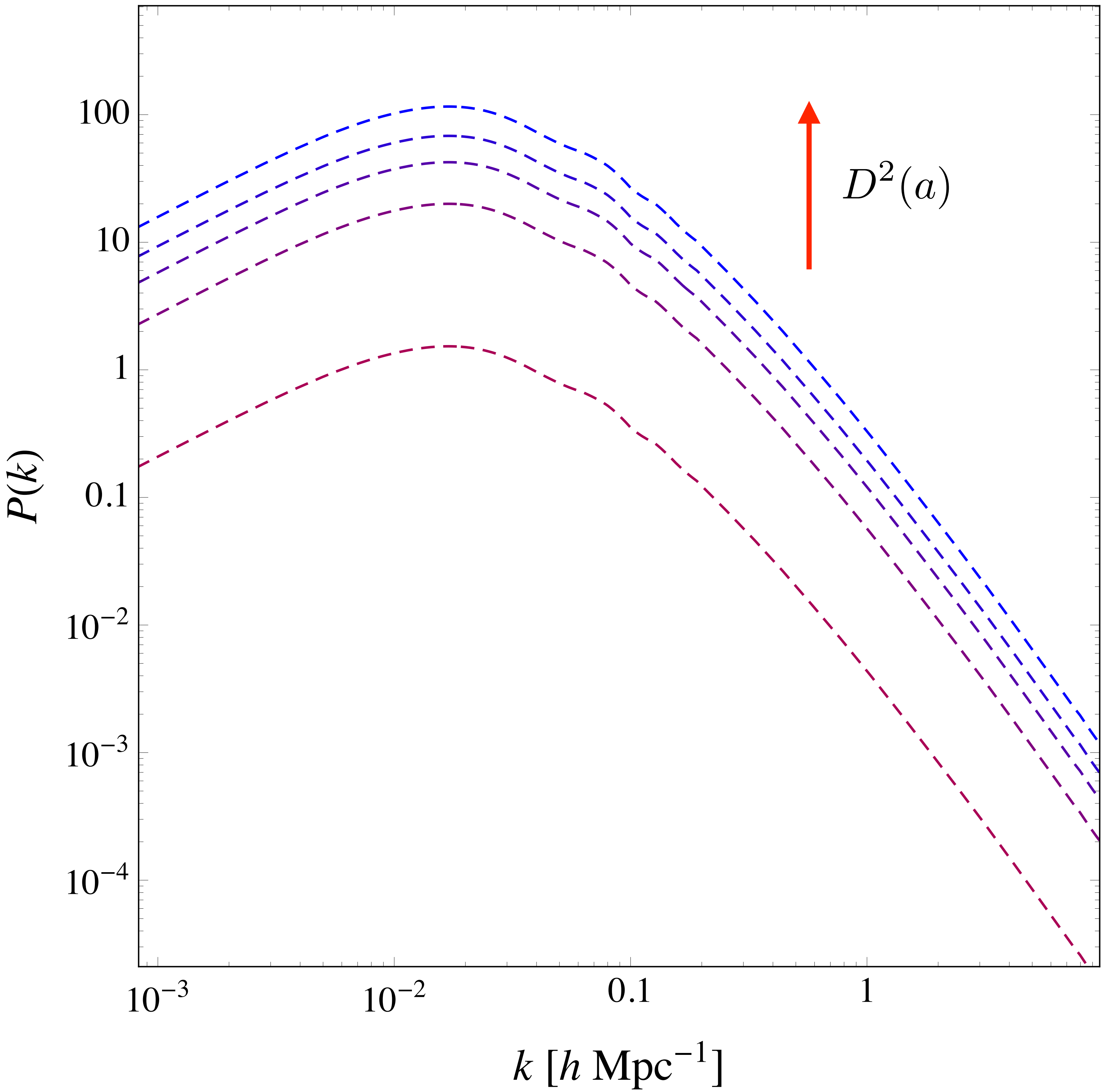
Linear vs Nonlinear evolution

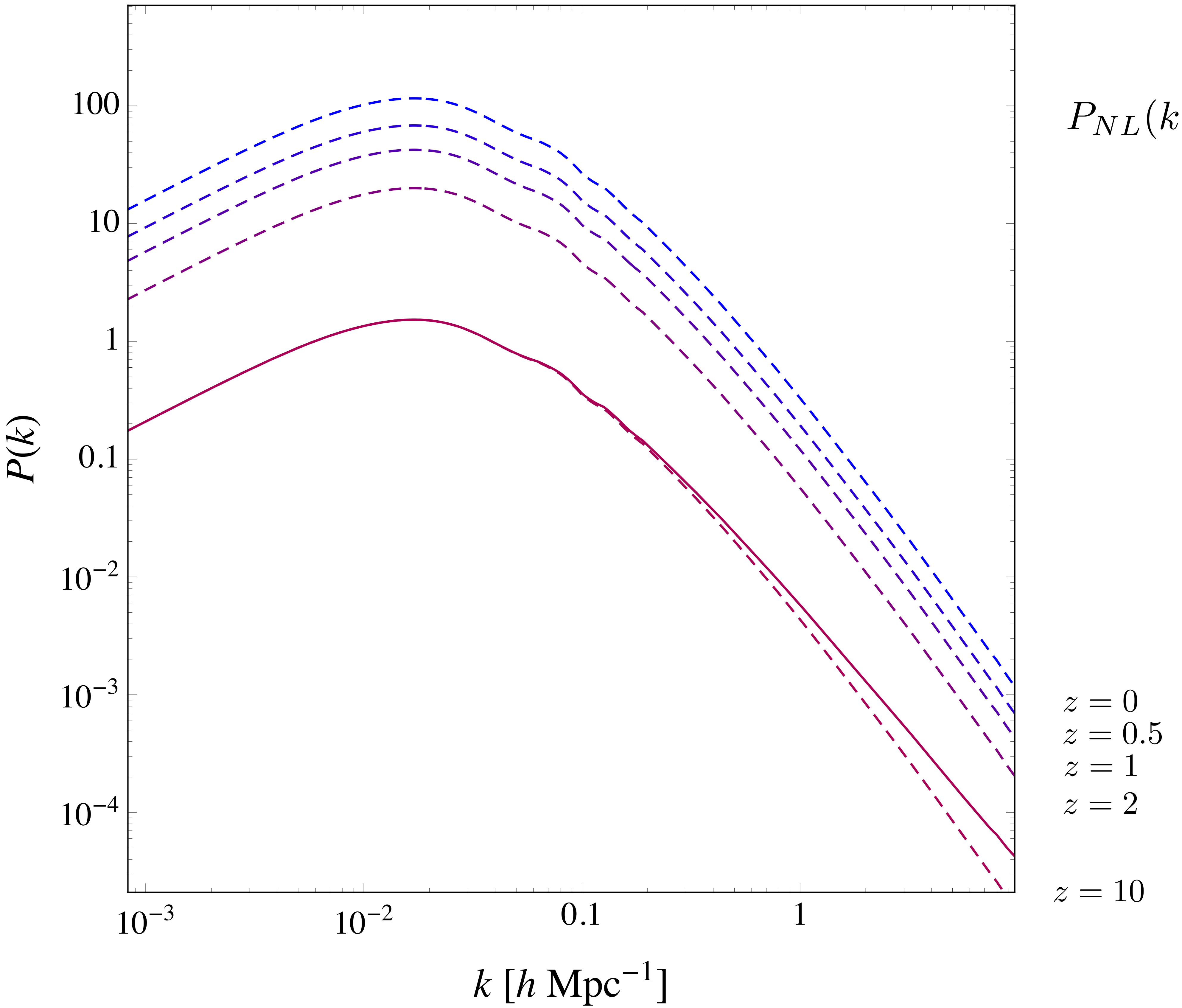


$$\langle\, \delta_{\vec{k}_1}\ \delta_{\vec{k}_2} \rangle = \delta_D(\vec{k}_1 + \vec{k}_2)\, P(k_1)$$

$$\boxed{P_L(k,a) = D^2(a)\, P_0(k)}$$

$z = 0$
$z = 0.5$
$z = 1$
$z = 2$
$z = 10$

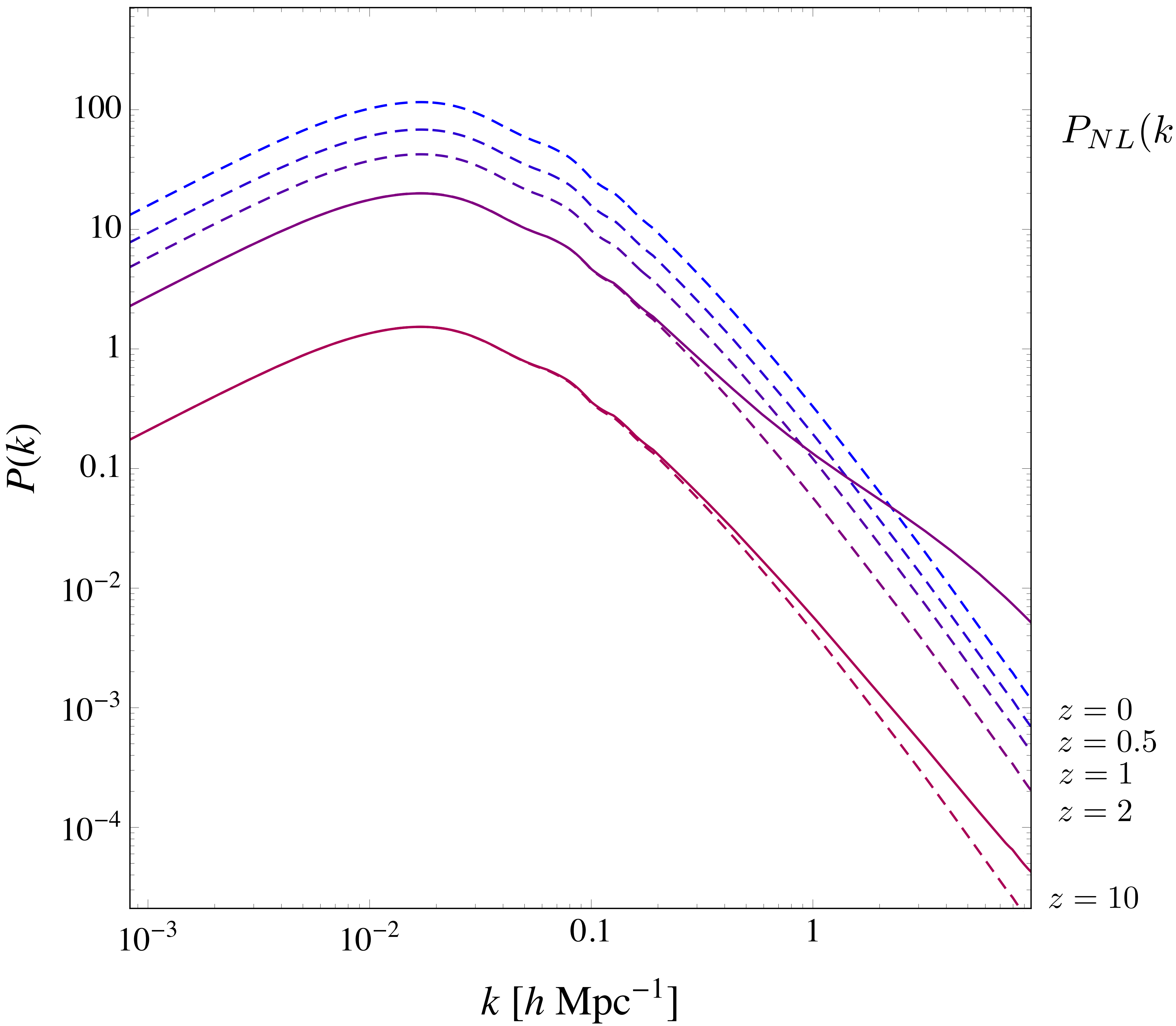
Linear vs Nonlinear evolution



$$P_{NL}(k) = P_L(k) + \Delta P_{NL}(k)$$

nonlinear corrections!

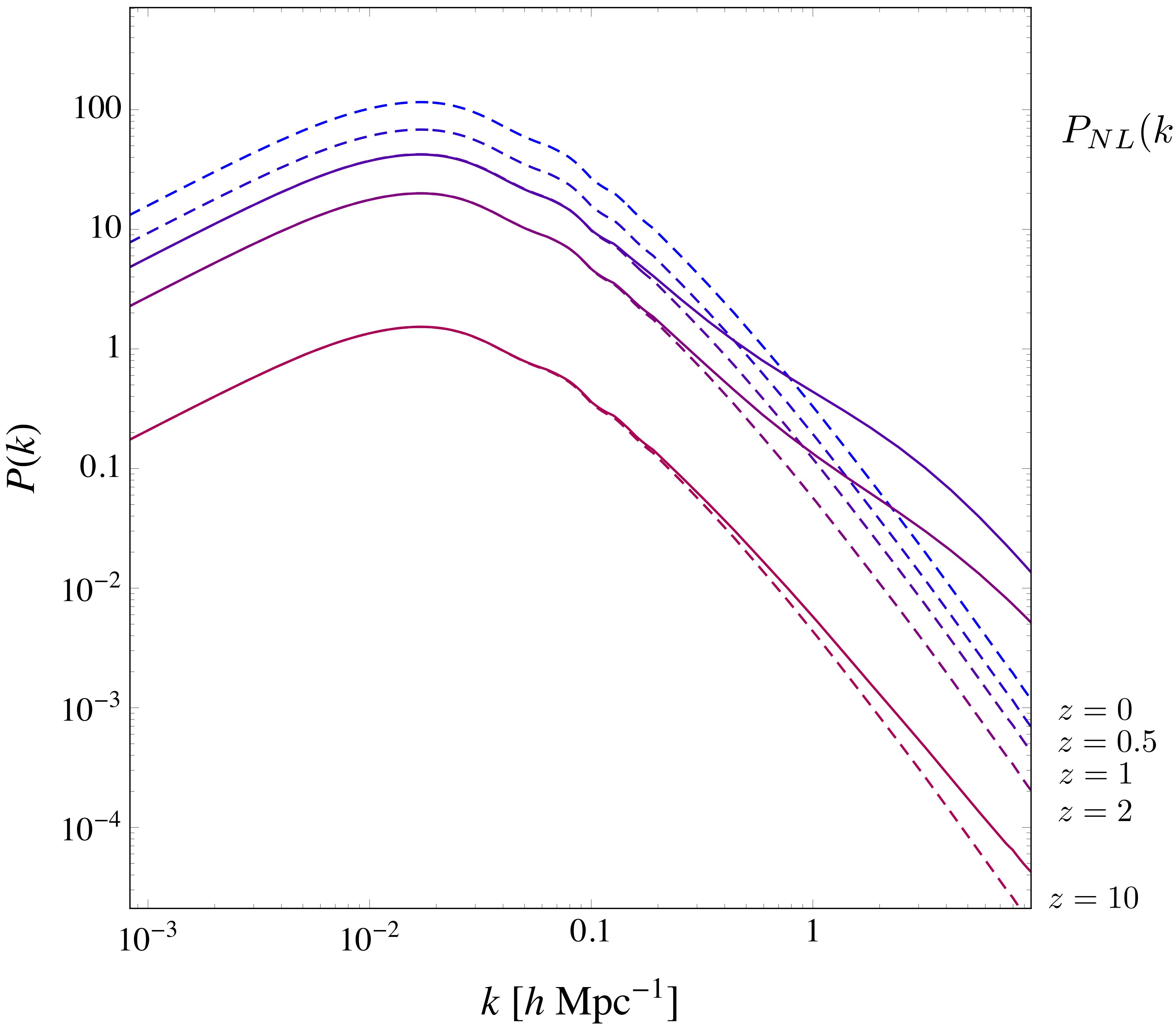
Linear vs Nonlinear evolution



$$P_{NL}(k) = P_L(k) + \Delta P_{NL}(k)$$

nonlinear corrections!

Linear vs Nonlinear evolution



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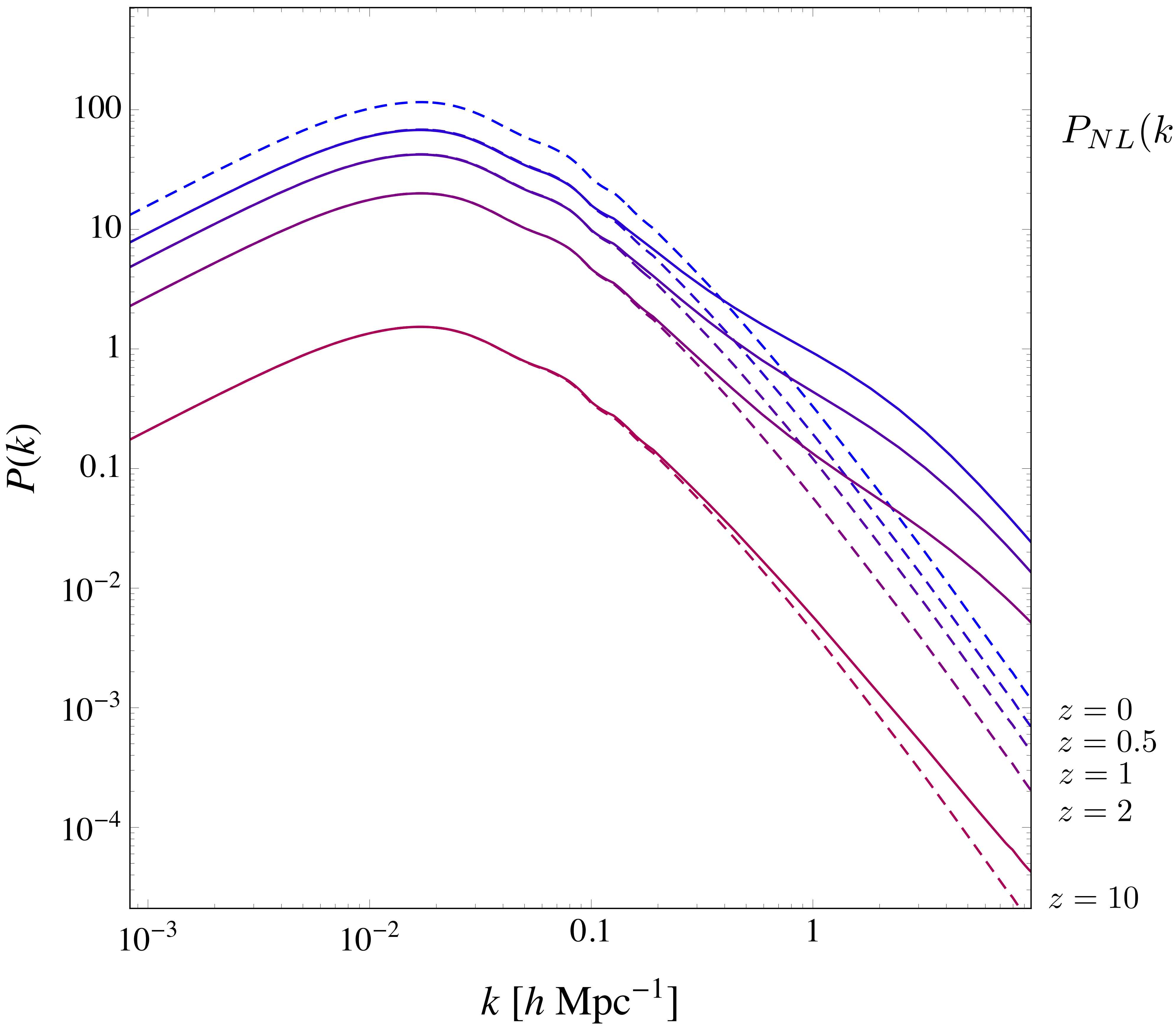

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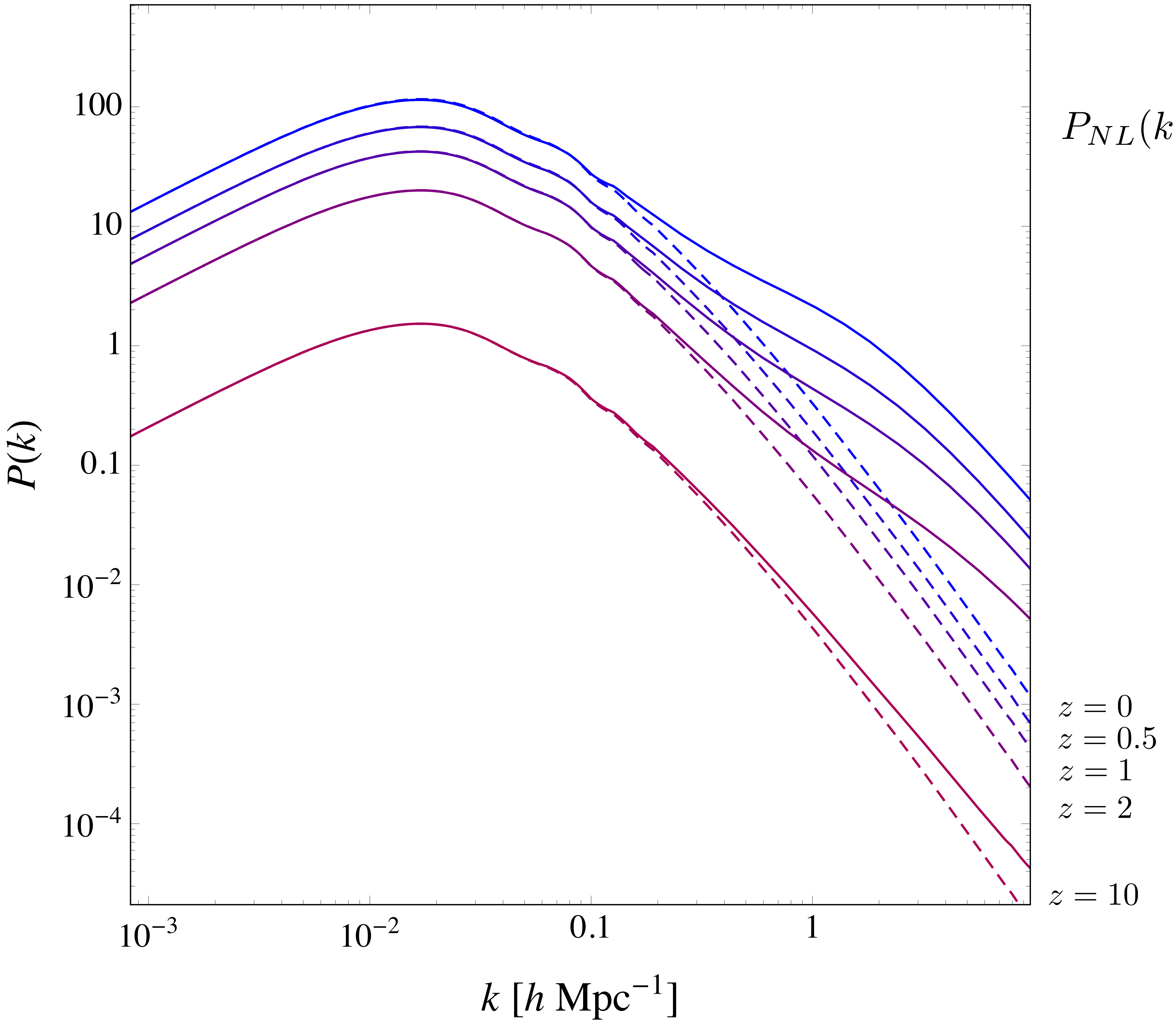
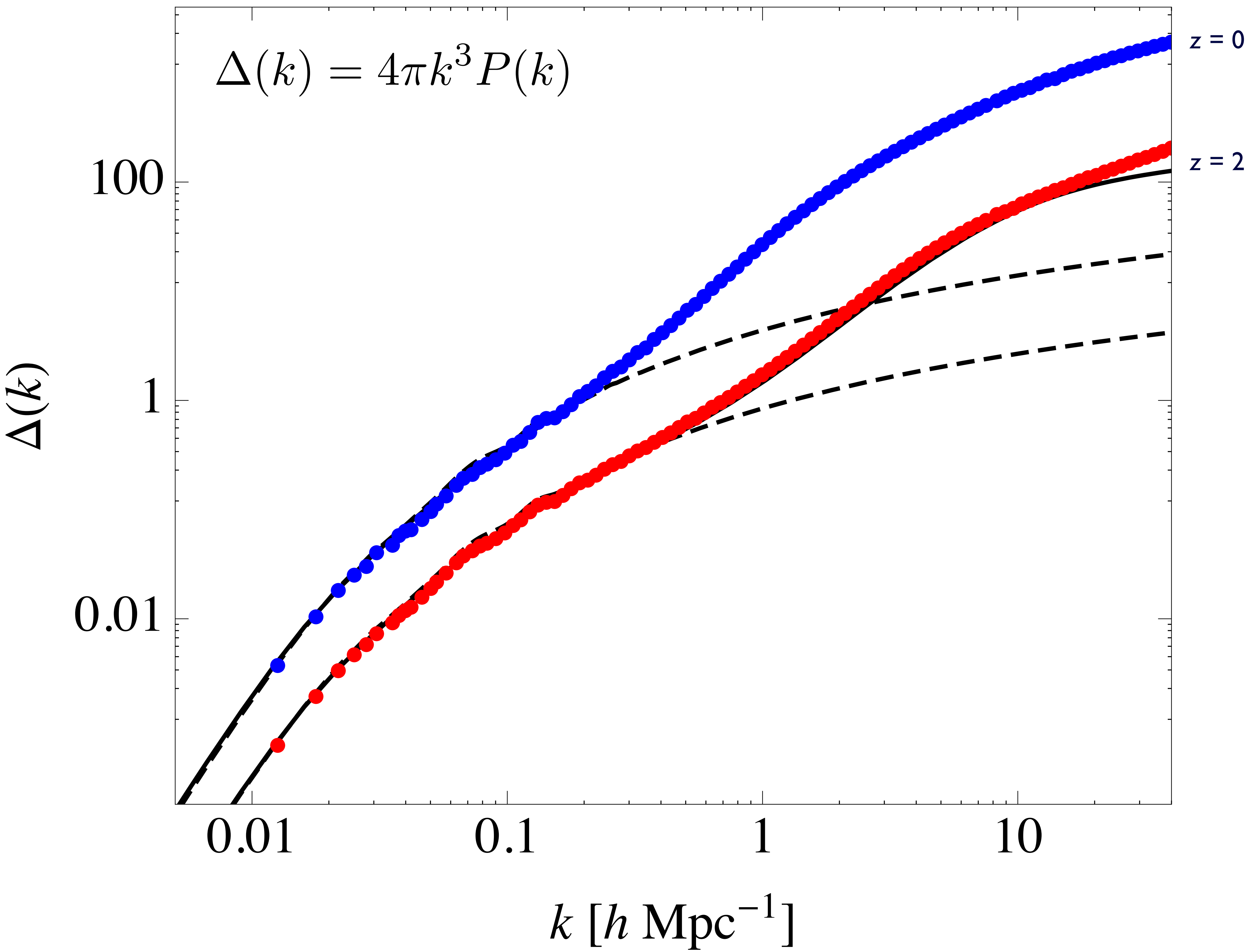
Linear vs Nonlinear evolution



$$P_{NL}(k) = P_L(k) + \Delta P_{NL}(k)$$

nonlinear corrections!

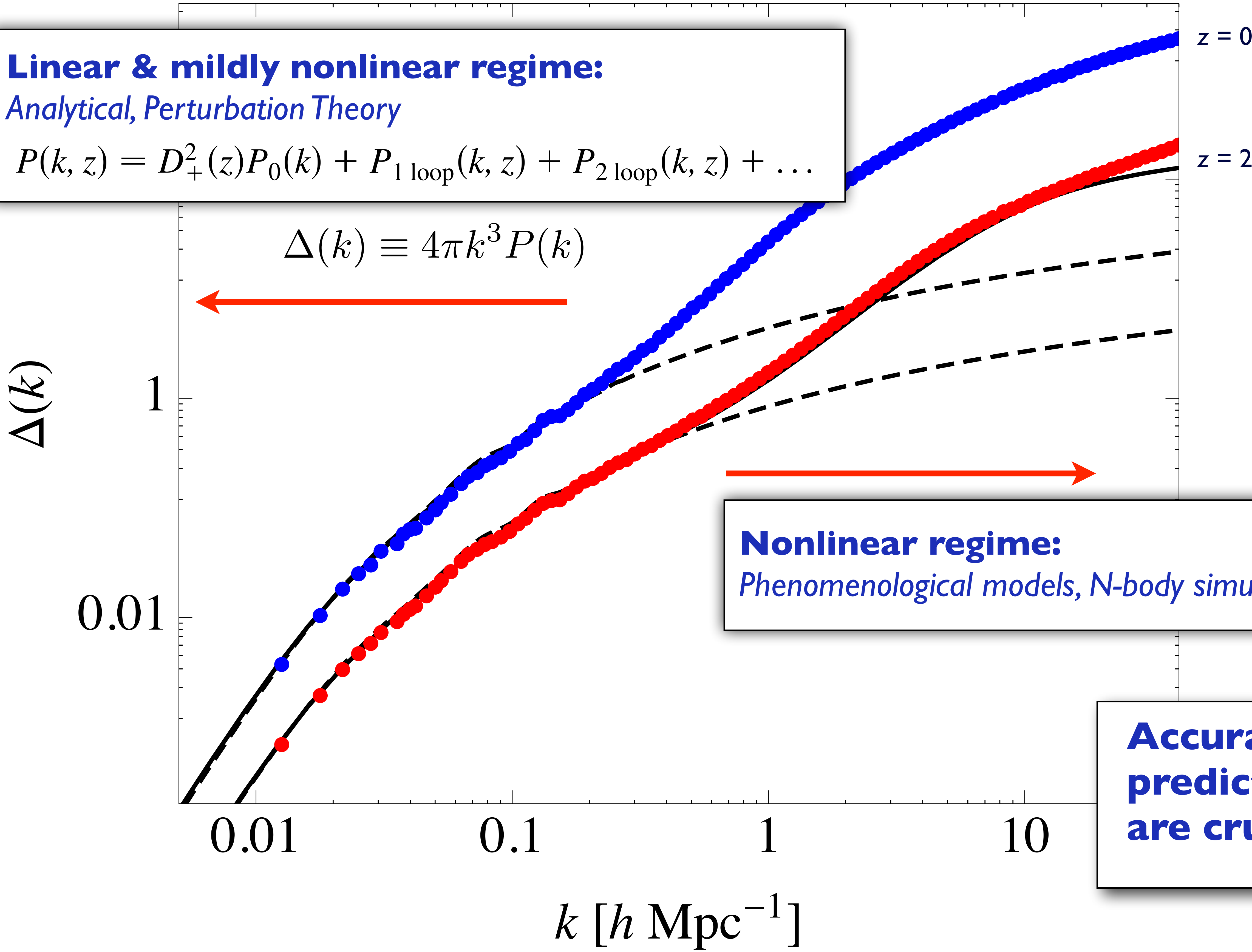
The growth of matter perturbations



The growth of matter perturbations

Linear & mildly nonlinear regime:
Analytical, Perturbation Theory

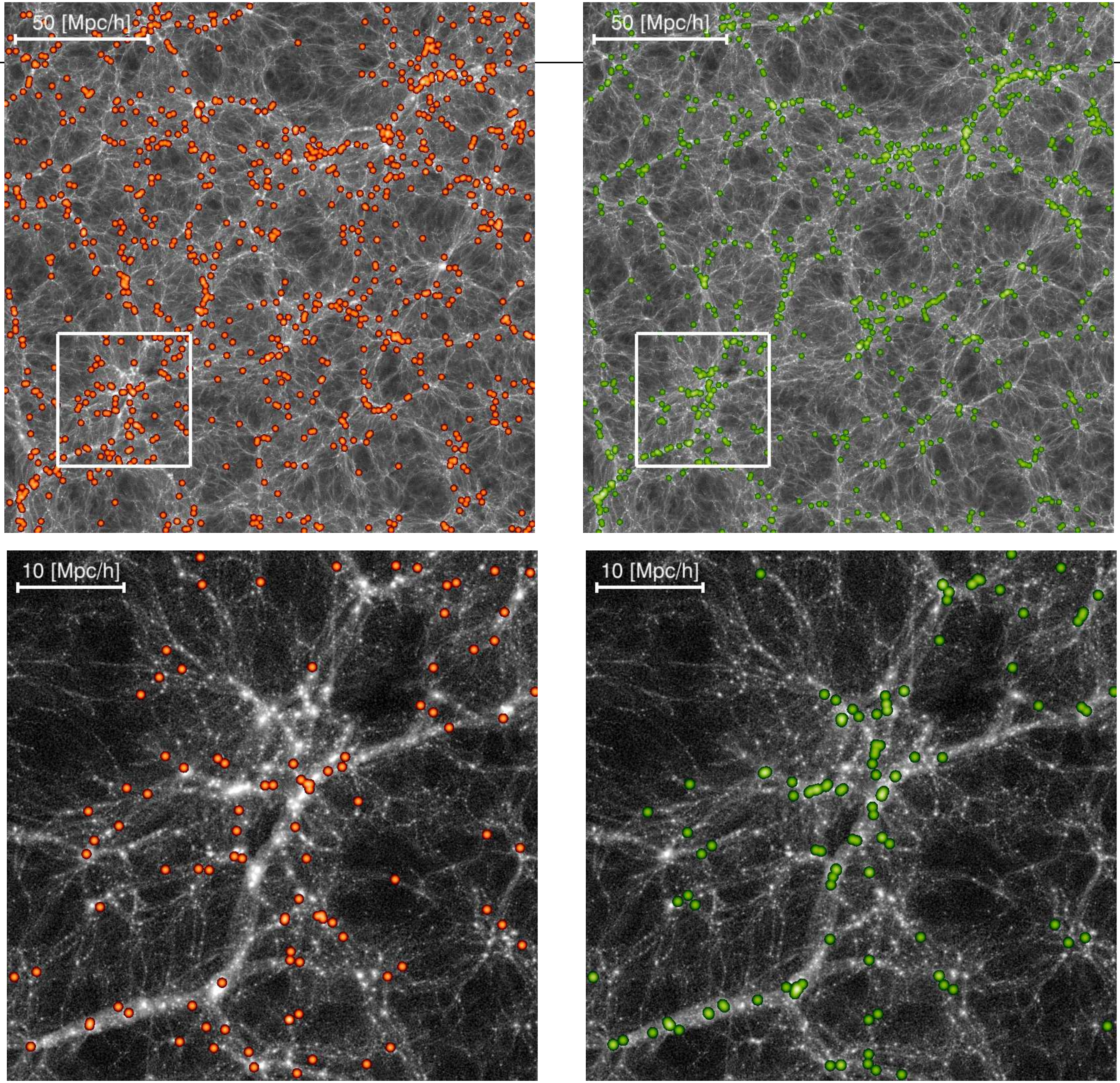
$$P(k,z) = D_+^2(z)P_0(k) + P_{1\,\mathrm{loop}}(k,z) + P_{2\,\mathrm{loop}}(k,z) + \ldots$$



Nonlinear regime:
Phenomenological models, N-body simulations

Accurate predictions are crucial!

Galaxies



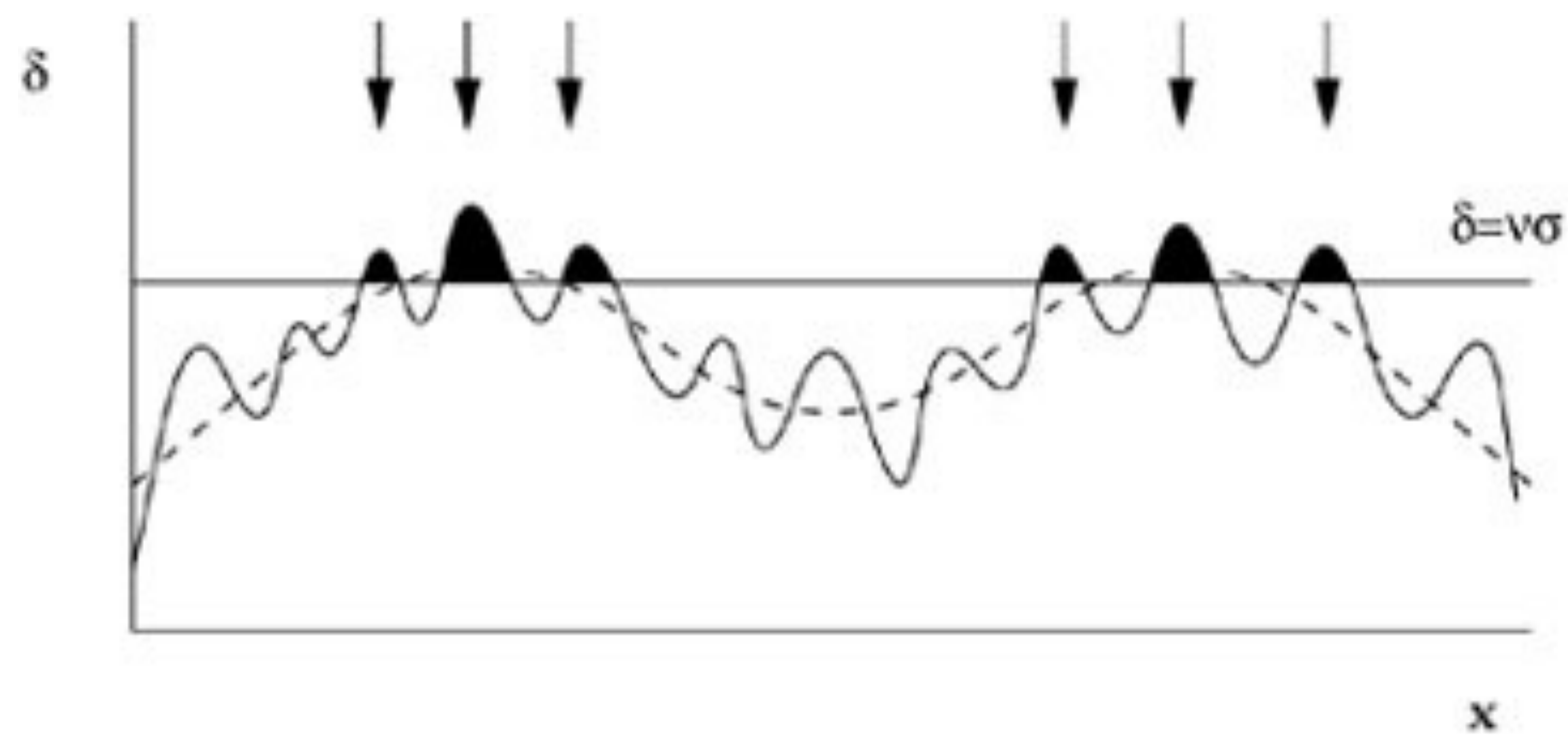
[Orsi *et al.* (2009)]

Local galaxy bias

A very simple assumption …

local galaxy bias
$$\delta_g(x) \equiv \frac{n_g(x) - \bar{n}_g}{\bar{n}_g} = f\left[\delta(x)\right]$$

If galaxies form in regions of large dark matter density, I can at least expect a direct dependence of the **galaxy overdensity** on the **matter overdensity**



Local galaxy bias

A very simple assumption …

local galaxy bias

$$\delta_g(x) \equiv \frac{n_g(x) - \bar{n}_g}{\bar{n}_g} = f\left[\delta(x)\right]$$

At large scales, we can expand it in a Taylor series

$$\delta_g(x) = b\,\delta(x) + \frac{1}{2}\,b_2\,\delta^2(x) + \ldots$$

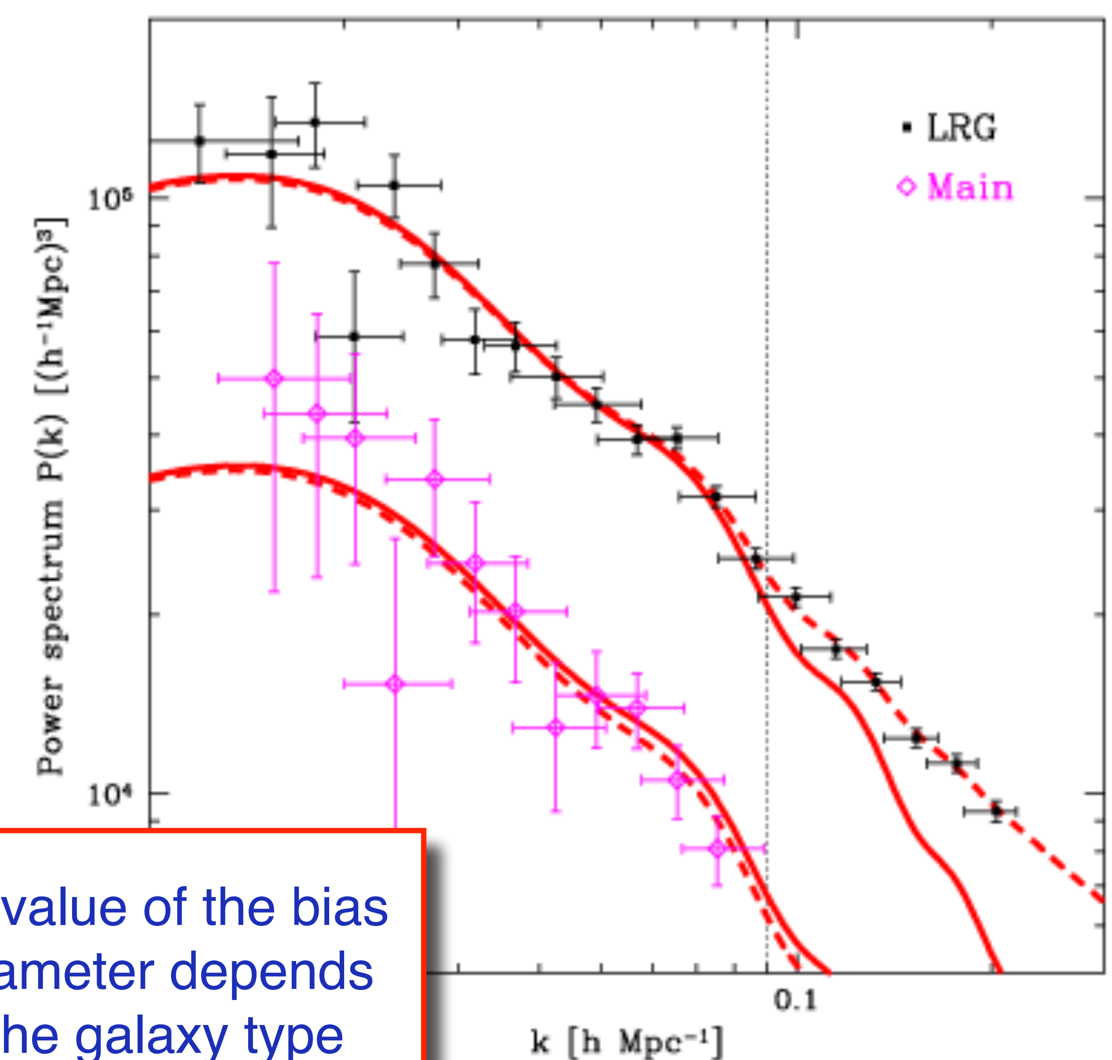
linear bias   nonlinear bias corrections

$$\langle \delta_g \delta_g \rangle = b^2\,\langle \delta\delta \rangle$$

$$\xi_g(x) \simeq b^2 \xi(x)$$

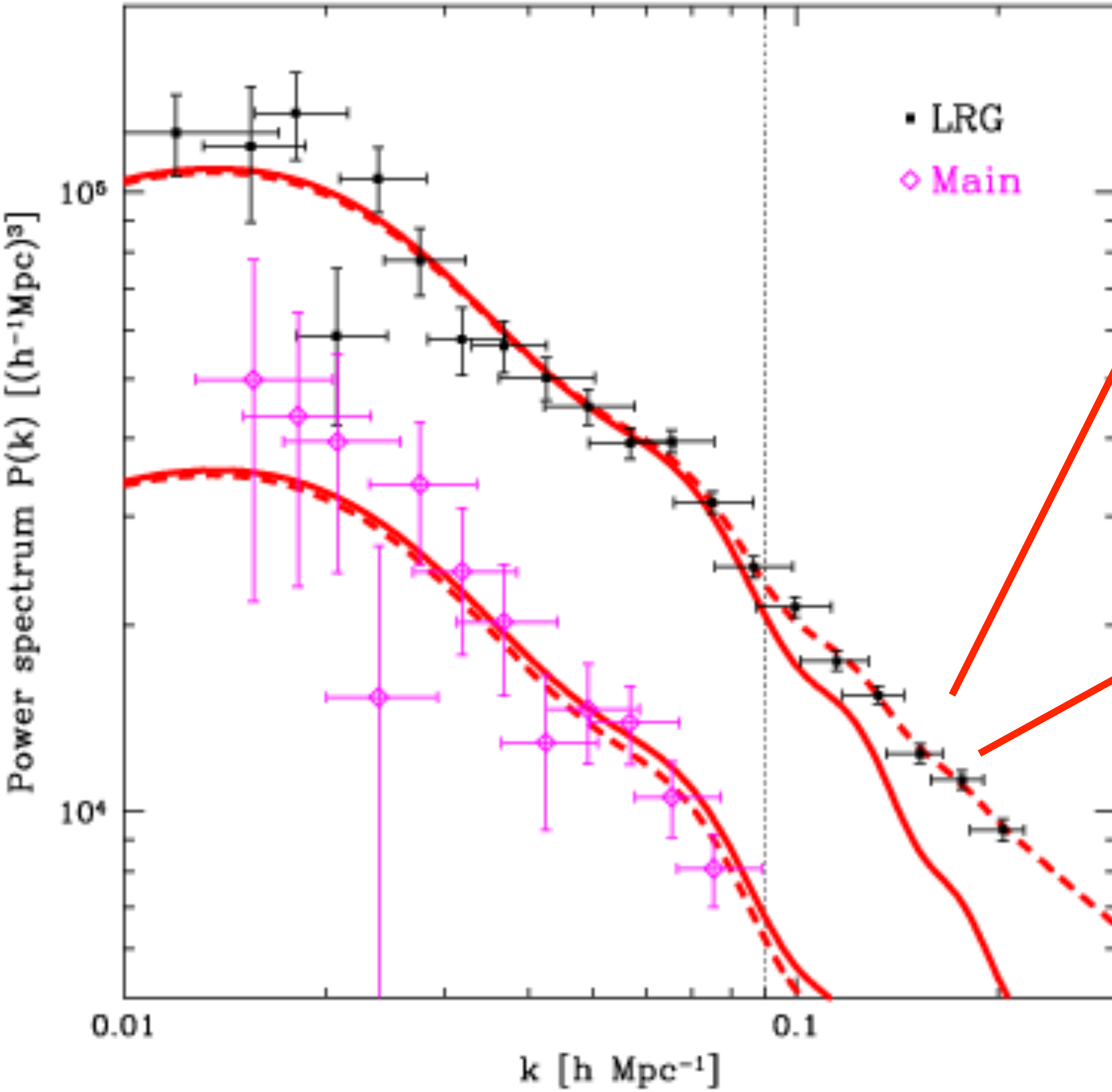
$$P_g(k) \simeq b^2 P(k)$$

At *large* scales, we expect a very **simple, linear relation between galaxy and matter correlation functions**



the value of the bias parameter depends on the galaxy type

Non-linear bias *and* non-linear gravitational instability

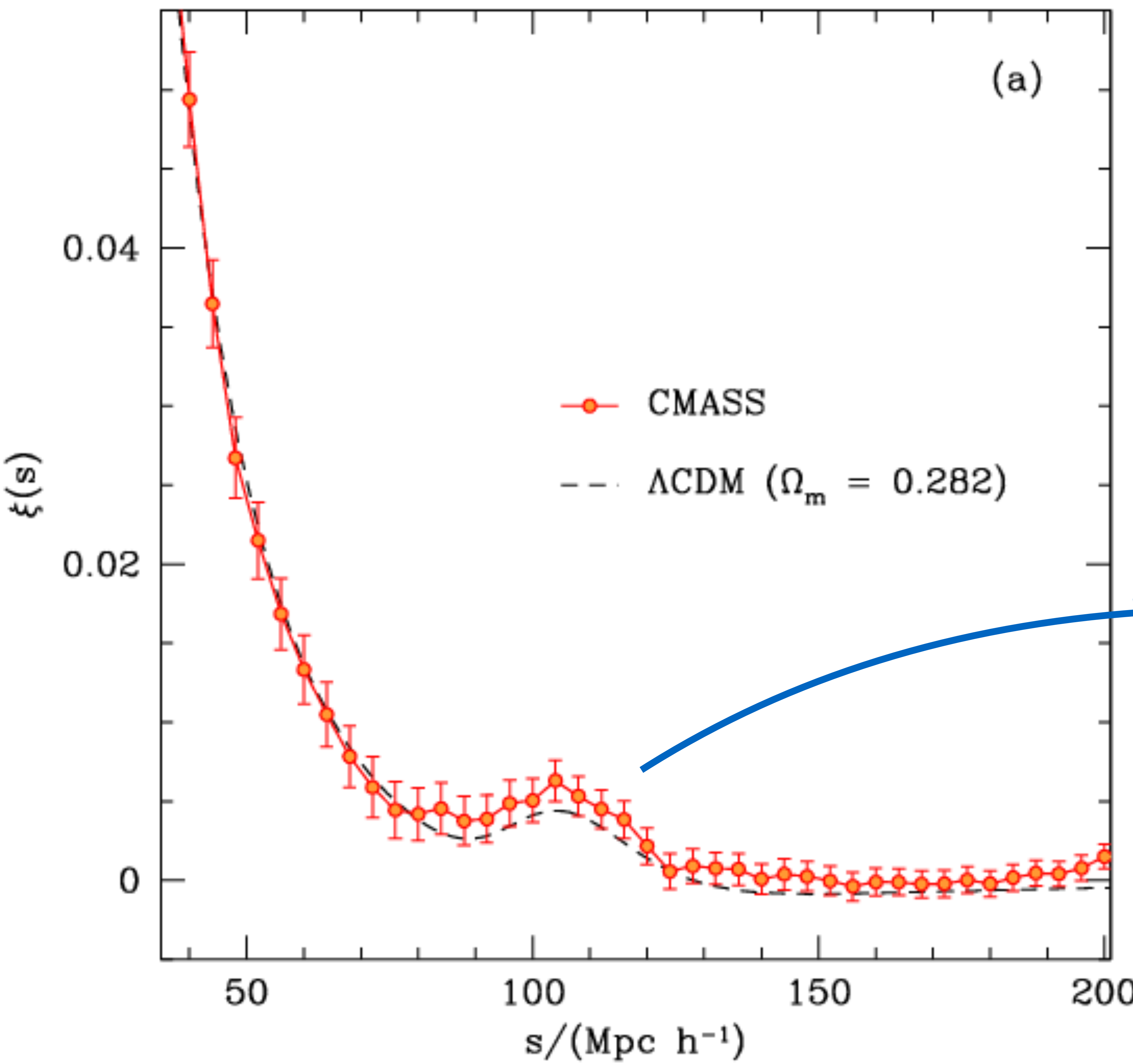


at small scales, non-linear bias is degenerate with non-linear corrections to the matter power spectrum!

at small scales we also have better statistics (smaller error bars)

Baryonic Acoustic Oscillations
in the galaxy distribution

Baryonic Acoustic Oscillations



Baryon Oscillation Spectroscopic Survey
BOSS

1.5 million galaxies
up to z = 0.7

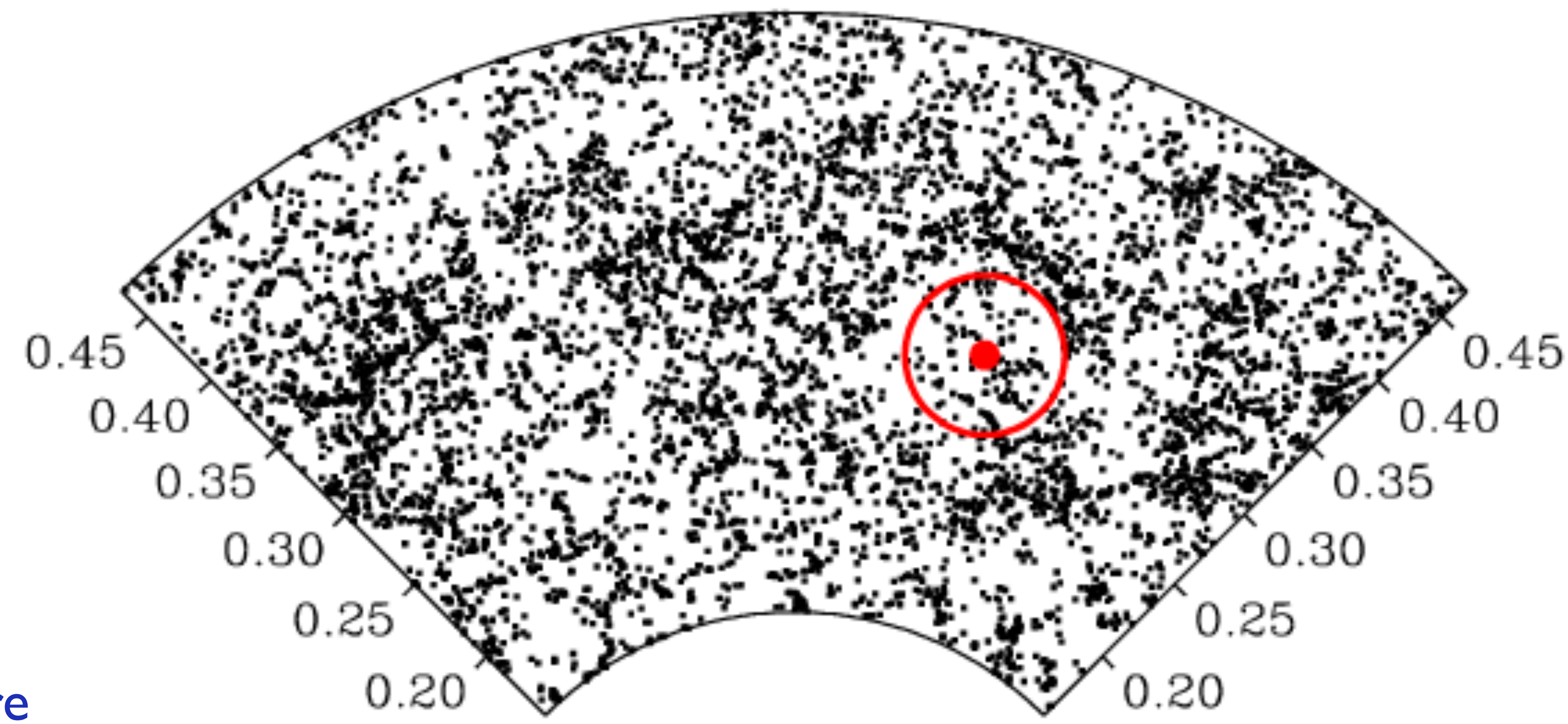
Anderson *et al.* (2012)

This is corresponds a "spherical ring" around a large overdensity

A standard ruler

we know the size of the "oscillation ring" very well from CMB observations

the galaxy 2-point function provides an "isotropic" measurement of the feature (if we get the cosmology right!)



SDSS LRG sample:
first detection of the BAO peak
Eisenstein *et al.* (2005)

$$D_V(z) = \left[D_M(z)^2 \frac{cz}{H(z)} \right]^{1/3}$$

A standard ruler

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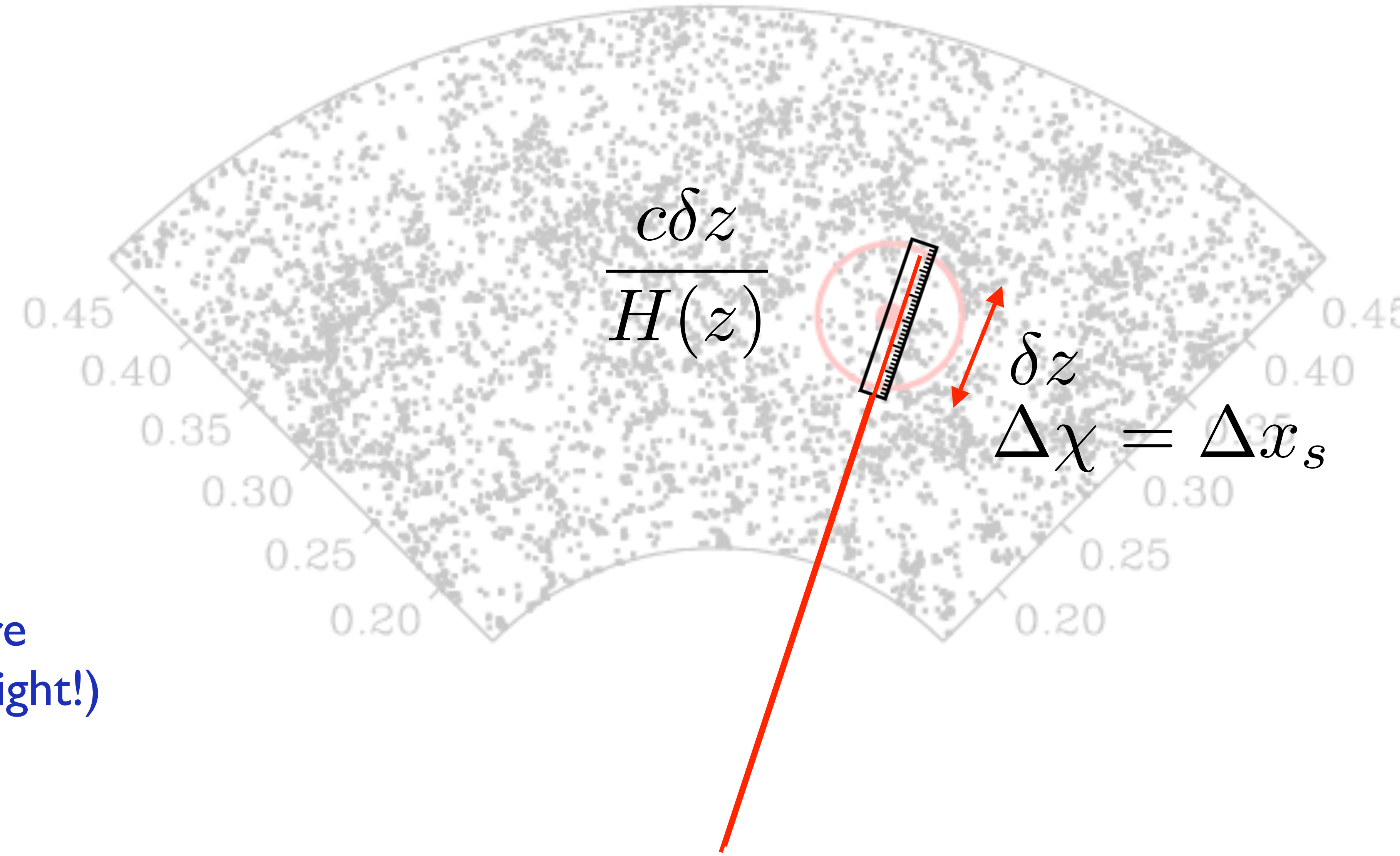


$$D_V(z) = \left[D_M(z)^2 \frac{cz}{H(z)} \right]^{1/3}$$

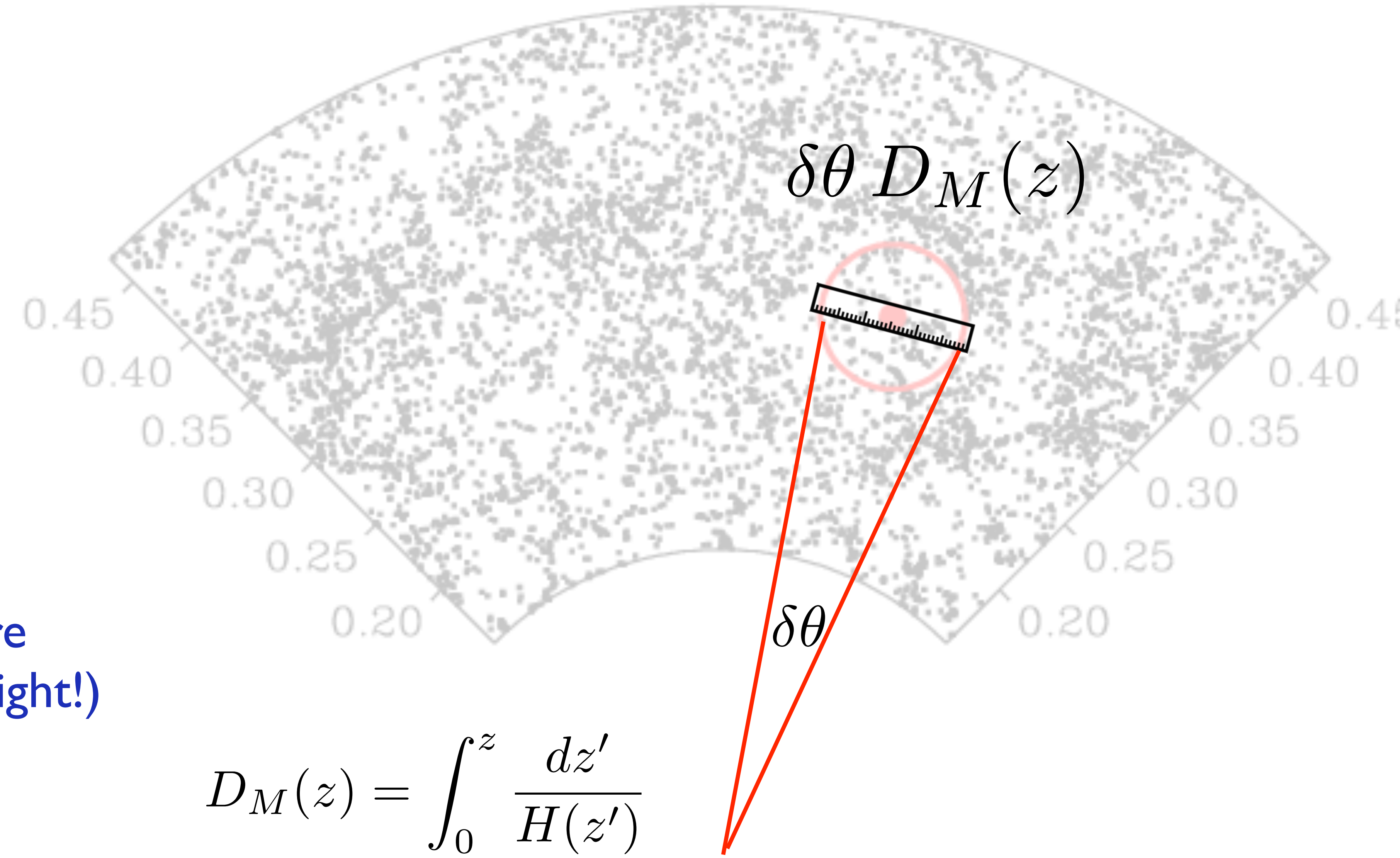
comoving distance along the line-of-sight

$$\chi = \int_{t_e}^{t_o} \frac{dt'}{a(t')} = \int_{a_e}^{a_o} \frac{da'}{H(a')} = \int_0^z \frac{dz'}{H(z')}$$

A standard ruler

we know the size of the "oscillation ring" very well from CMB observations

the galaxy 2-point function provides an "isotropic" measurement of the feature (if we get the cosmology right!)

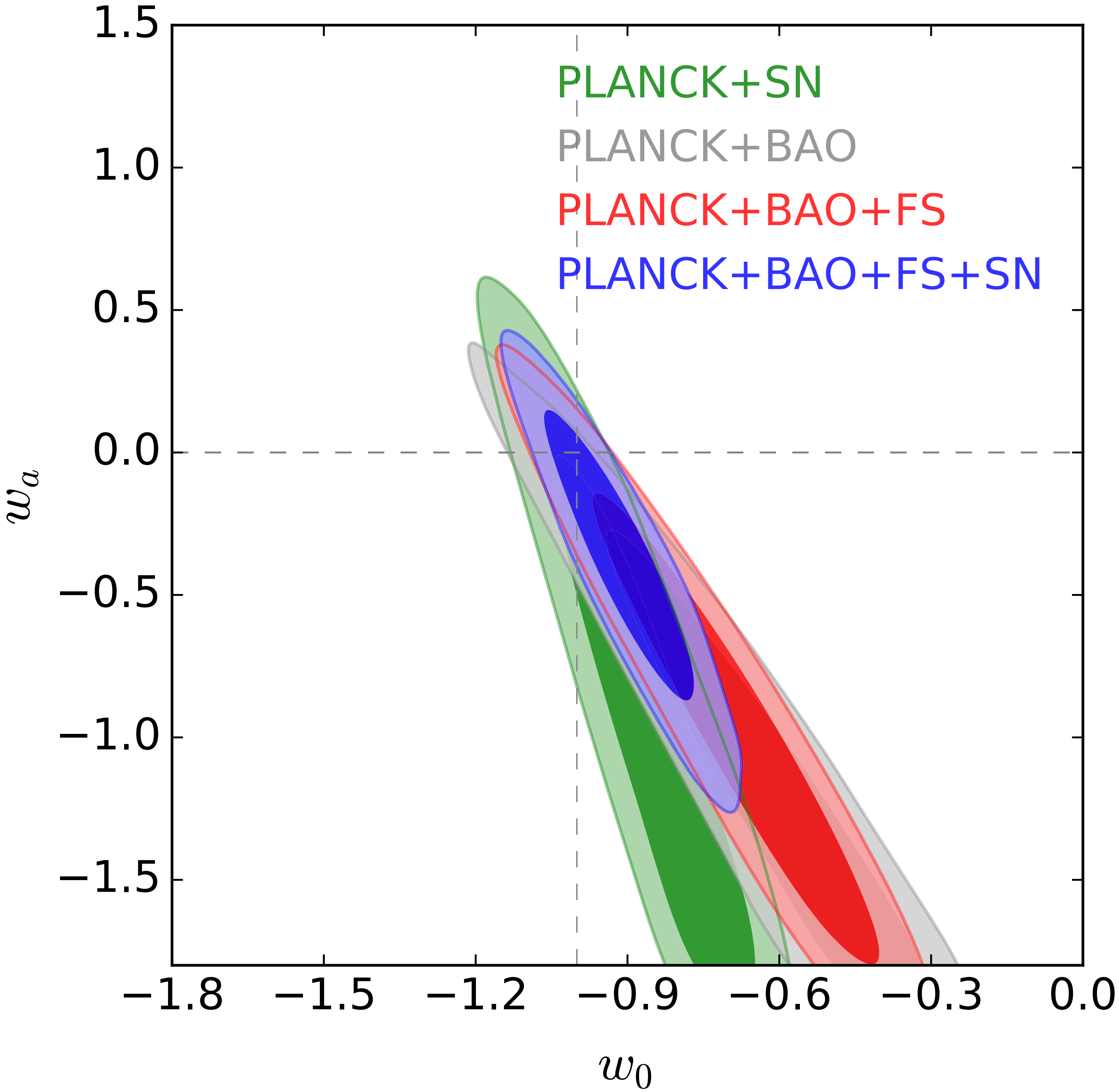


$$D_M(z) = \int_0^z \frac{dz'}{H(z')}$$

comoving angular diameter distance

$$D_V(z) = \left[D_M(z)^2 \, \frac{cz}{H(z)} \right]^{1/3}$$

Constraints from BAO



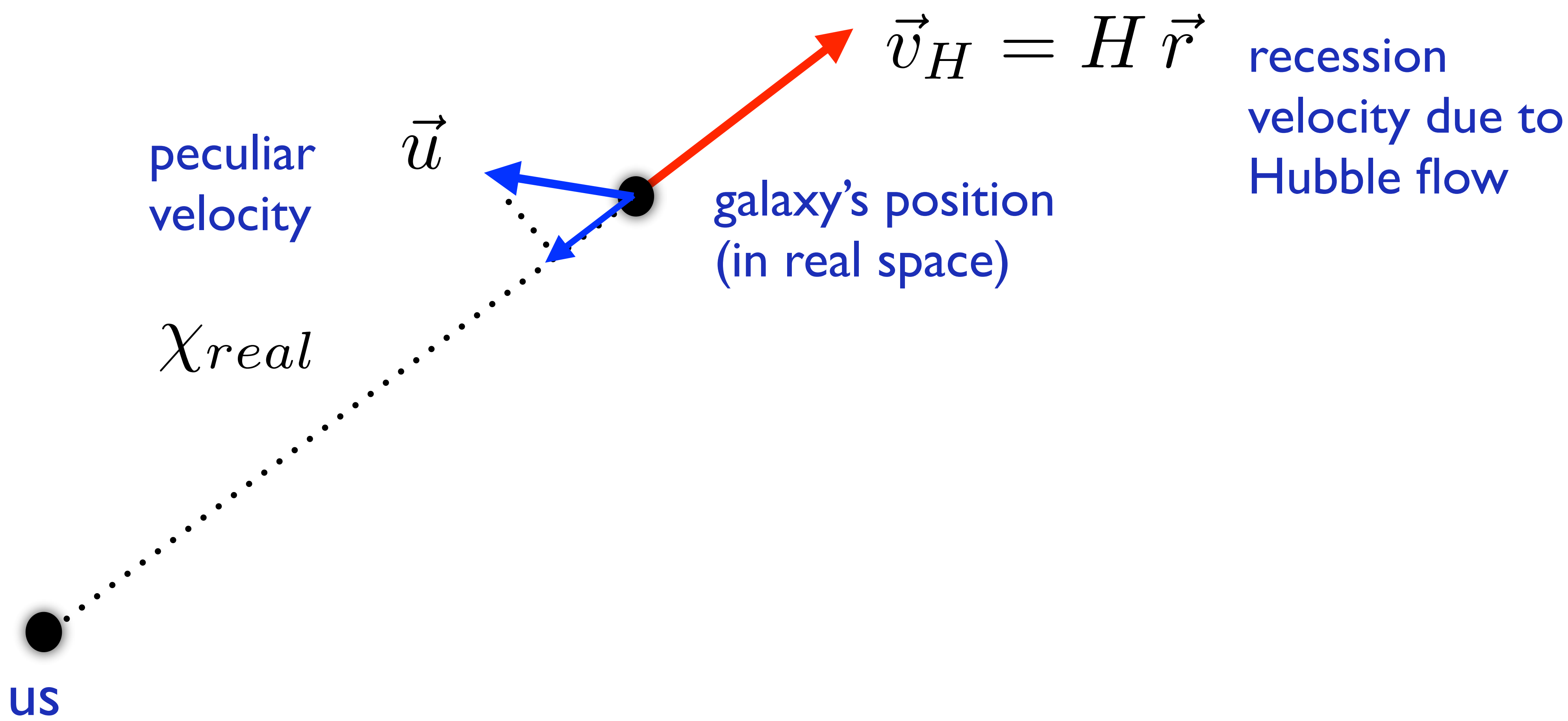
Constraints on a the **Dark Energy equation of state**

Alam *et al.* (2017)
BOSS collaboration

Redshift-space distortions

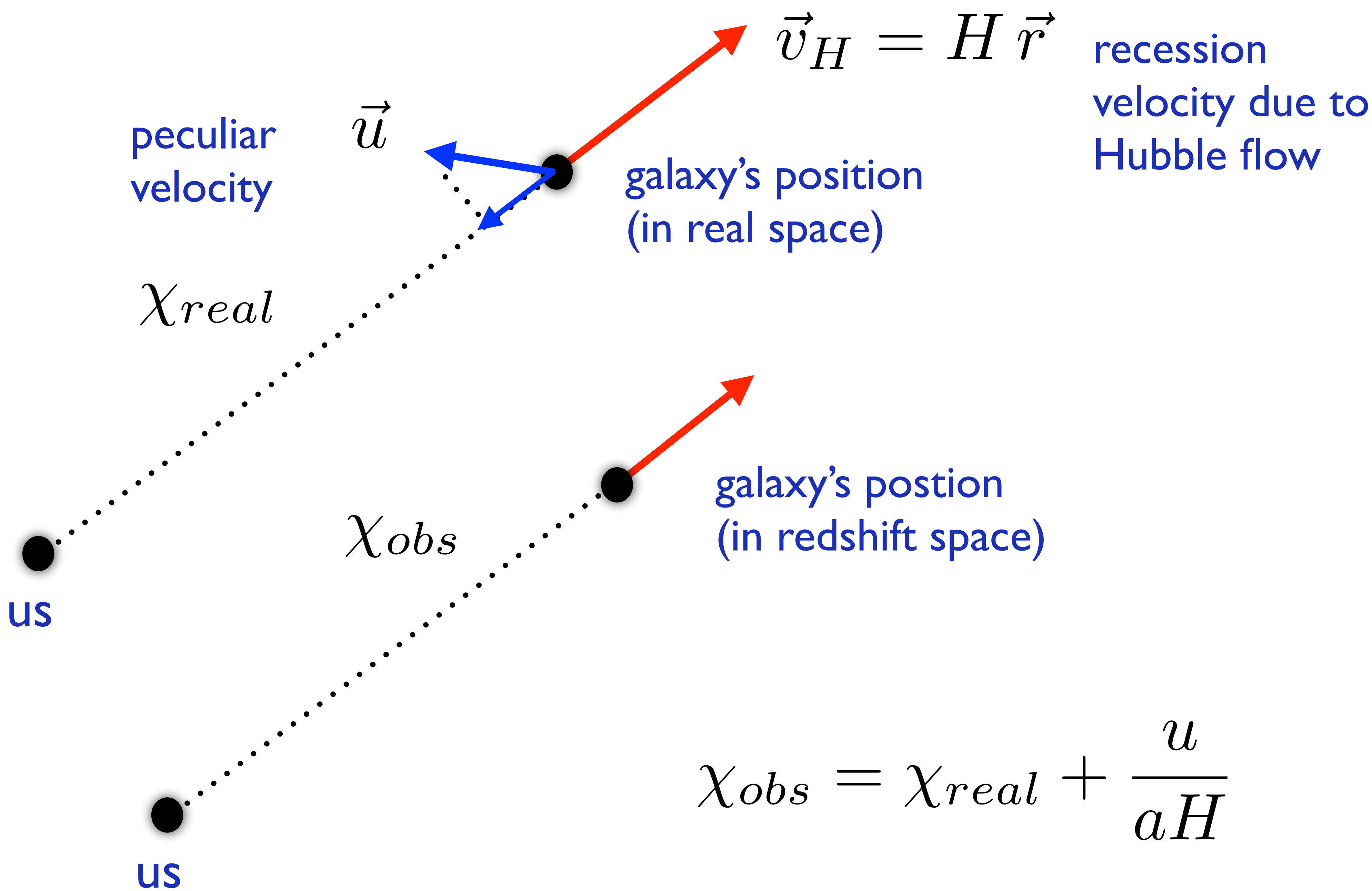
Redshift-space

Galaxies are observed in **redshift space** *not* in real space



Redshift-space

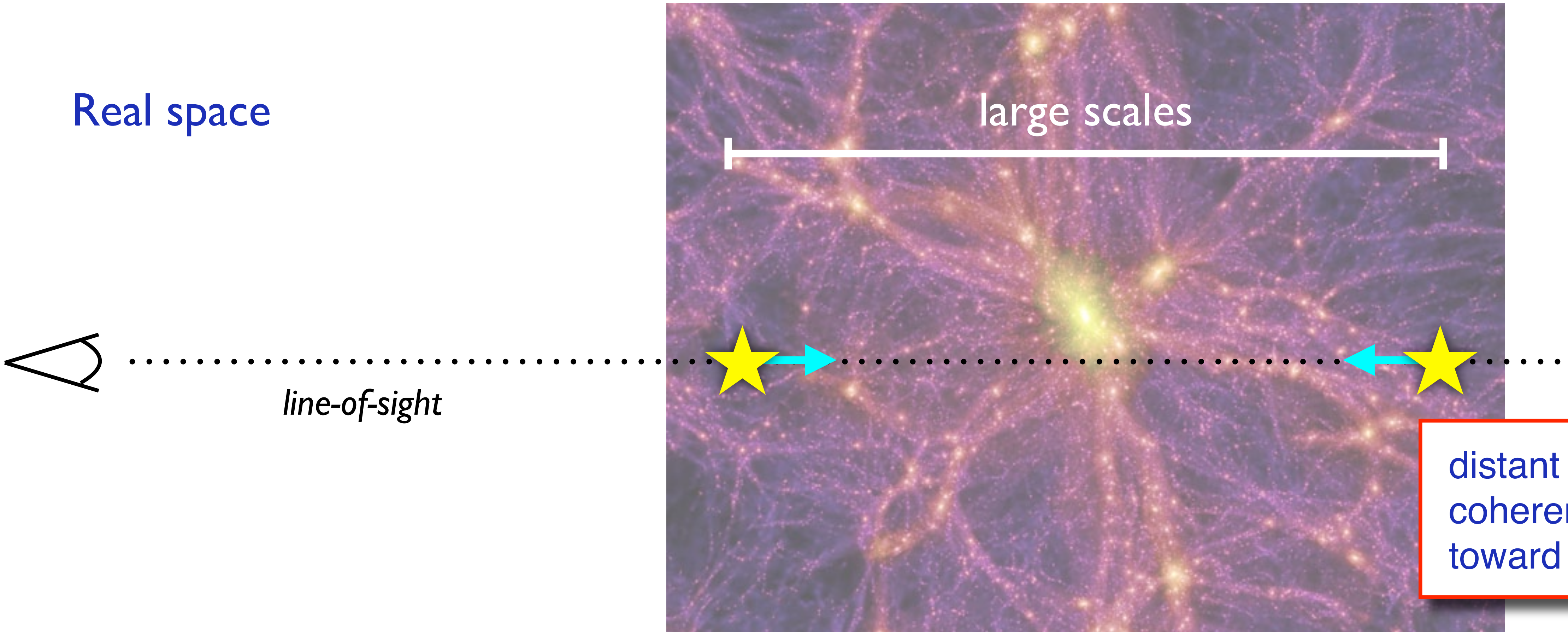
Galaxies are observed in **redshift space** *not* in real space



$$\chi_{obs} = \chi_{real} + \frac{u}{aH}$$

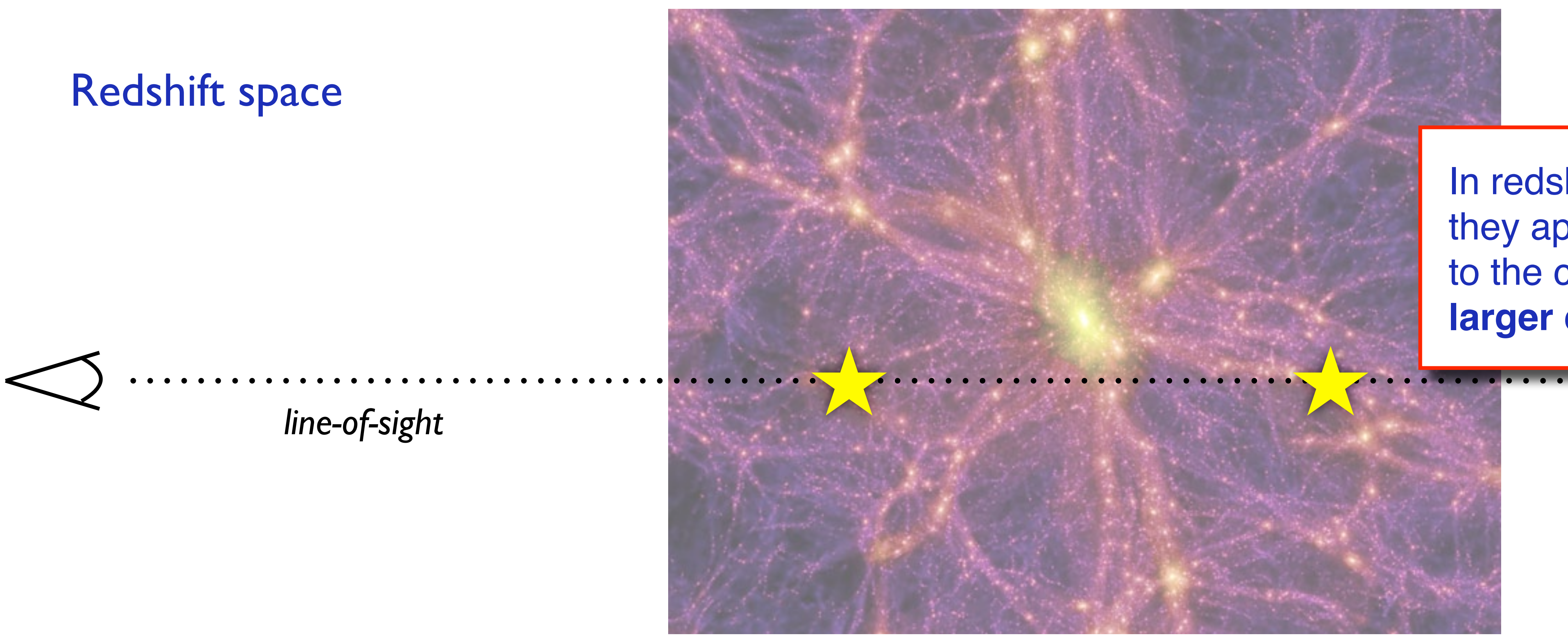
Kaiser effect

Real space



distant galaxies coherently in-falling toward a cluster

Redshift space



In redshift space they appear **closer** to the cluster: **larger clustering!**

Kaiser effect

$$\boxed{P_s(\vec{k}) = \left(1 + \frac{f}{b}\,\mu_k^2\right)^2 P_g(k)}$$

the **redshift-space power spectrum** is **anisotropic**!
(and so is the correlation function)

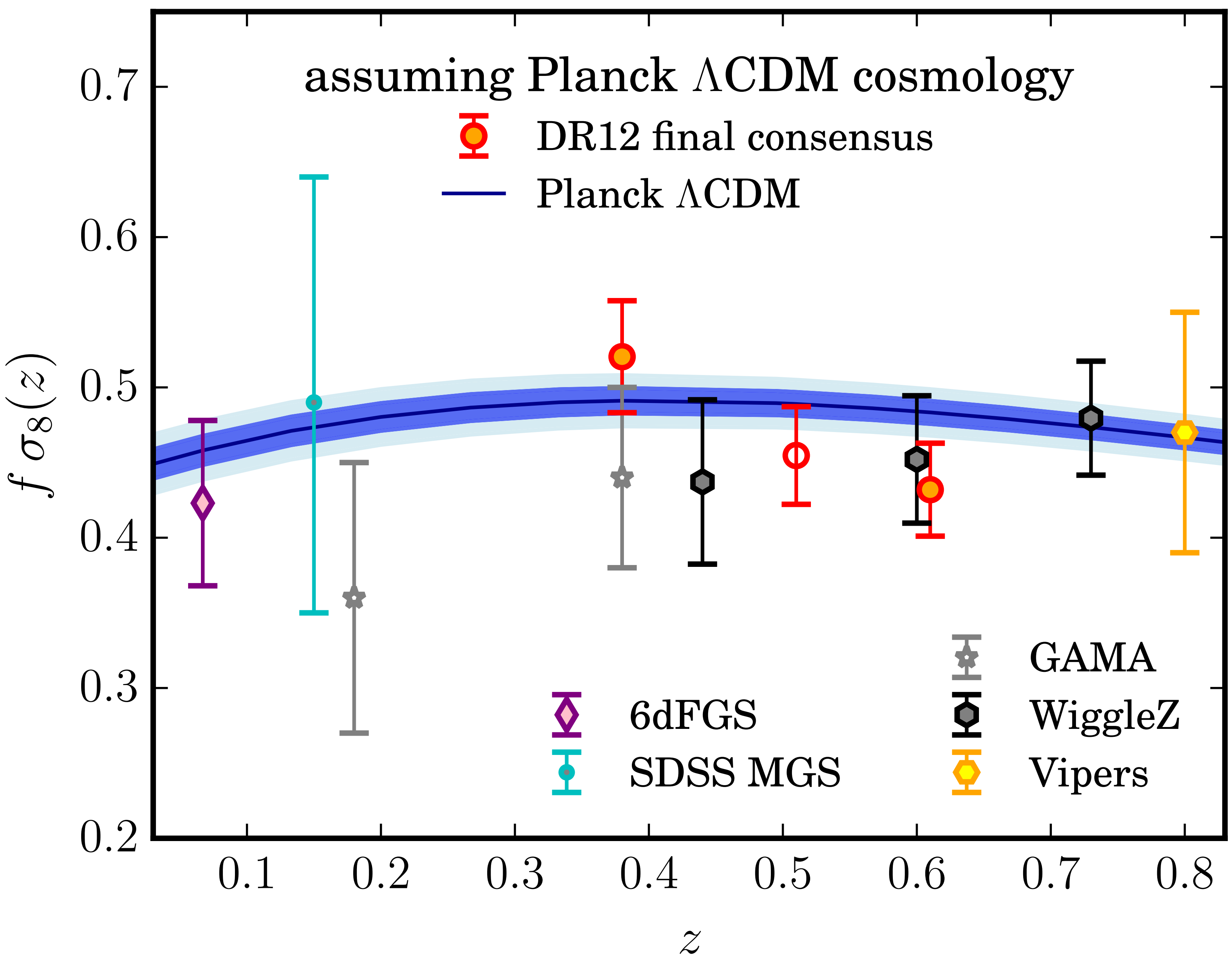
$$f = \frac{d\ln D(a)}{d\ln a}$$

enhanced clustering along the line-of-sight, proportional to the **growth rate**

We measure power spectrum **multipoles**

$$P_\ell(k) \equiv \frac{2\ell+1}{2}\int_{-1}^{1} L_\ell(\mu) P(\mu, k)\,\mathrm{d}\mu$$

Constraining the growth rate



Recent RSD measurements against CMB expectation (blu)

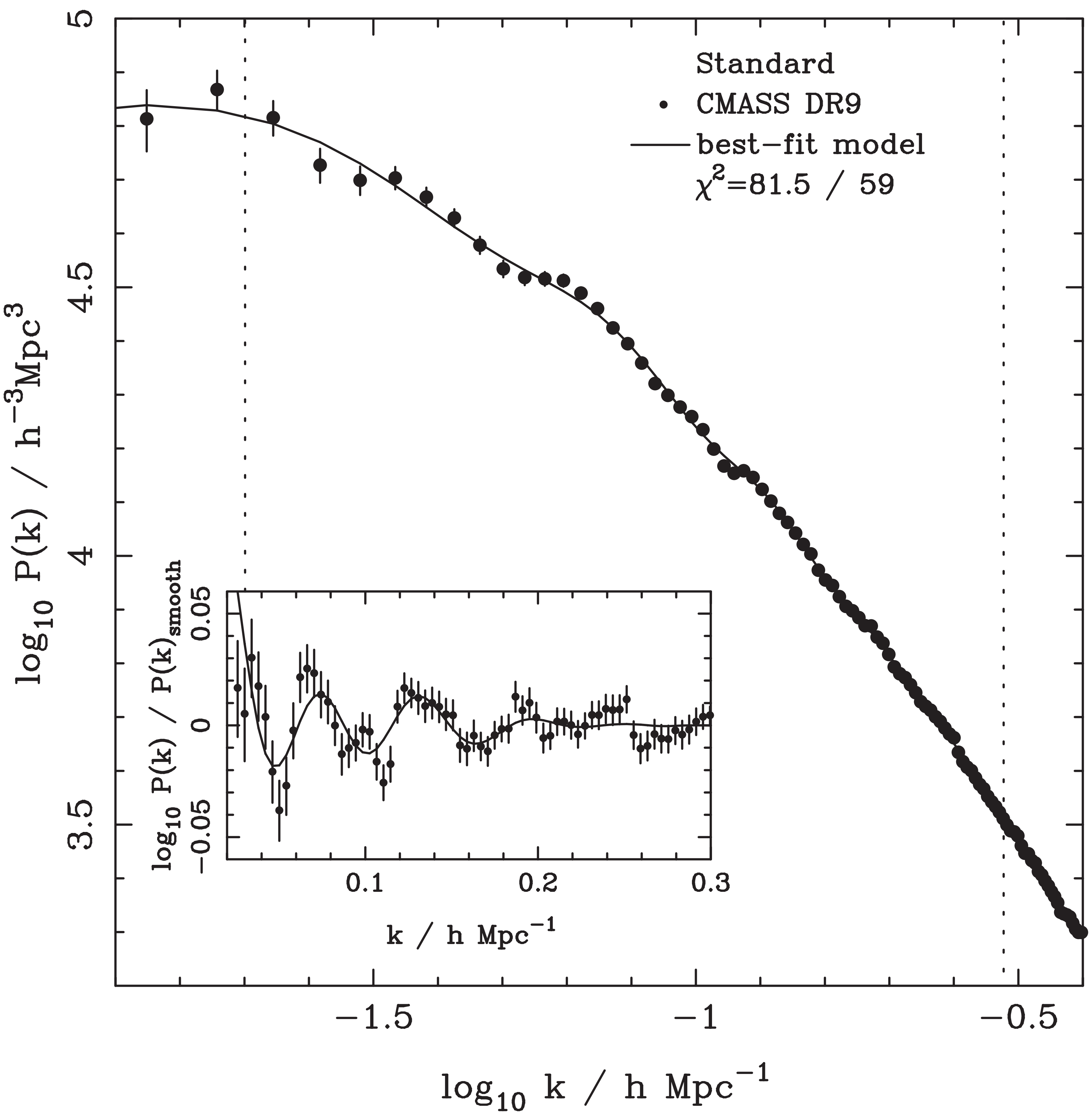
Alam *et al.* (2017) **BOSS collaboration**

$$f \equiv \frac{d\ln D(a)}{d\ln a} = \Omega_m^\gamma(z)$$

This is a test of GR on large scales

Full-shape analysis

Full-shape analysis

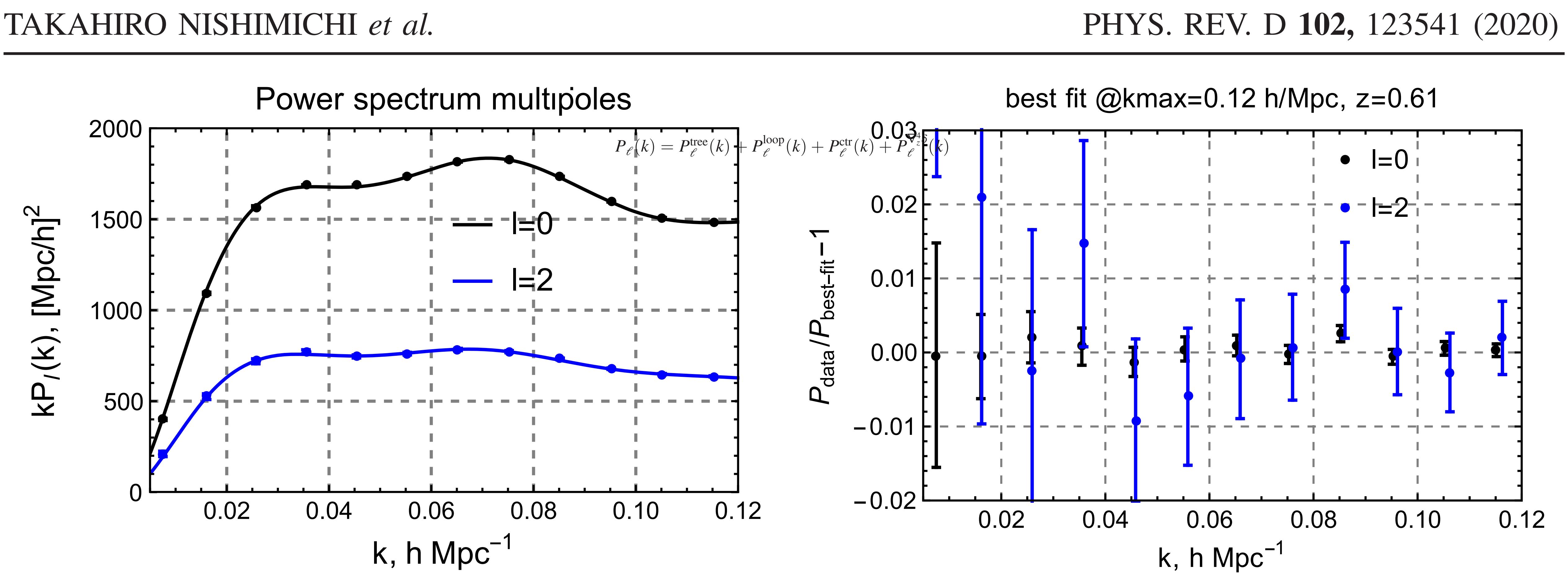


We now think we understand the power spectrum well enough:

we should have a systematics-free model of the nonlinear galaxy power spectrum in redshift space

Full-shape analysis

We should have a systematic-free model of the nonlinear galaxy power spectrum in redshift space



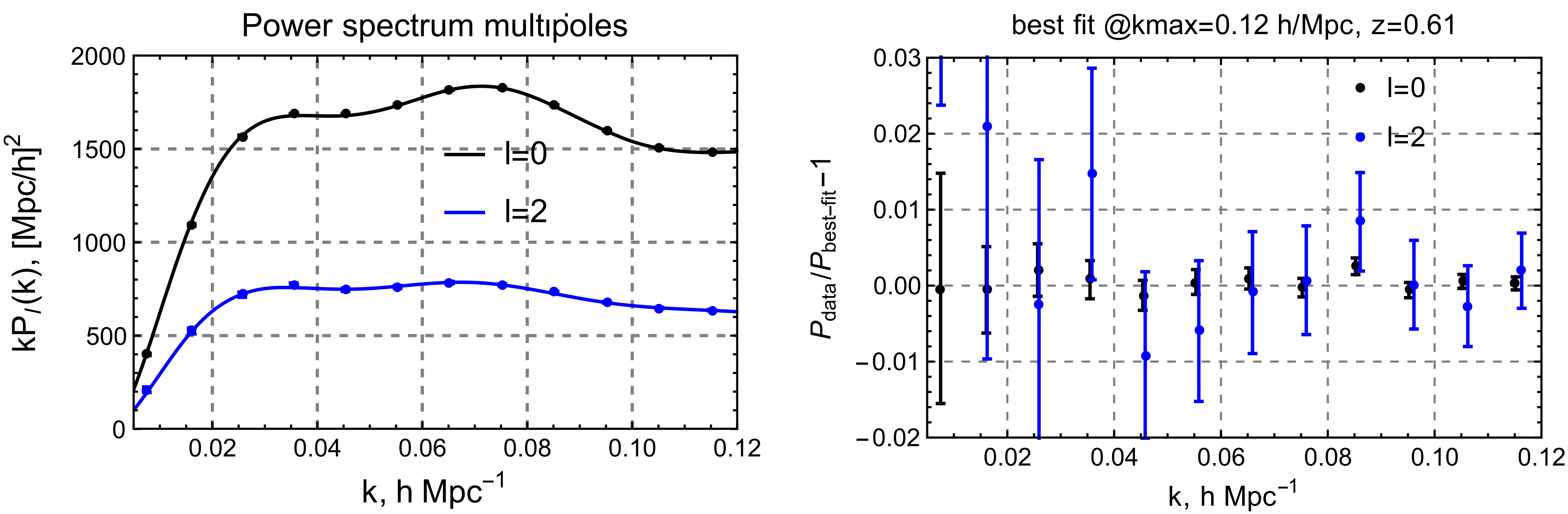
Test on a very large volume: 566 h^{-3}Gpc^3 ~ 100 times BOSS

Full-shape analysis

$$P_\ell(k) = P_\ell^{\rm tree}(k) + P_\ell^{\rm loop}(k) + P_\ell^{\rm ctr}(k) + P_\ell^{\nabla_z^4\delta}(k)$$

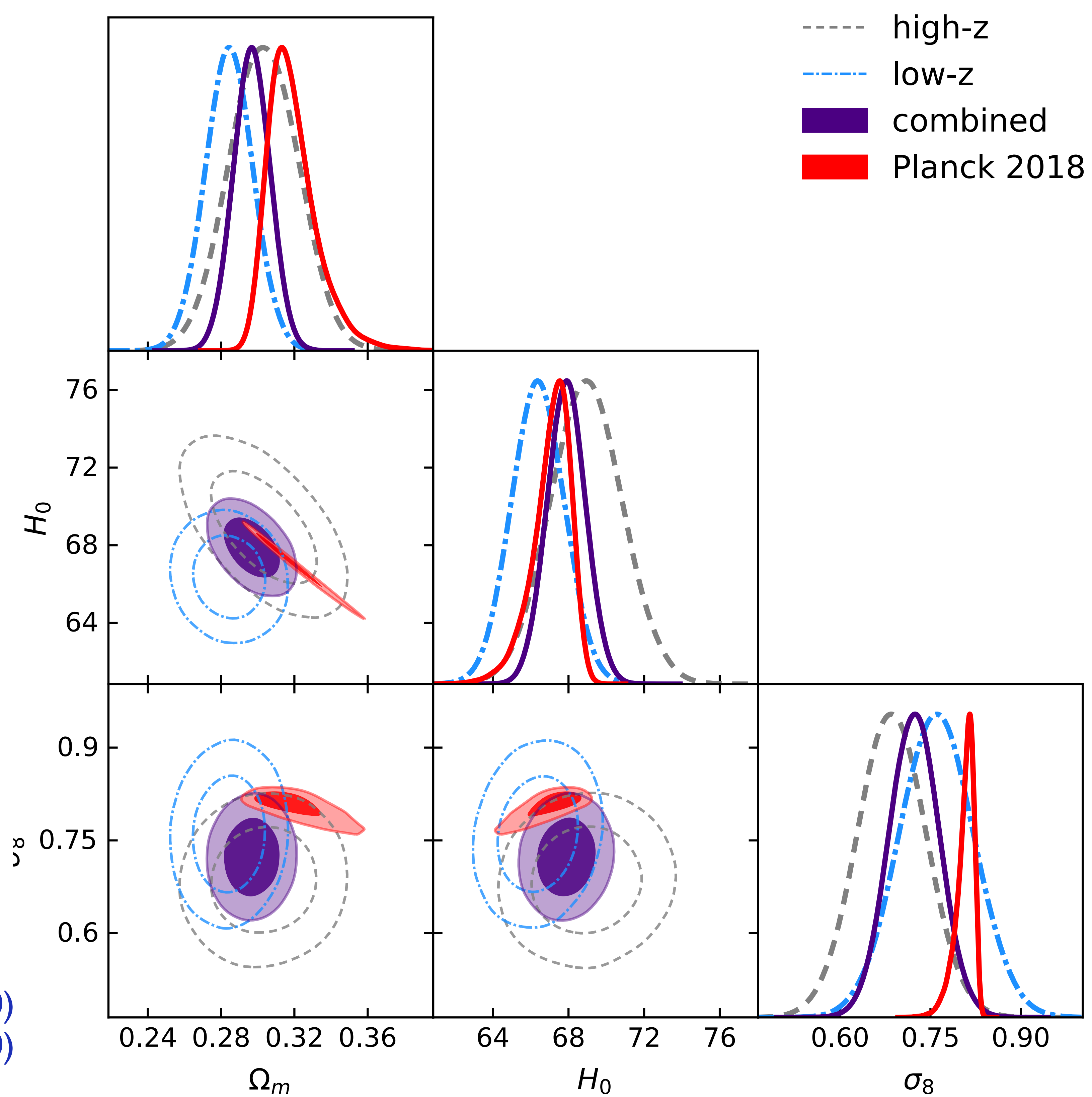
EFTofLSS: 3 cosmological + 8 nuisance parameters (more or less)

TAKAHIRO NISHIMICHI *et al.* PHYS. REV. D **102,** 123541 (2020)



Full-shape analysis

Independent analysis of the BOSS data



Ivanov *et al.* (2020)
see also D'Amico et al. (2020)

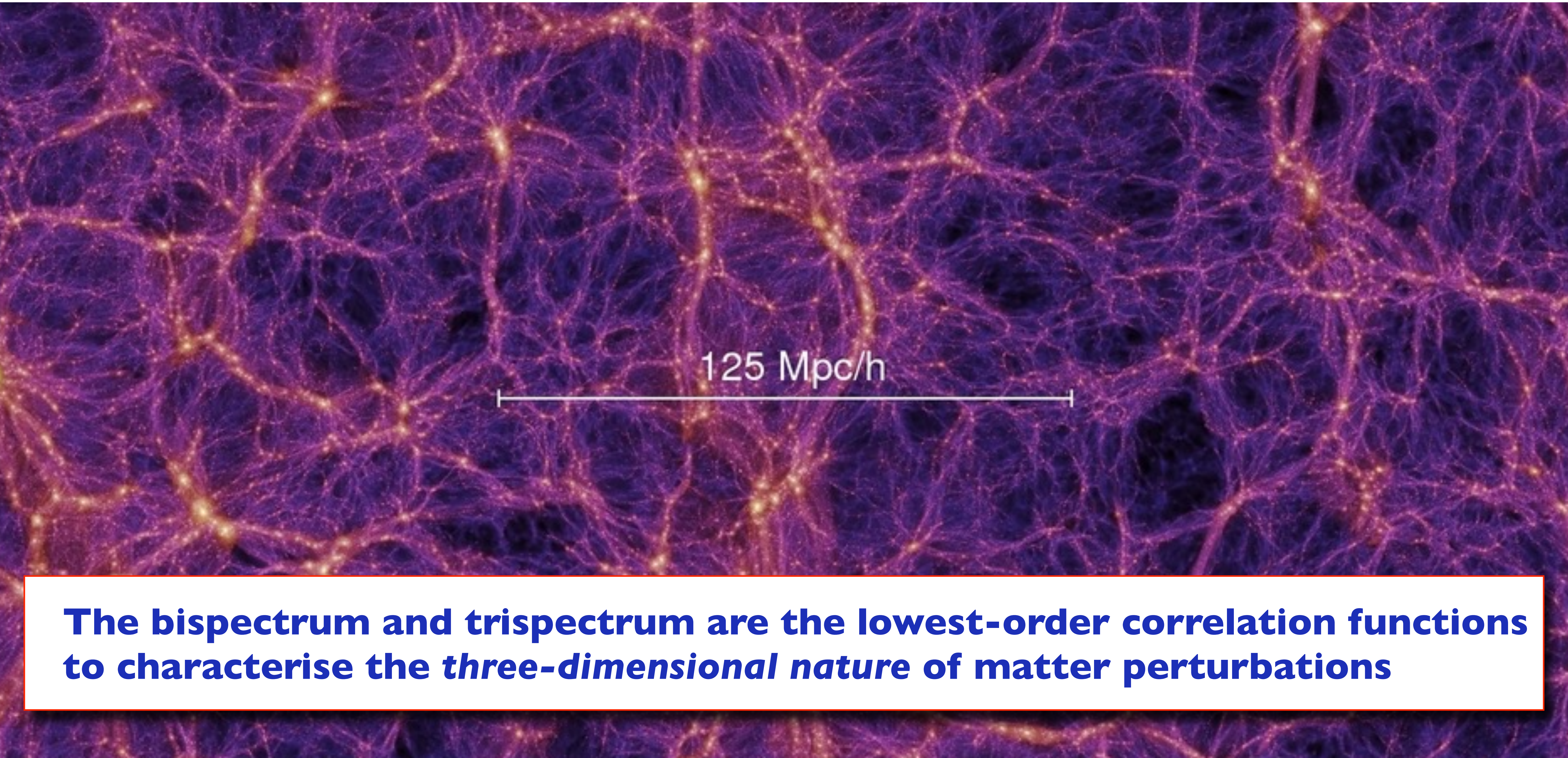
Beyond the power spectrum

Fourier space: correlation functions

Higher-order correlation functions:

$$\langle \delta_{\vec{k}_1} \delta_{\vec{k}_2} \delta_{\vec{k}_3} \rangle \equiv \delta_D(\vec{k}_1 + \vec{k}_2 + \vec{k}_3)\, B(k_1, k_2, k_3)$$ **bispectrum**

$$\langle \delta_{\vec{k}_1} \delta_{\vec{k}_2} \delta_{\vec{k}_3} \delta_{\vec{k}_4} \rangle \equiv \delta_D(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 + \vec{k}_4)\, T(\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{k}_4)$$ **trispectrum**



The bispectrum and trispectrum are the lowest-order correlation functions to characterise the *three-dimensional nature* of matter perturbations

The matter bispectrum

Perturbative solution for the matter density, in Fourier space

$$\delta_{\vec{k}} = \delta^{(1)}_{\vec{k}} + \delta^{(2)}_{\vec{k}} + \ldots$$

linear solution

quadratic correction

$$\delta^{(2)}_{\vec{k}} = \int d^3q\, F_2(\vec{k}-\vec{q},\vec{q})\, \delta^{(1)}_{\vec{k}-\vec{q}}\, \delta^{(1)}_{\vec{q}}$$

Perturbative solution for the matter 3-point function

$$\langle\delta\delta\delta\rangle = \langle\delta^{(1)}\delta^{(1)}\delta^{(1)}\rangle + \langle\delta^{(1)}\delta^{(1)}\delta^{(2)}\rangle + \ldots$$

loop corrections

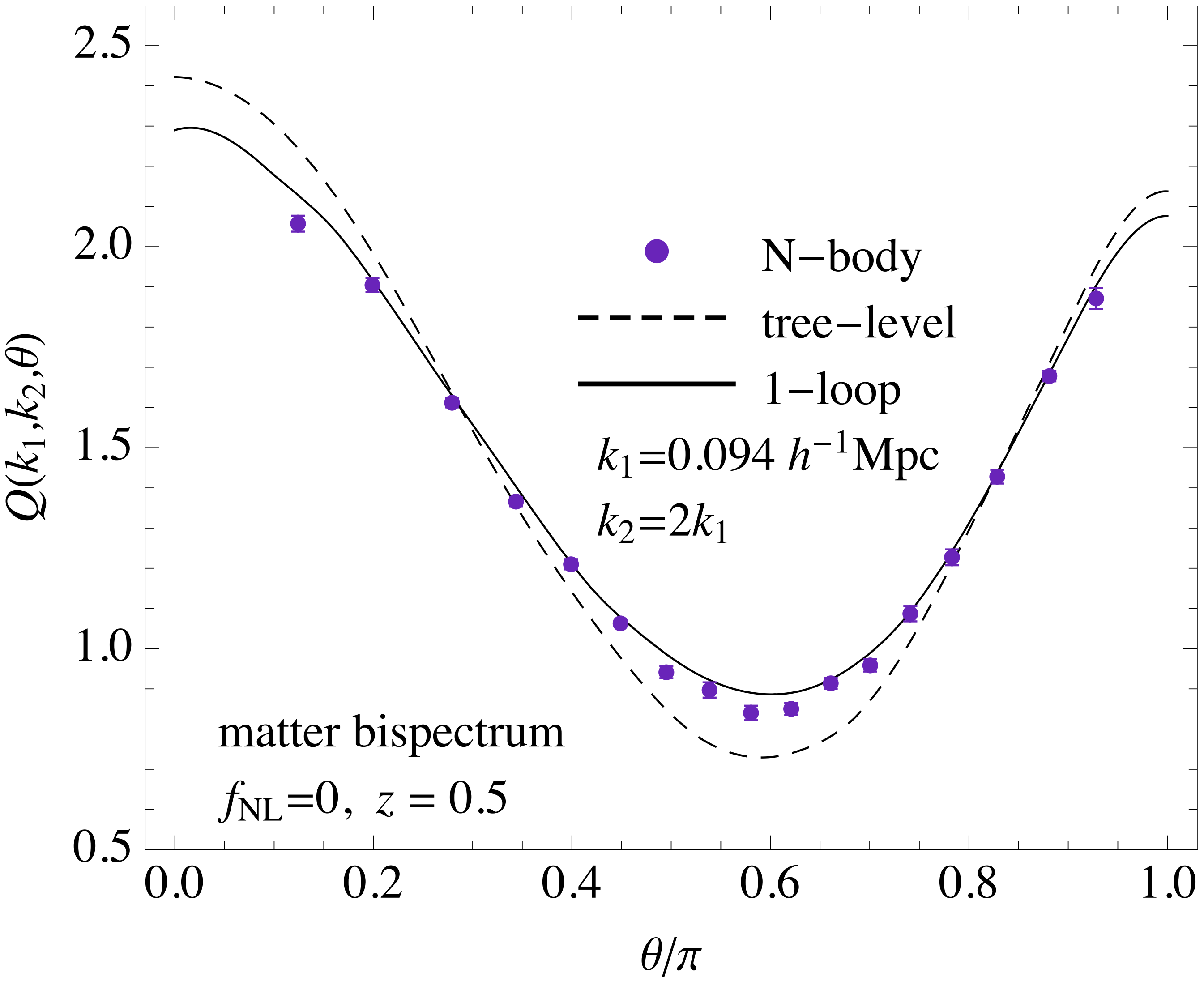
= 0 for Gaussian initial conditions

non-zero bispectrum *induced by gravity*

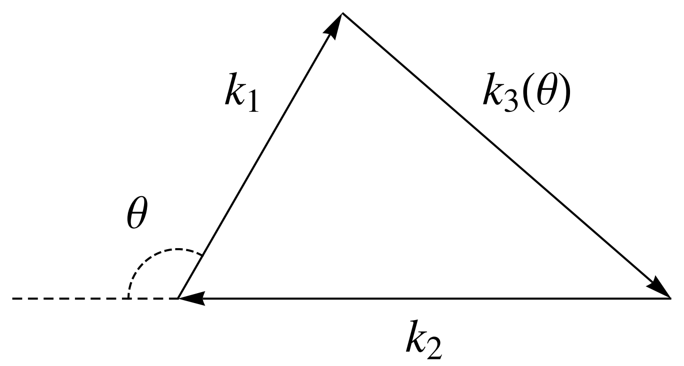
The matter bispectrum

$$B_G^{tree}(k_1, k_2, k_3) = 2\, F_2(\vec{k}_1, \vec{k}_2)\, P_0(k_1)\, P_0(k_2) + 2 \text{ perm.}$$

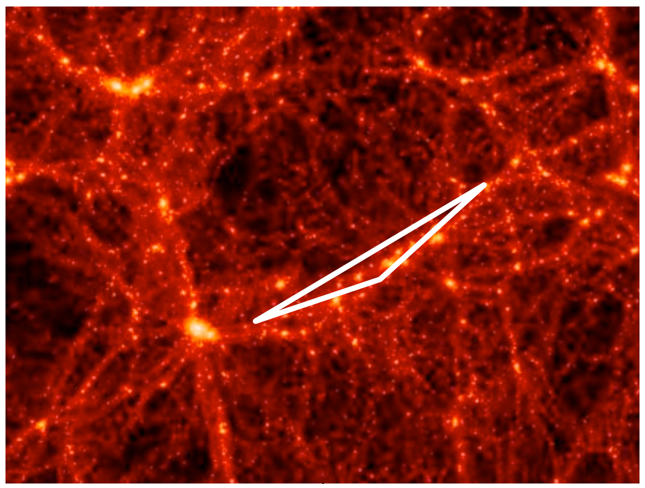
$$Q(k_1, k_2, k_3) = \frac{B(k_1, k_2, k_3)}{P(k_1)P(k_2) + P(k_1)P(k_3) + P(k_2)P(k_3)}$$



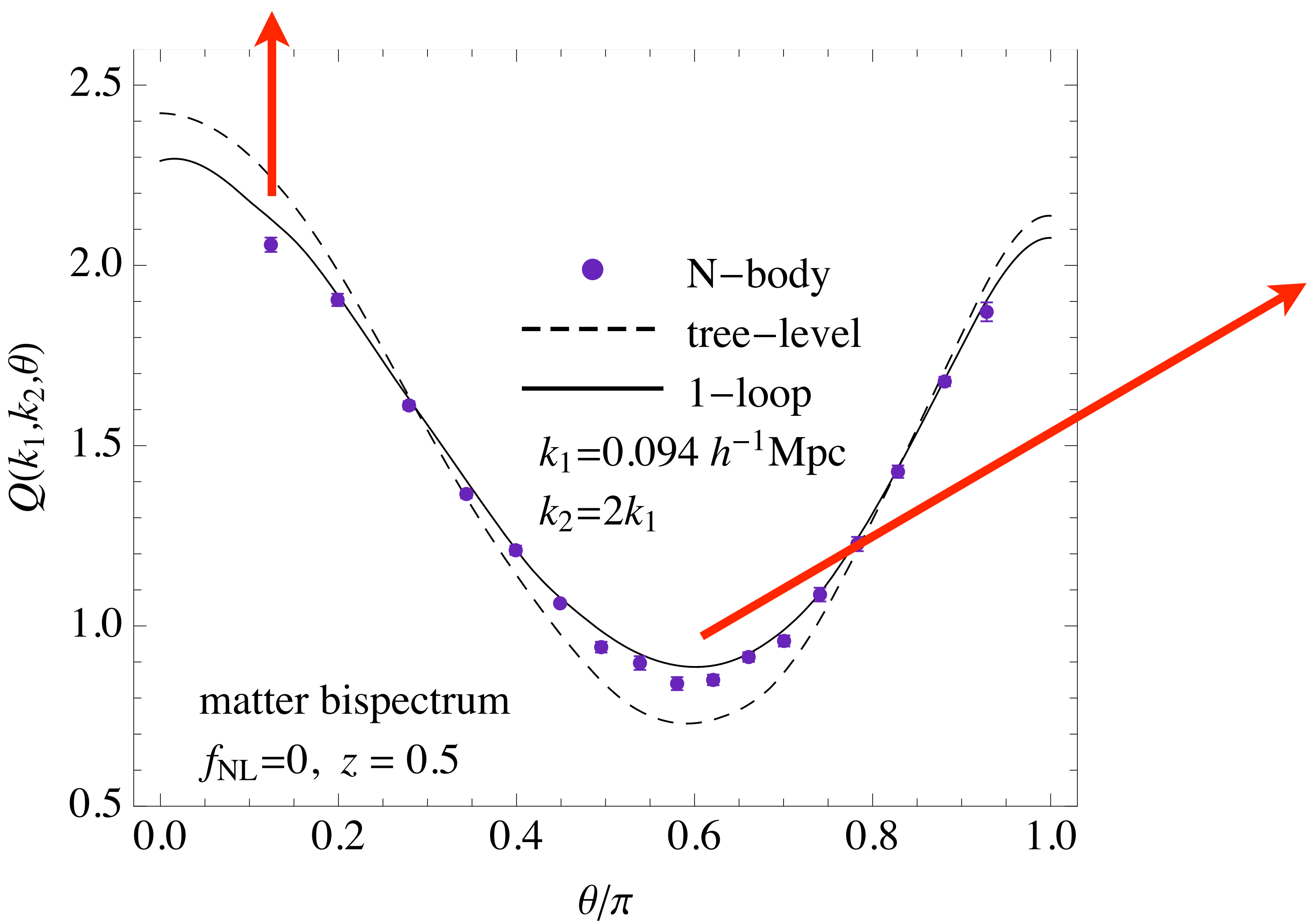
Plot of the reduced bispectrum with **fixed** k_1 and k_2 as a function of the **angle** between the two wavenumbers

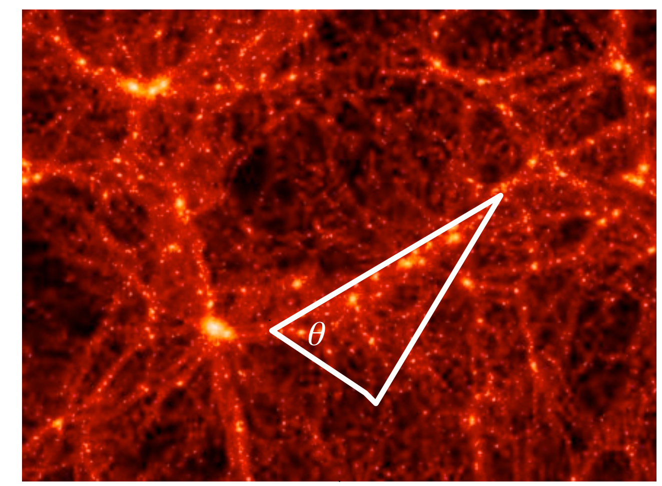


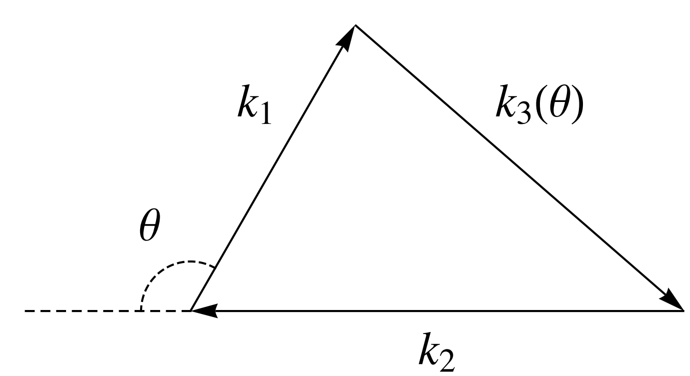
The galaxy bispectrum



$$Q(k_1, k_2, k_3) = \frac{B(k_1, k_2, k_3)}{P(k_1)P(k_2) + P(k_1)P(k_3) + P(k_2)P(k_3)}$$







The galaxy bispectrum

local bias

$$\delta_g(x) \equiv \frac{n_g(x) - \bar{n}_g}{\bar{n}_g} = f\left[\delta(x)\right]$$

expand it in a Taylor series ...

$$\delta_g(x) = b_1\, \delta(x) + \frac{1}{2}\, b_2\, \delta^2(x) + \ldots$$

Linear bias

Quadratic bias correction

Perturbative solution for the **galaxy** 3-point function

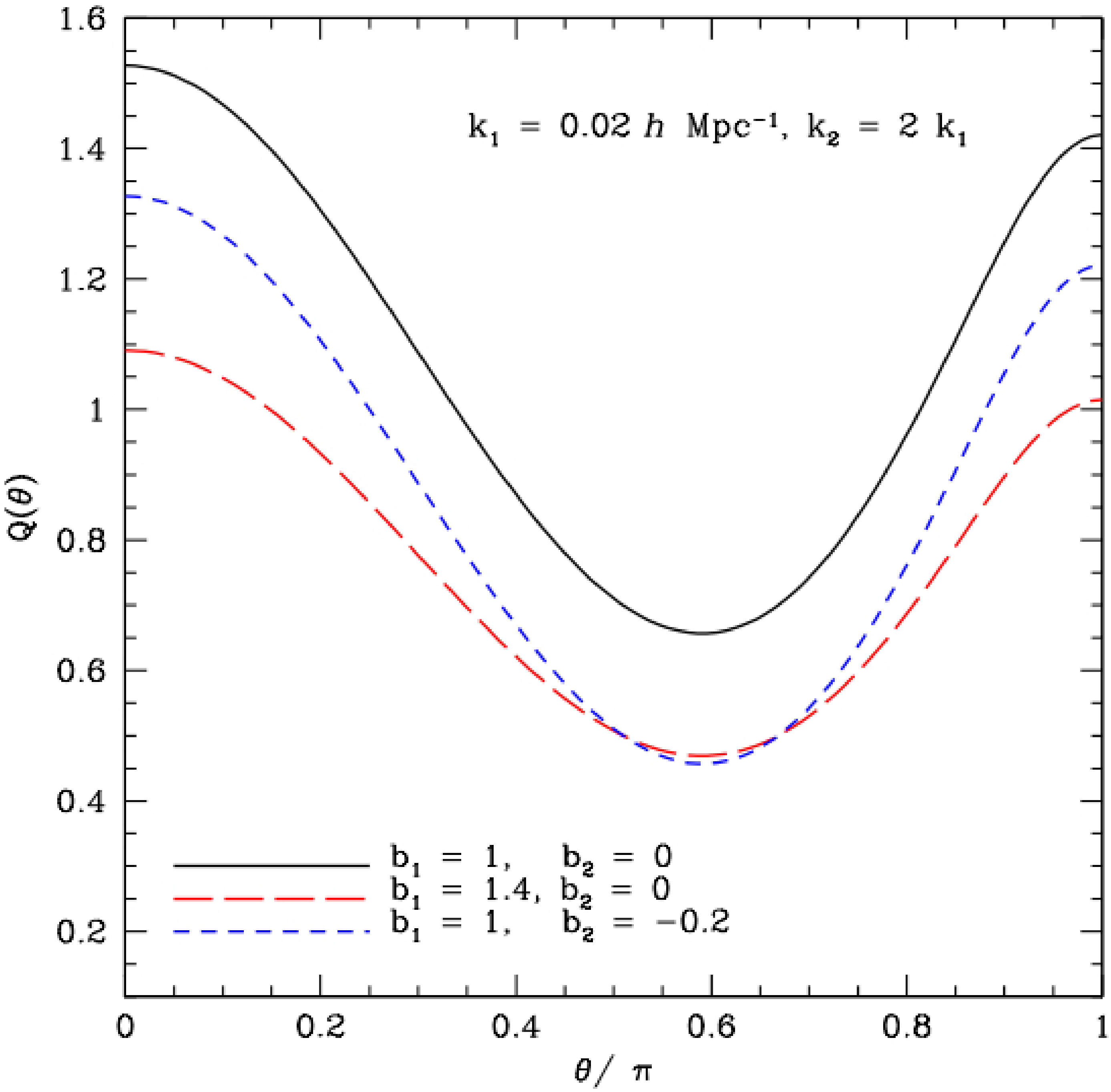
$$\langle \delta_g \delta_g \delta_g \rangle = b_1^3\, \langle \delta\delta\delta \rangle + b_1^2\, b_2\, \langle \delta\delta\delta^2 \rangle + \ldots$$

matter bispectrum

bispectrum induced by nonlinear bias

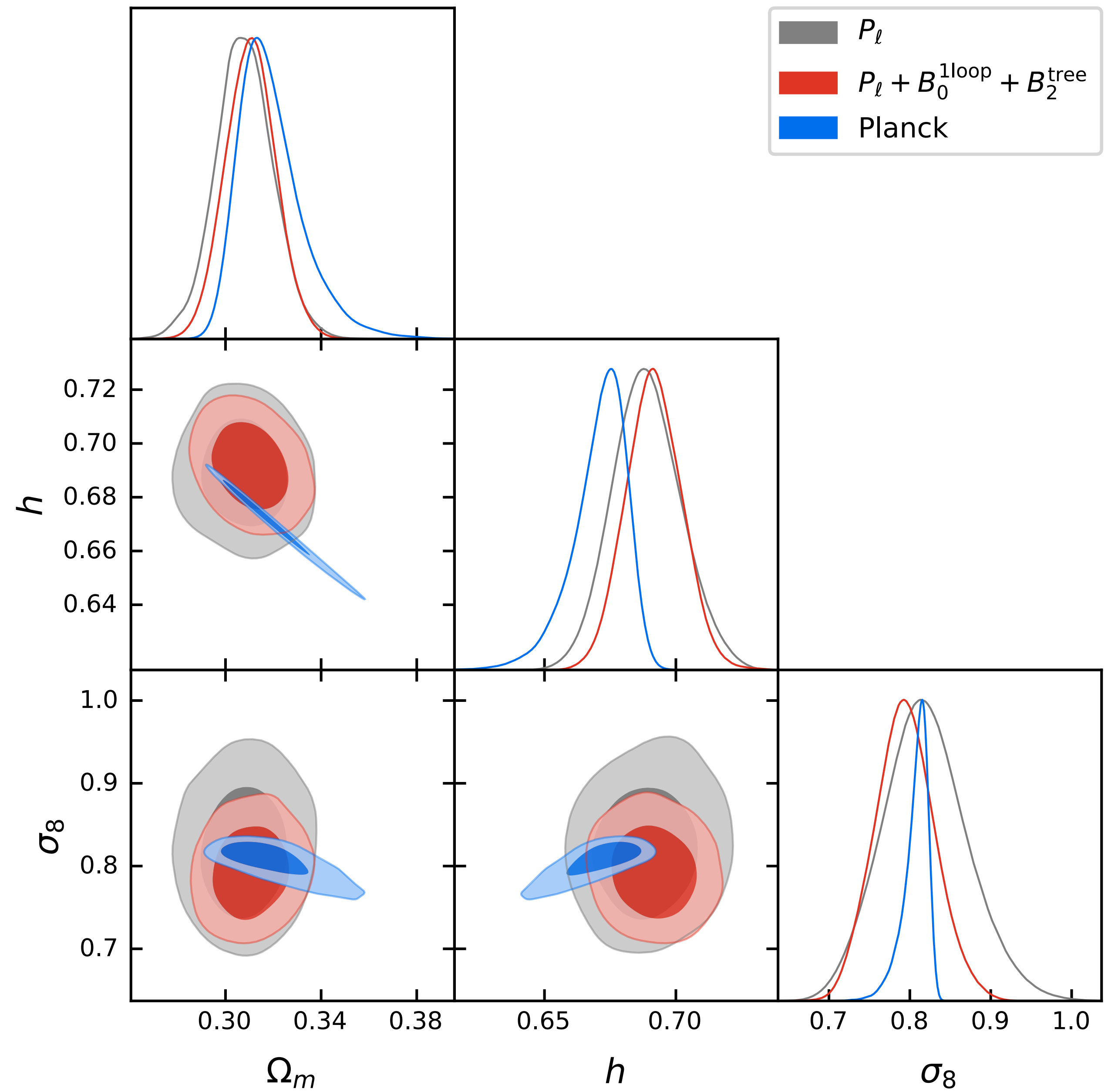
$$B_g(k_1, k_2, k_3) = b_1^3\, B(k_1, k_2, k_3) + b_1^2\, b_2\, P(k_1)\, P(k_2) + 2\ \text{perm.} + \ldots$$

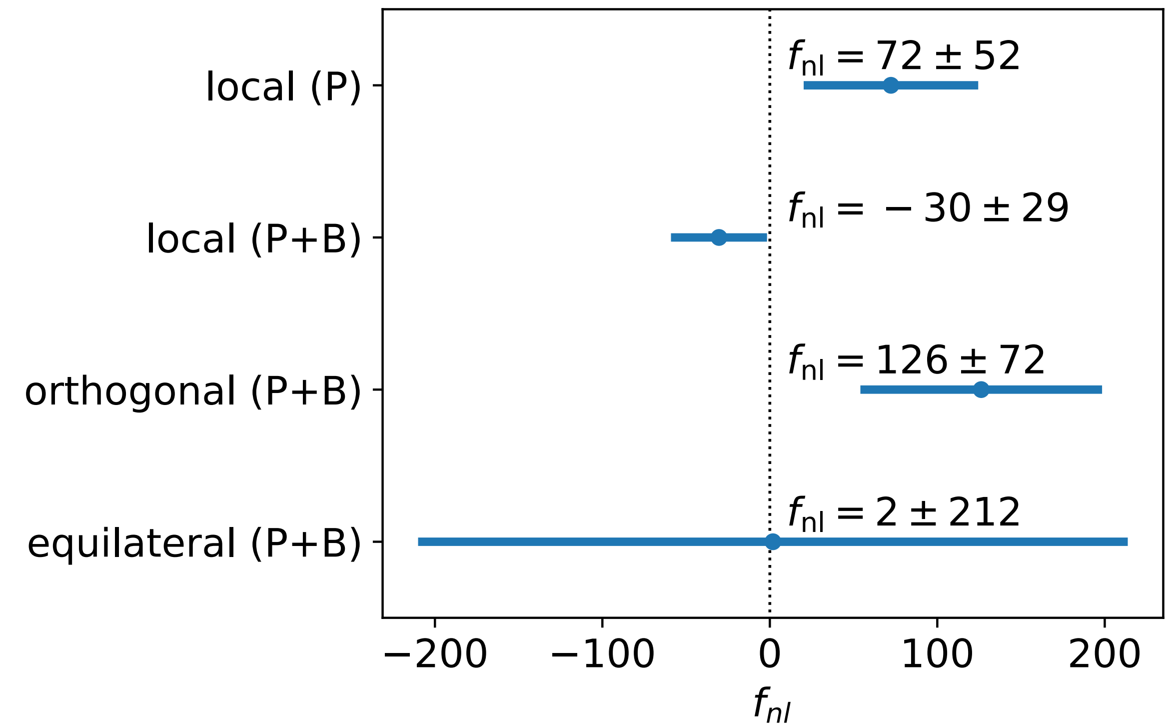
The galaxy bispectrum



$$Q_g(k_1, k_2, k_3) = \frac{1}{b_1} Q(k_1, k_2, k_3) + \frac{b_2}{b_1^2}$$

The galaxy bispectrum





D'Amico et al. (2022A, B)

Conclusions

Cosmological constraints from spectroscopic surveys

Future galaxy redshift surveys (e.g. DESI from the ground or Euclid from the sky) will continue an on-going effort to map the **large-scale galaxy distribution**

Different **features of the galaxy power spectrum** provide different constraint on the cosmological model:

- **BAO are a *standard ruler***, a geometrical probe of the expansion history
- The **anisotropy** of the galaxy power spectrum (Redshift-Space Distortions) measure instead the *growth of structure*
- The **"shape" of the power spectrum** provide an upper bound on neutrino masses

Current efforts are aimed at extracting **all available information** in 2-point *and higher-order correlation functions* and extend PT predictions *beyond* the Standard Cosmological Model

But a lot more is brewing …