Cosmology Beyond the Average with 1pt Statistics

Cora Uhlemann (see also Oliver Friedrich's talk)
many others in Paris, Munich, Durham, Bonn, …

Newcastle University

IFPU Focus Week, June 2022

Newcastle University

Cosmology & Observational Astro

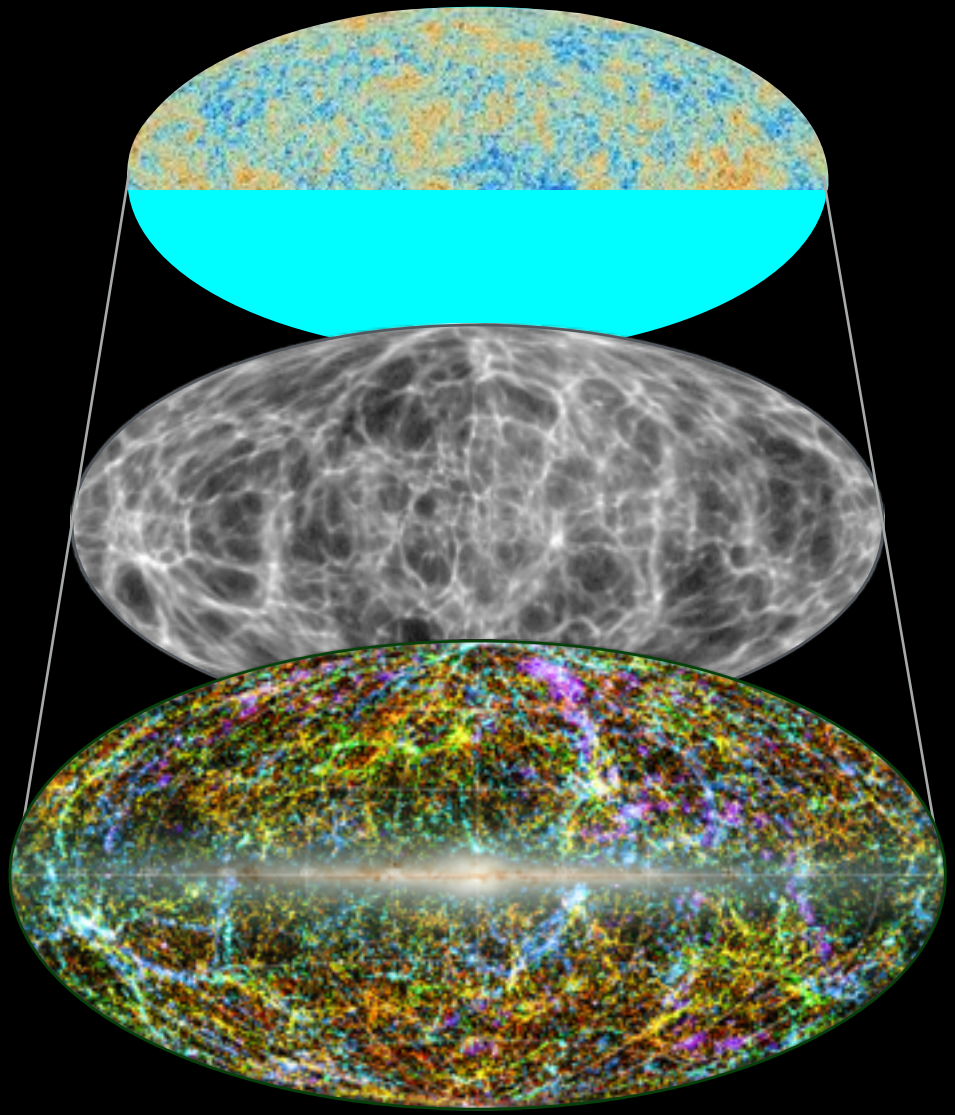
blogs.ncl.ac.uk/cosmology/

blogs.ncl.ac.uk/astro-obs/

Lectureship in Astrophysics open

Cosmic Laboratory

beginning
nearly uniform



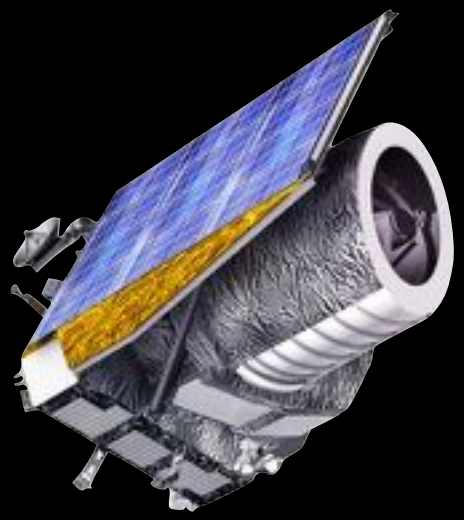
AFTERGLOW
early universe

SKELETON
Dark Matter

today
rich structure

COSMIC WEB
galaxies

Cosmic Laboratory



Euclid

~1 billion €, 1/3 sky
10 billion years look-back
galaxy positions,
redshifts & shapes

LSST

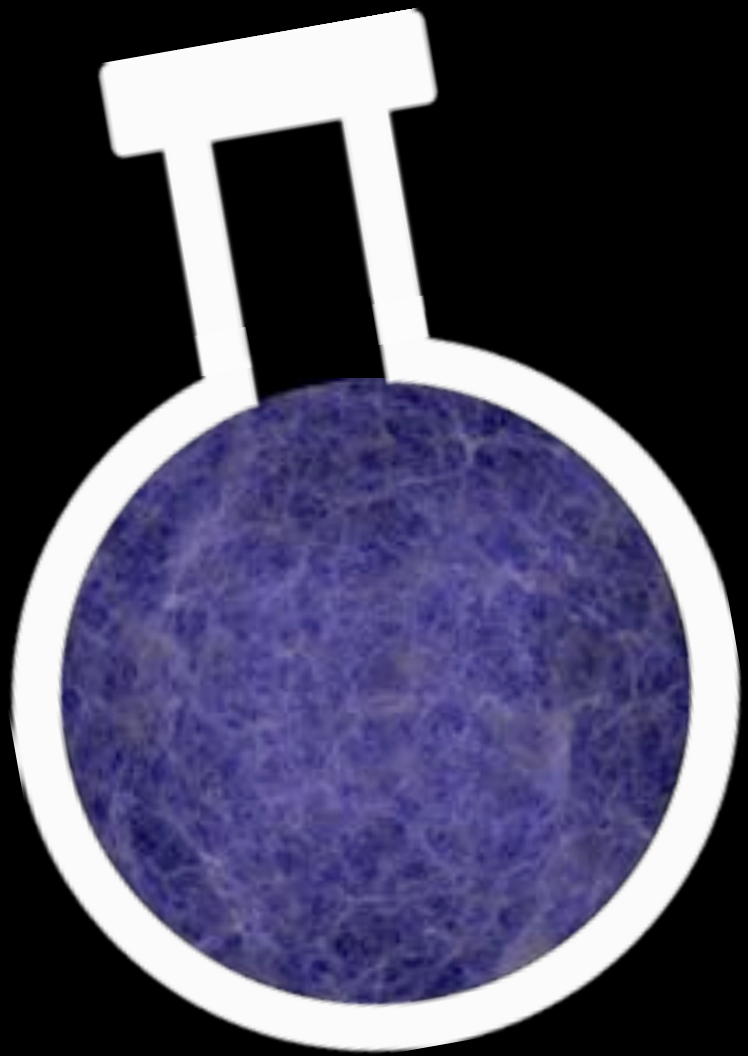
DESI

Cosmic Laboratory

dark energy & gravity,
dark matter, neutrinos,
early universe

Predict Clustering
nonlinear dynamics

Extract Information
non-Gaussian statistics



Good Old Days

CMB: one snapshot

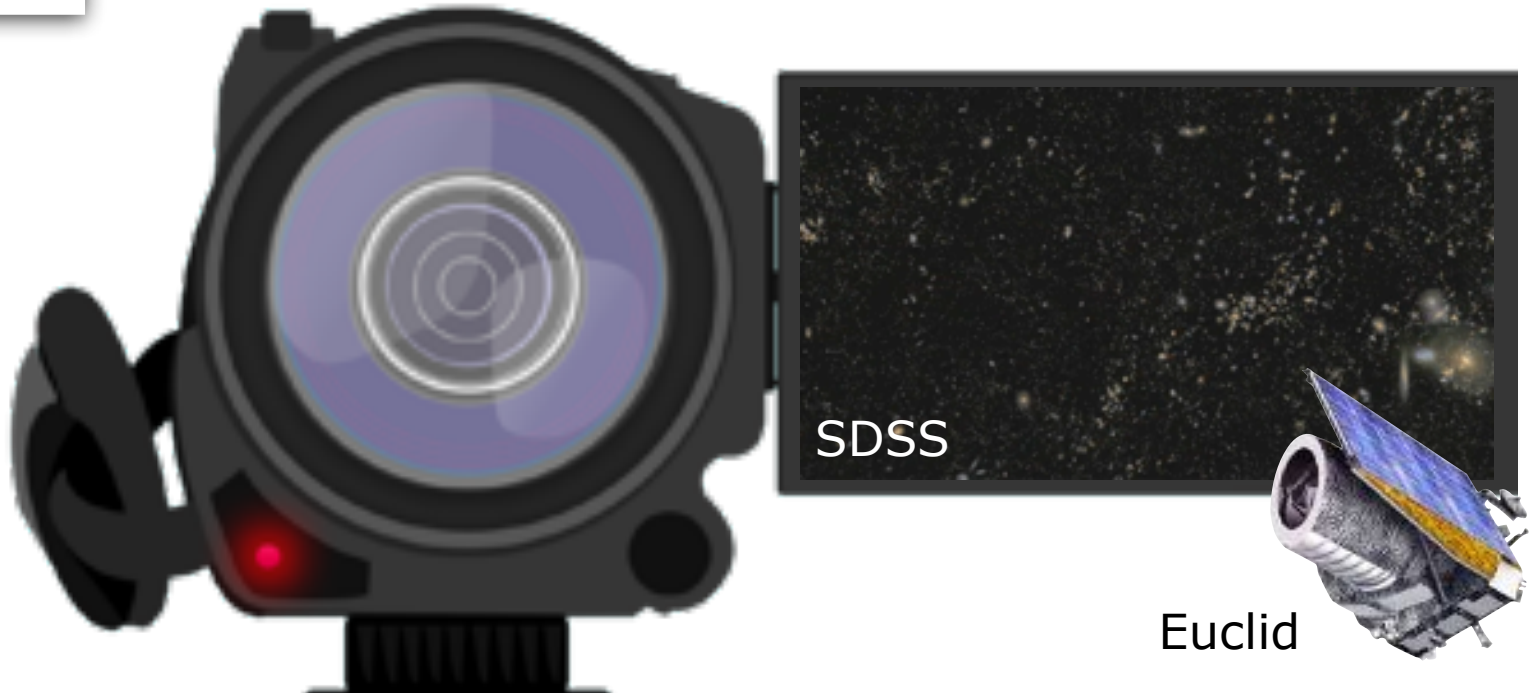
linear, almost Gaussian

captured by 2-point statistics

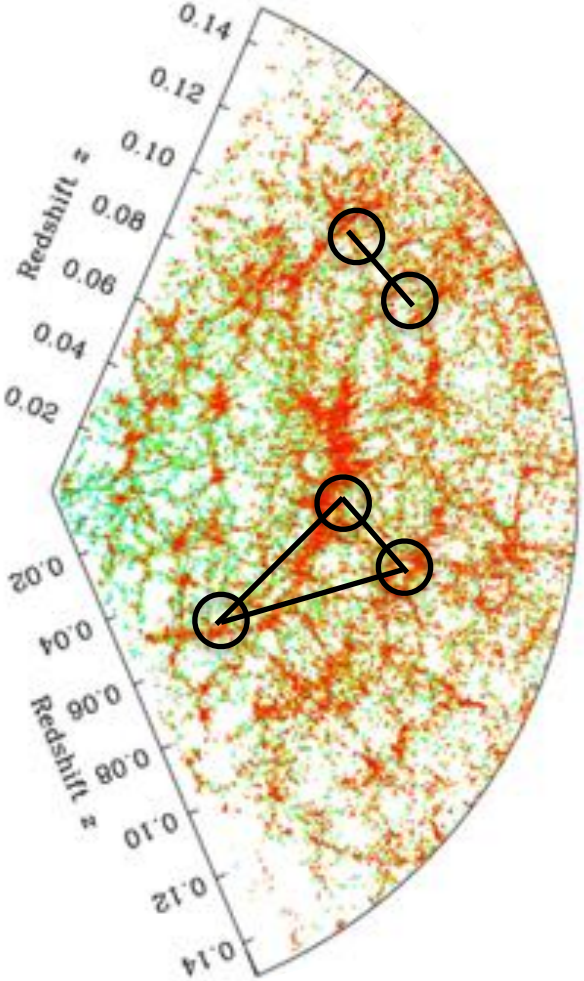
Good Old Days -> Future

CMB: one snapshot
linear, almost Gaussian

LSS: motion picture
nonlinear, non-Gaussian



Traditional Statistics



Gaussian: 2-pt correlation

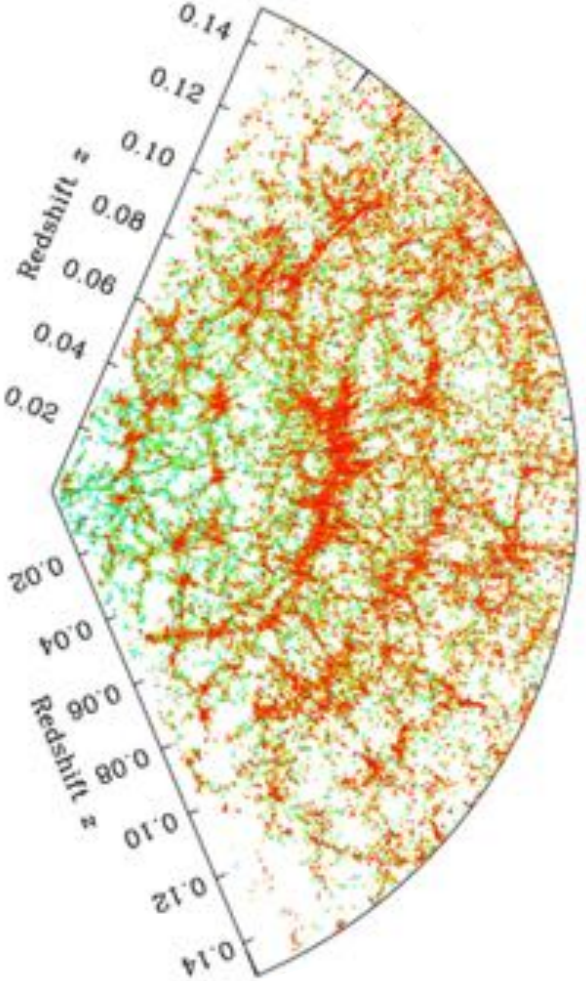
$$\xi(r) = \langle \delta(\boldsymbol{x}) \delta(\boldsymbol{x} + \boldsymbol{r}) \rangle$$

nonlinear → non-Gaussian

Higher N-pt correlations

N≥3 hard to measure

Statistics Wishlist



powerful statistics

beyond 2pt correlation

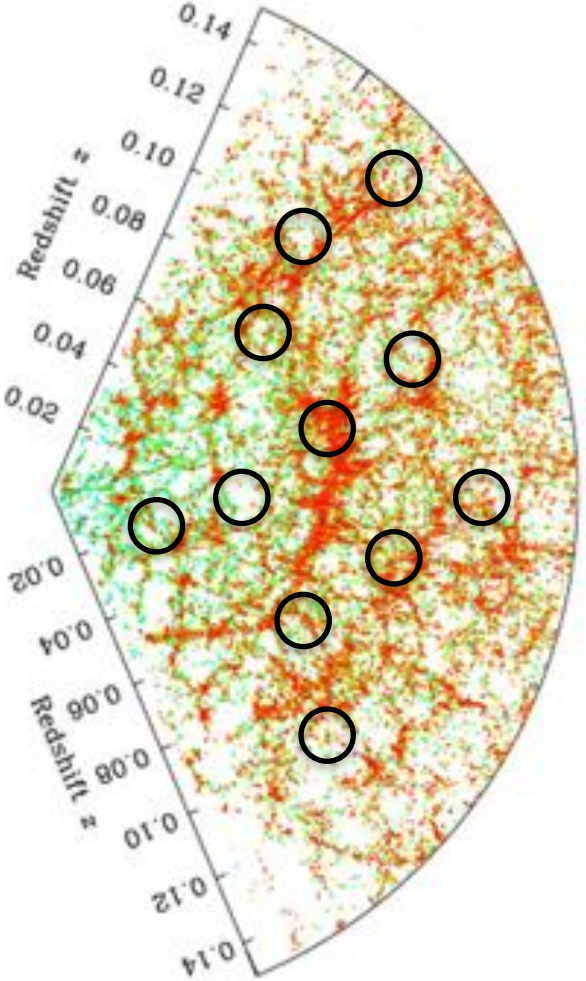
interesting regime

beyond average

predictive dynamics

beyond perturbative

Statistics Compromise



powerful statistics

1-point density PDF

interesting regime

large deviations

predictive dynamics

spherical collapse

Retro Statistics: 1-Point

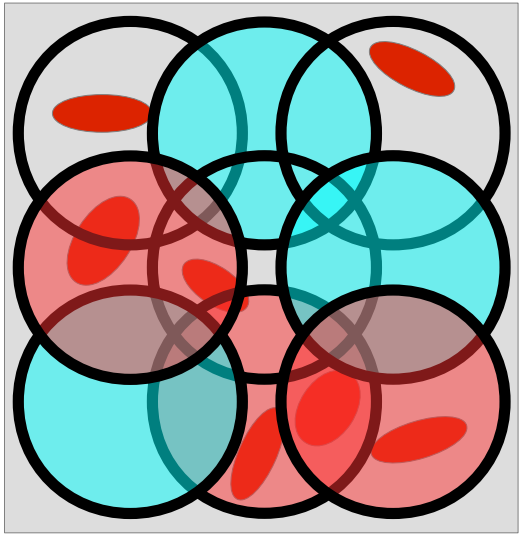
long history

first: Hubble `34
many thereafter

links to moments,
void probability, kNN

Leap in predictions

Leap in observations →



DES & KiDS, Euclid, LSST, DESI

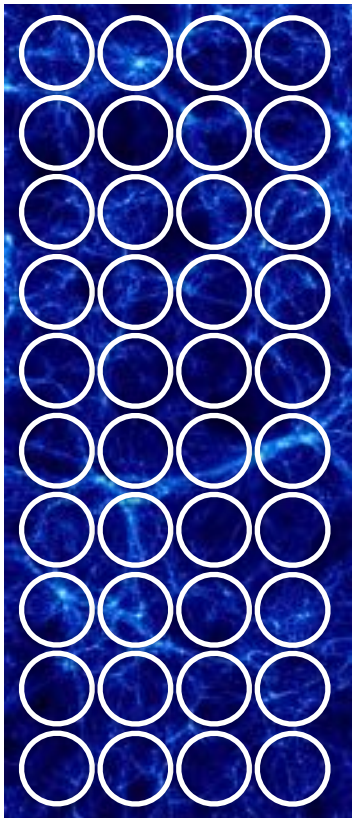
Matter 1-Point Statistics

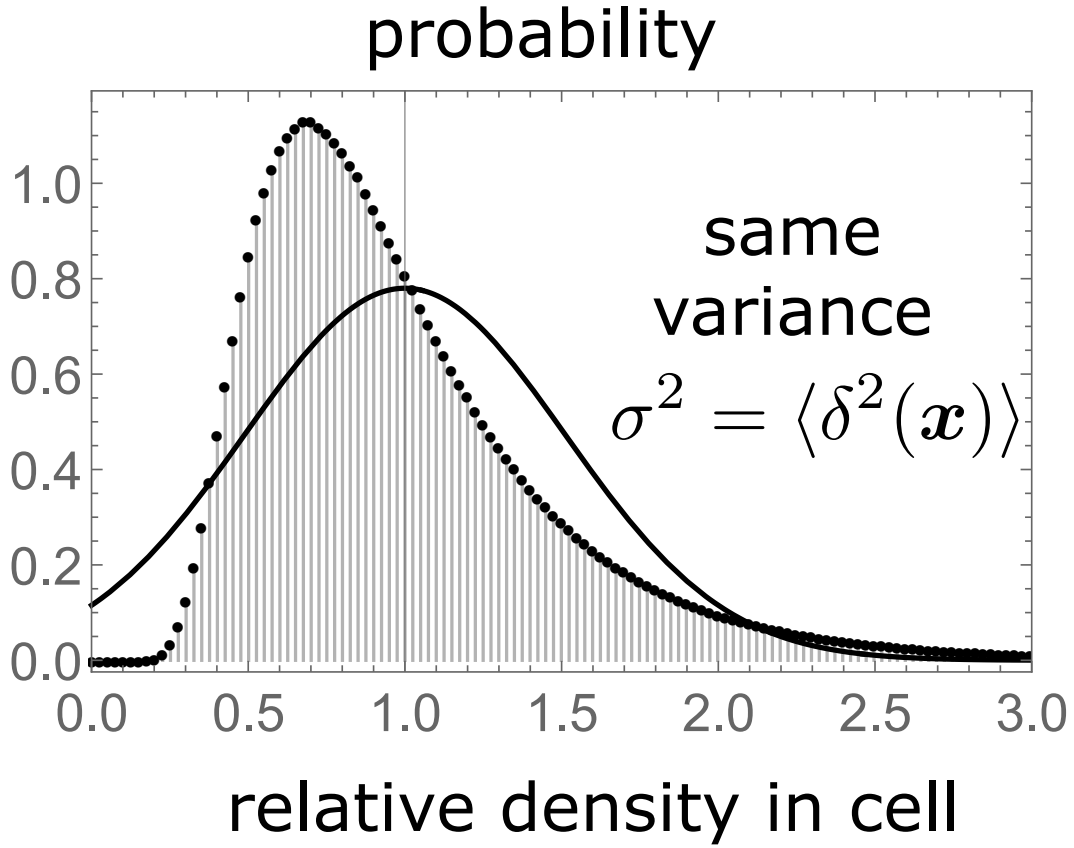
if we only could observe dark matter

1-Point Statistics Idea

Recapture lost information

smooth & plot histogram

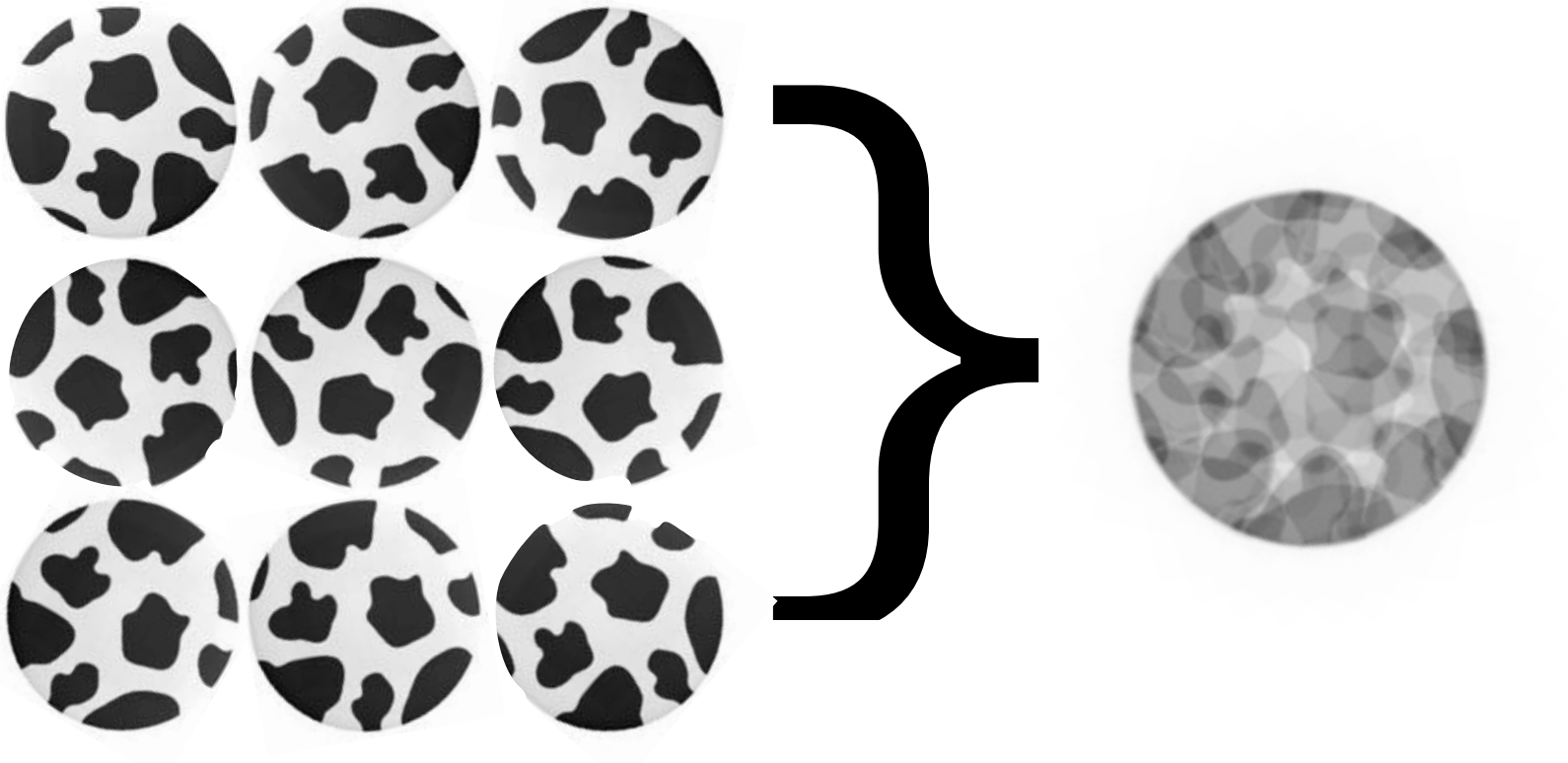




1-Point Statistics Idea

matter density in symmetric cells

symmetry statistics ↔ dynamics

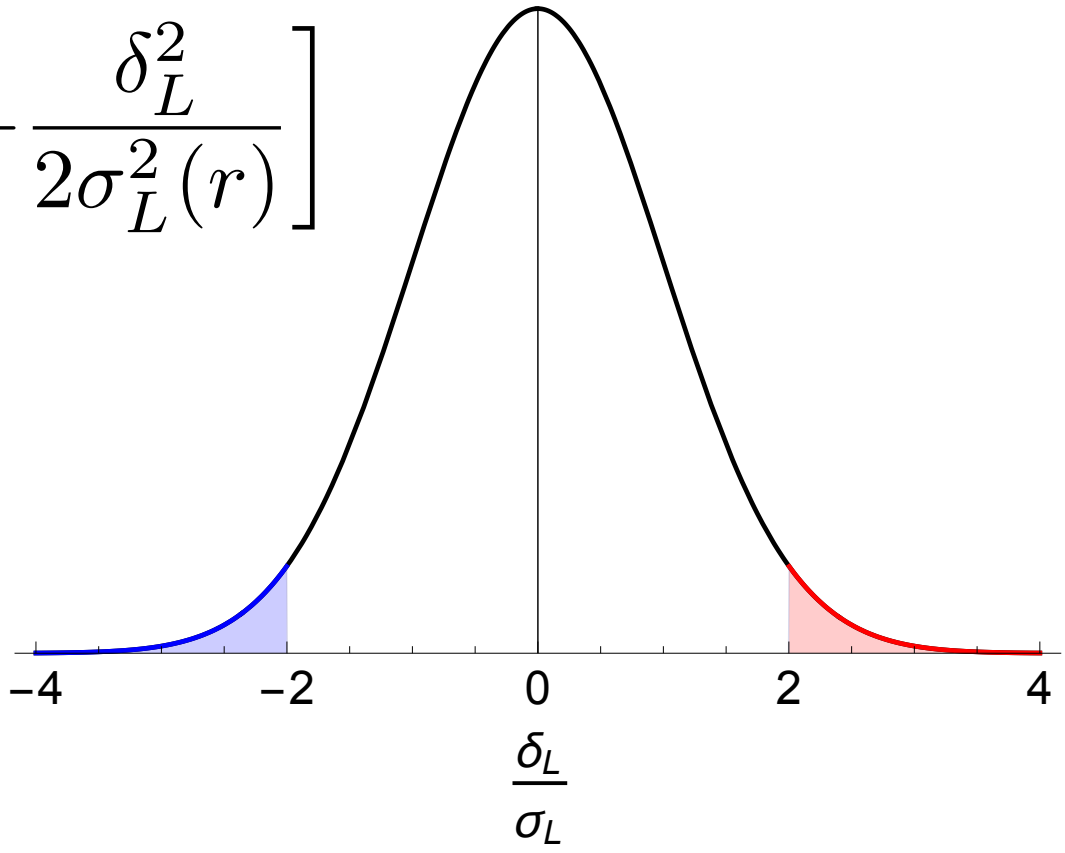


1-Point Statistics Idea

Large-deviation statistics

large deviations exponentially unlikely

$$\mathcal{P}_r^{\rm ini}(\delta_L) \sim \exp\left[-\frac{\delta_L^2}{2\sigma_L^2(r)}\right]$$



1-Point Statistics Theory

Large-deviation statistics

most likely path dominates $\rho = 1 + \delta_{\rm NL}$

spherical collapse

$$\mathcal{P}_{R,z}(\rho) \sim \exp\left[-\frac{\delta_L(\rho)^2}{2\sigma_L^2(z, r(R,\rho))}\frac{\sigma_L^2}{\sigma_{\rm NL}^2}\right]$$

linear variance & growth

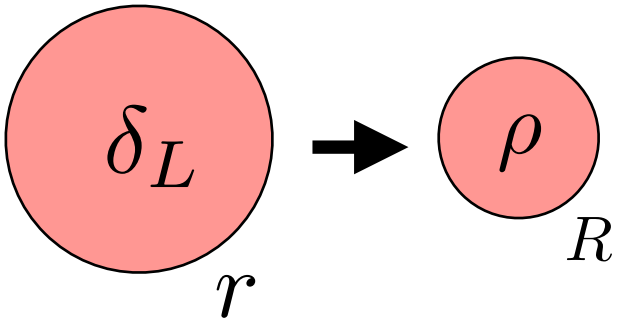
nonlinear variance

Bernardeau 94
CU++ 16

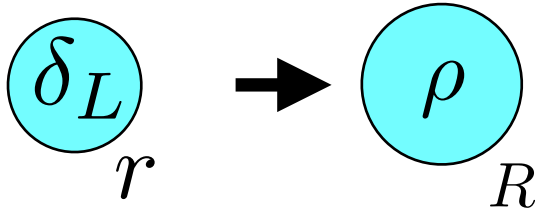
Spherical Collapse

mass conservation $r = \rho^{1/3} R$

large density



small density

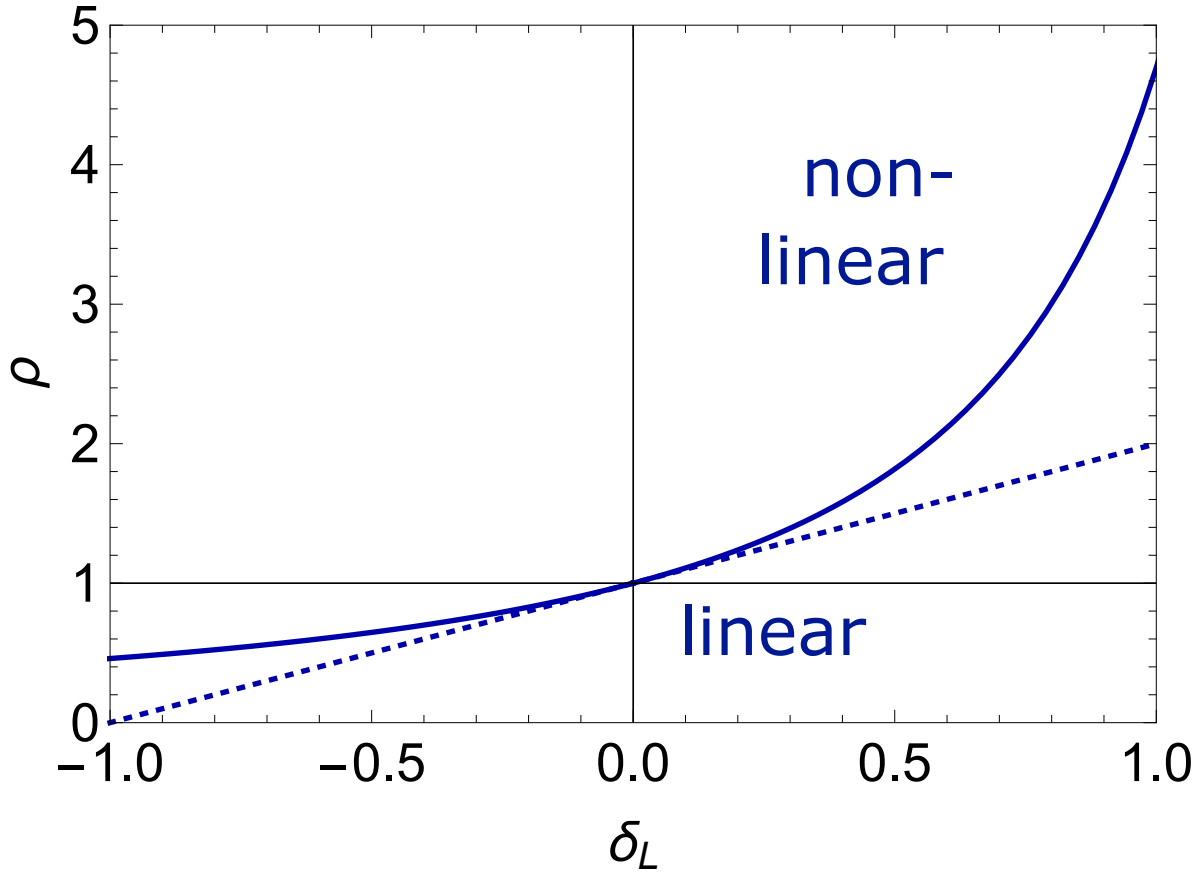


fixed final radius R mixes scales!

Spherical Collapse

density mapping $\delta_L(\rho) \simeq \frac{21}{13}\left(1-\rho^{-\frac{13}{21}}\right)$

~ cosmology independent

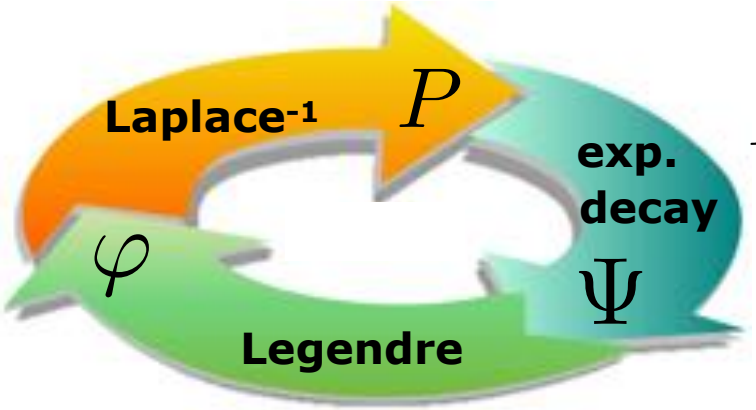


Large-Deviation Statistics

holy trinity: PDF, decay-rate & cumulant generator

$$\exp[\varphi(\lambda)] = \int d\rho\, P(\rho) \exp(\lambda\rho)$$

PDF



$$P(\rho) \propto \exp\left(-\Psi(\rho)\right)$$

cumulant generator

decay-rate

Gärtner-Ellis

$$\varphi(\lambda) = \sup_{\rho}[\lambda\rho - \Psi(\rho)]$$

Gärtner `77 & Ellis `84

Contraction principle

$$\Psi(\rho) = \inf_{\{\delta_L\}\to\rho} \Psi(\delta_L)$$

for any mapping

Bernardeau `92/94

validity: small variance

$$\sigma^2 = \langle\rho^2\rangle_c \to 0$$

Large-Deviation Statistics

Goal: final density PDF $P(\rho)$

3. Inverse Laplace

$$P(\rho) = \int_{-i\infty}^{+i\infty} \frac{d\lambda}{2\pi i} \exp\left[-\lambda\rho + \varphi_R(\lambda\sigma^2)/\sigma^2\right]$$

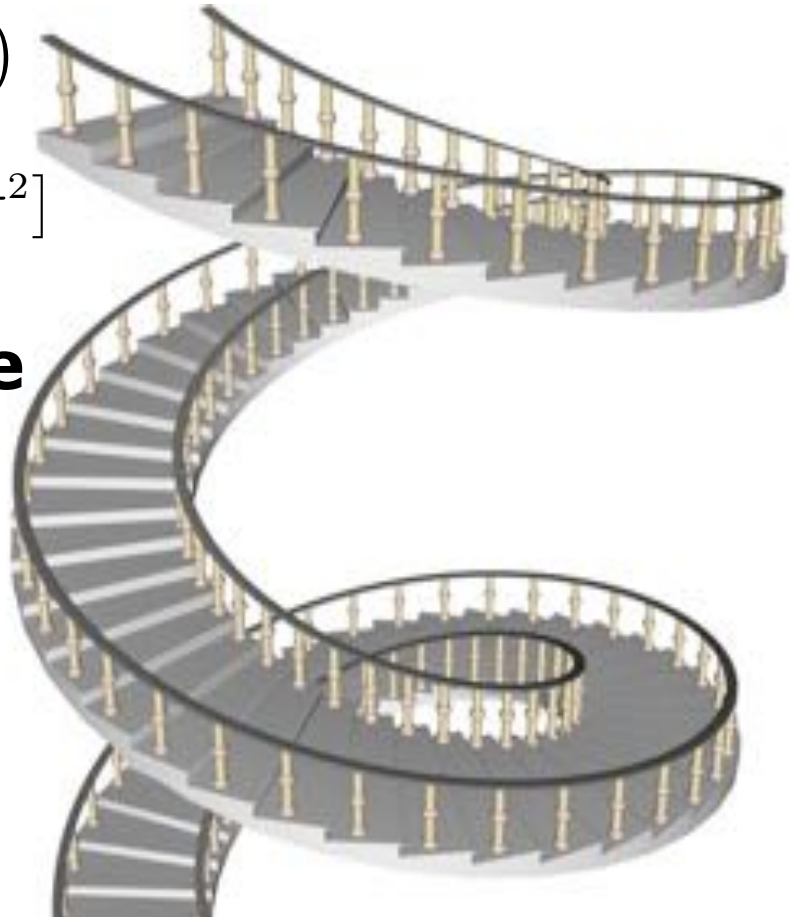
2. Large deviation principle

$$\varphi(\lambda) = \sup_{\rho}[\lambda\rho - \Psi(\rho)] \qquad \varphi(\lambda)$$

1. Contraction principle

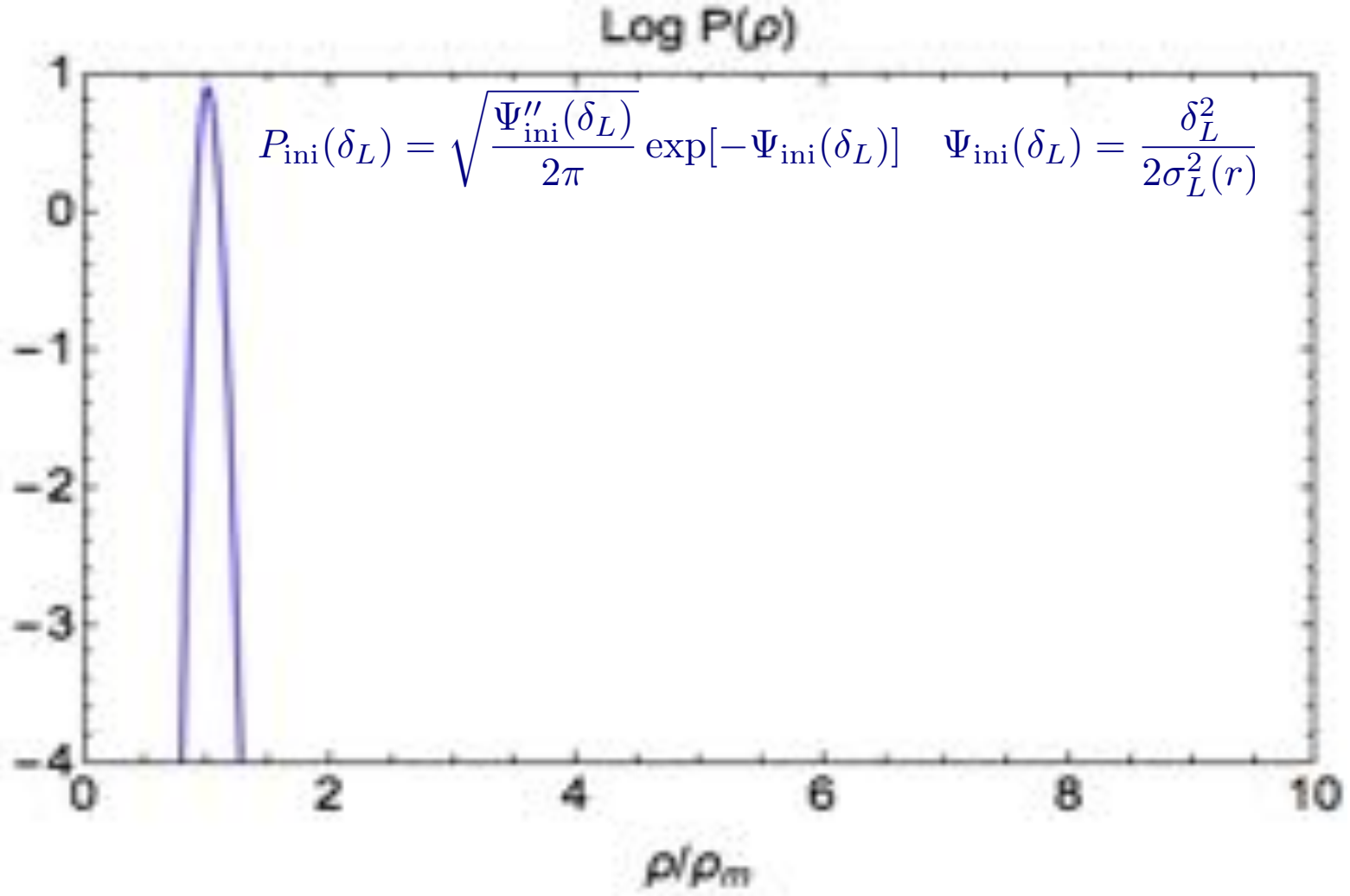
$$\delta_L = \delta_L(\rho) \qquad \Psi(\rho)$$

$$r = \rho^{1/3} R$$



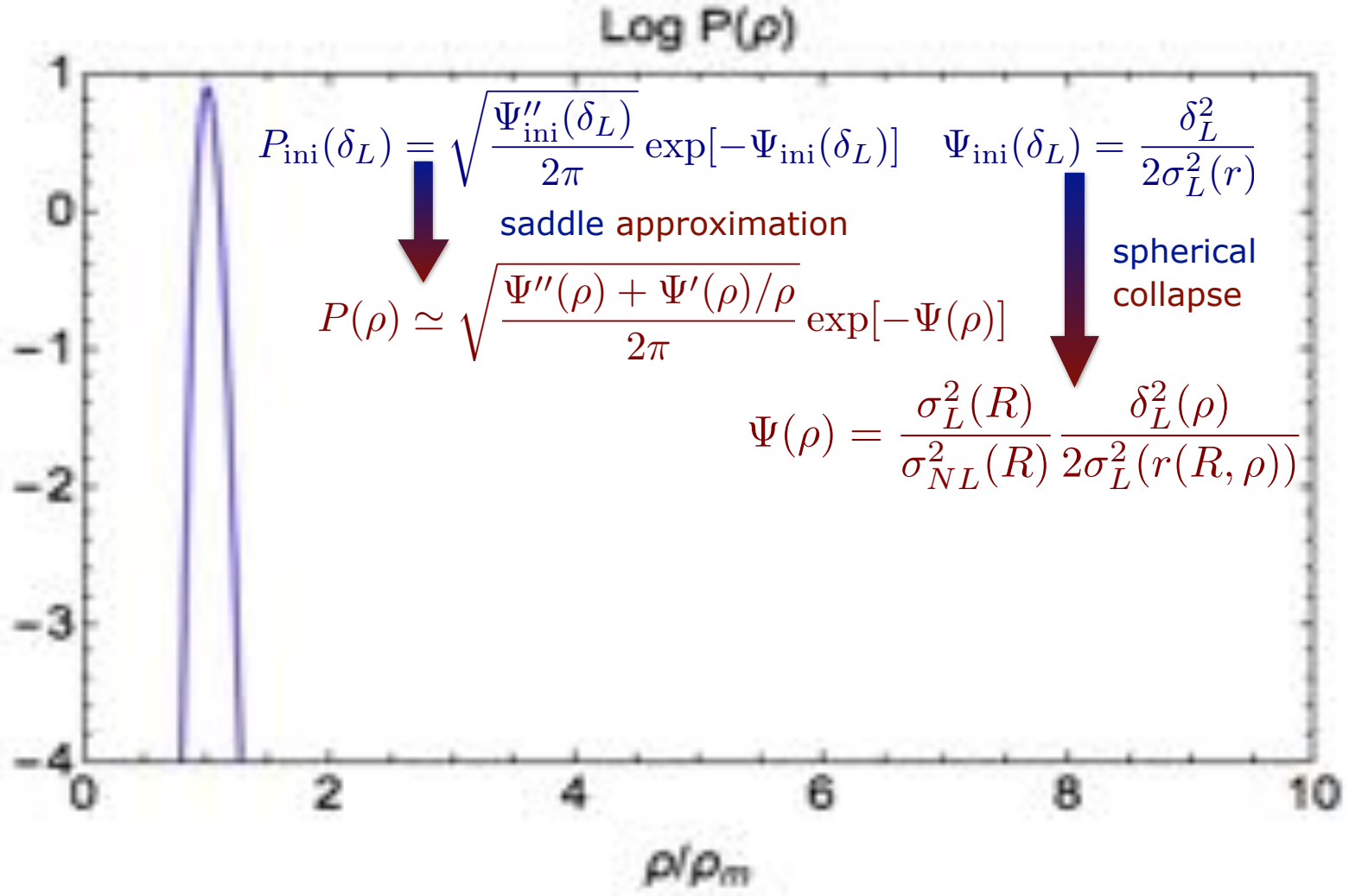
Knowledge: initial density PDF $\Psi(\delta_L) = \dfrac{\delta_L^2}{2\sigma_L^2(r)}$

Matter 1-Point PDF



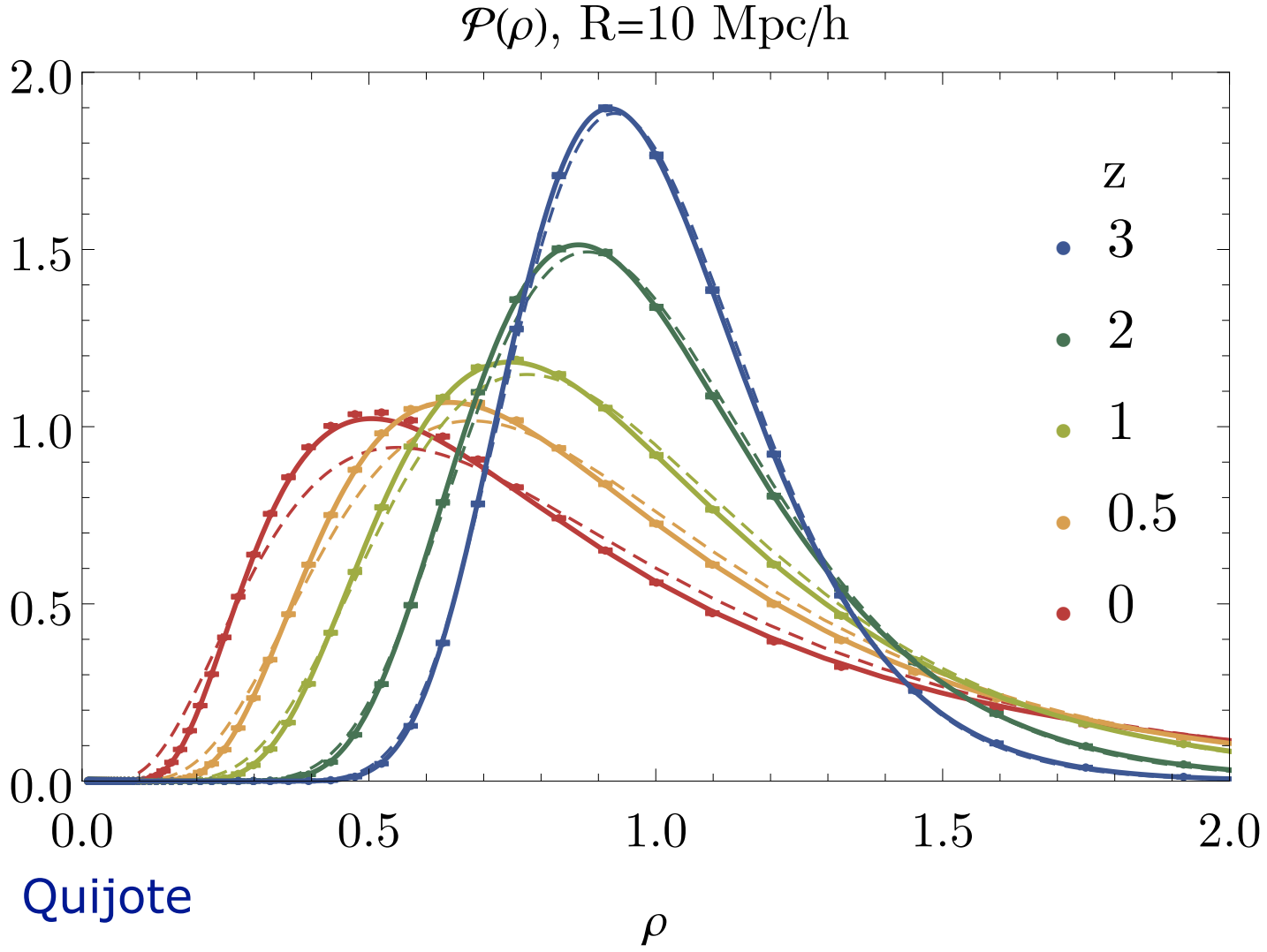
$$P_{\rm ini}(\delta_L) = \sqrt{\frac{\Psi''_{\rm ini}(\delta_L)}{2\pi}}\exp[-\Psi_{\rm ini}(\delta_L)] \quad \Psi_{\rm ini}(\delta_L) = \frac{\delta_L^2}{2\sigma_L^2(r)}$$

Matter 1-Point PDF



Matter 1-Point PDF

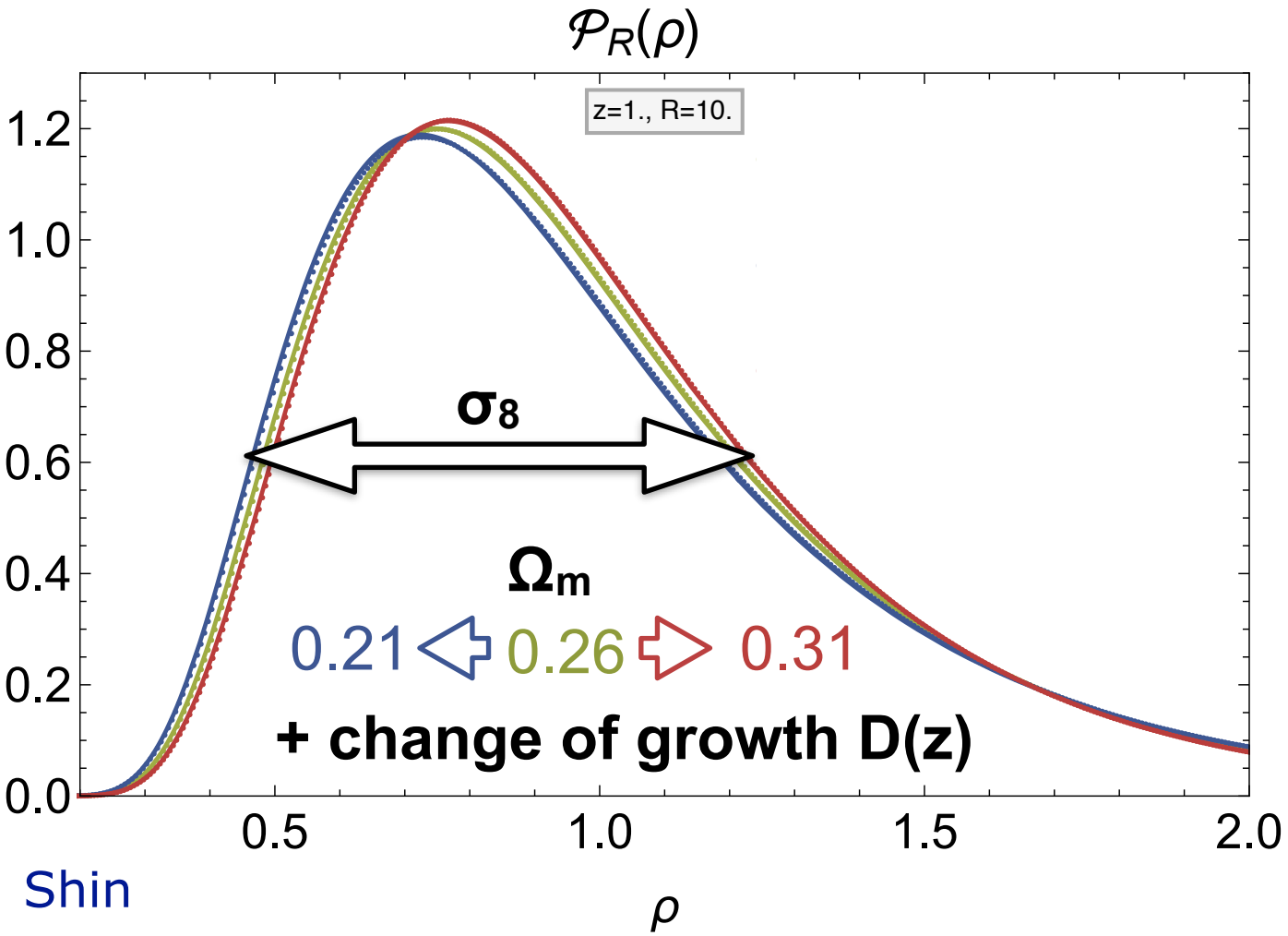
accurate PDF from first principles, not lognormal



sims: Quijote

1-Point PDF Cosmology

matter content σ_8 & Ω_m: width & tilt

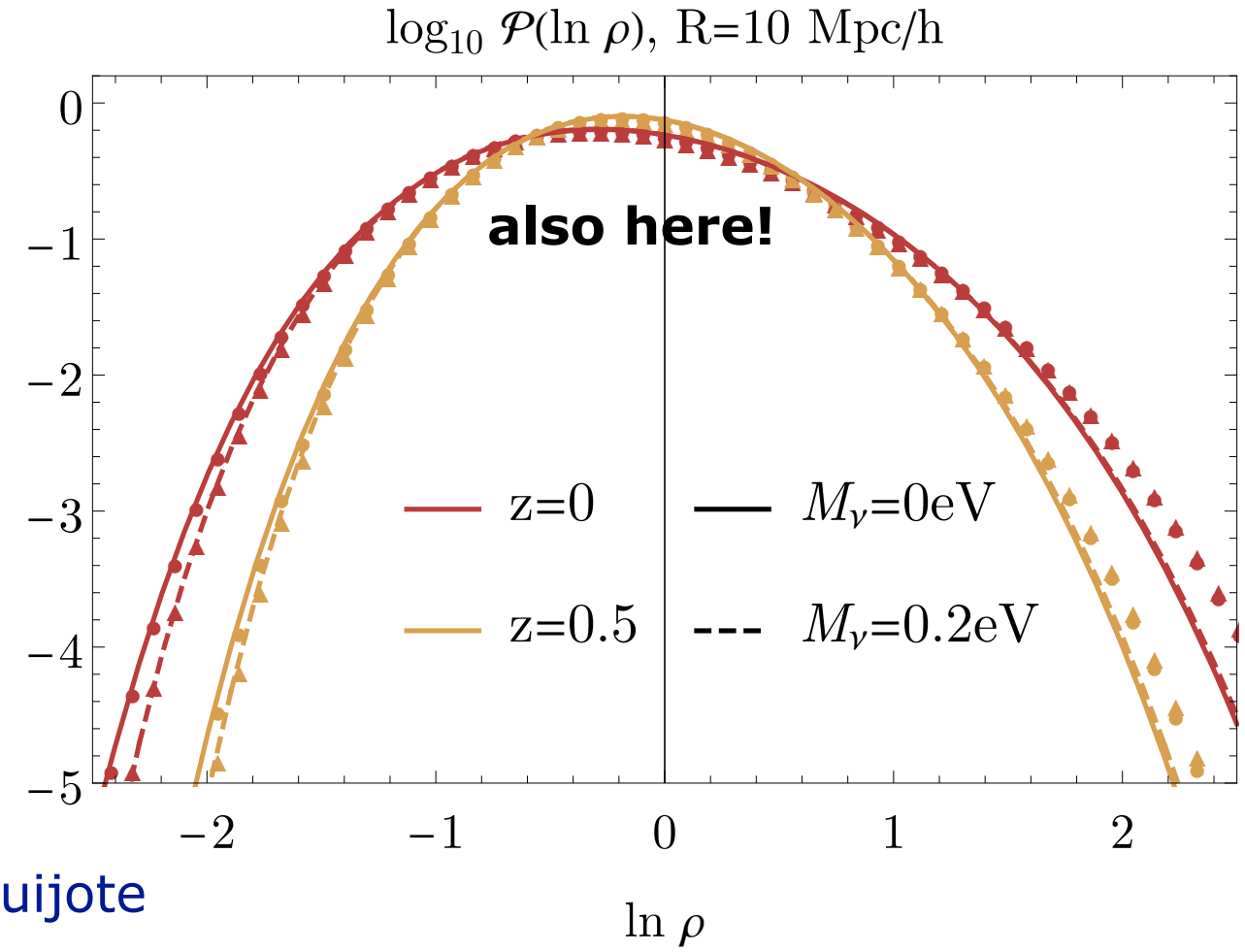


sims: J. Shin

1-Point PDF Cosmology

massive neutrinos M_ν: partial clustering

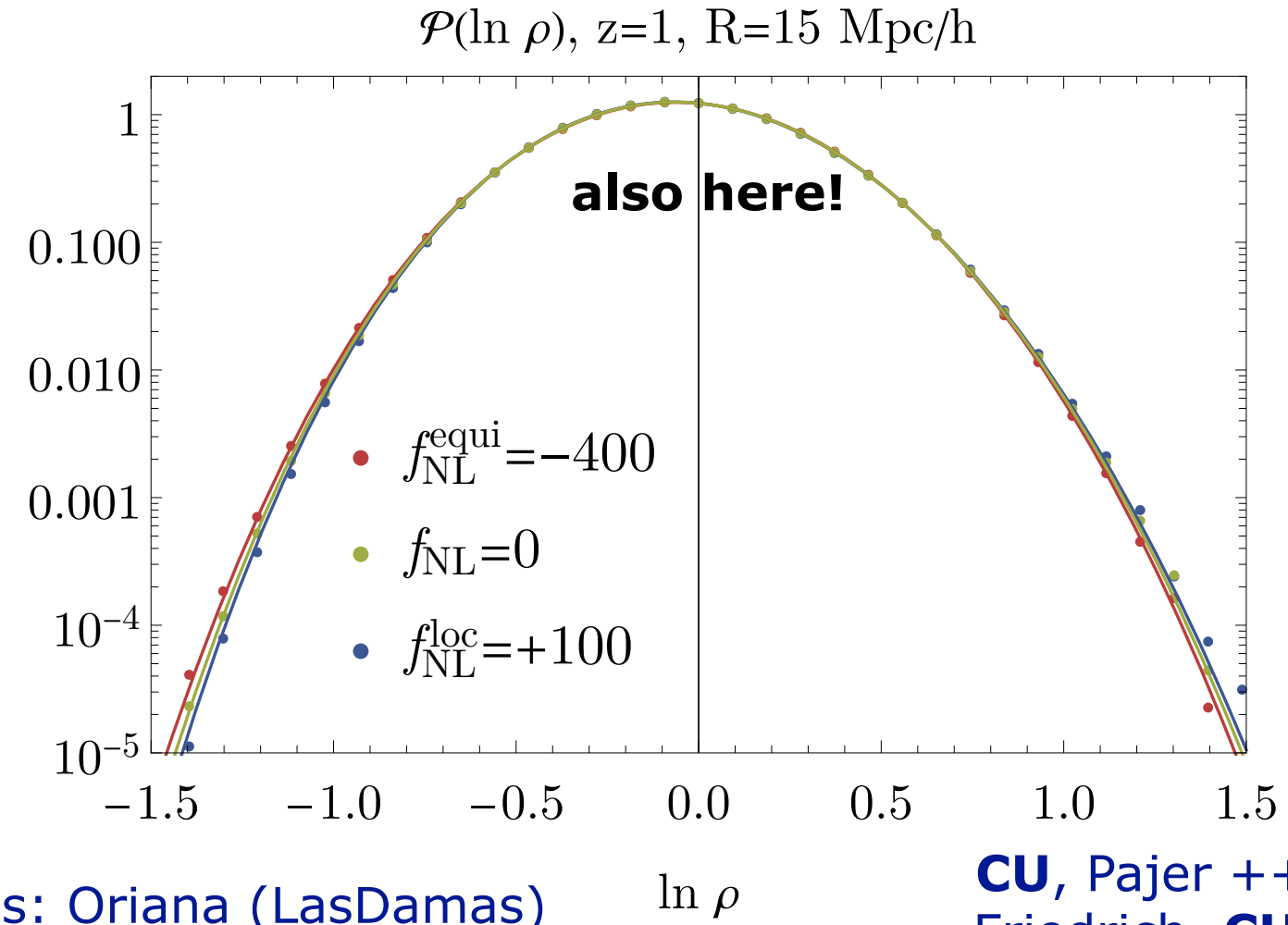
fixed σ_8



sims: Quijote

1-Point PDF Cosmology

prim. non-Gaussianity f_{NL}: extra tilt



sims: Oriana (LasDamas)

CU, Pajer ++ 18
Friedrich, **CU** ++ 19

Fisher for Complements

CU, Friedrich ++ 19

Quijote simulations

F. Villaescusa-Navarro ++ (incl **CU**) 19

15.000x fiducial cosmo

500x derivative cosmo

σ_8, Ω_m, Ω_b, h, n_s, M_v, w_0

>1 million PDFs

Fisher for Complements

Fisher matrix

covariance

$$F_{ij} = \sum_{\alpha,\beta} \frac{\partial S_\alpha}{\partial \theta_i} C^{-1}_{\alpha\beta} \frac{\partial S_\beta}{\partial \theta_j}$$

summary stats: PDF bins

derivatives w.r.t. cosmo

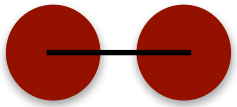
marginalised errors $\delta\theta_i \geqslant \sqrt{(F^{-1})_{ii}}$

Fisher for Complements

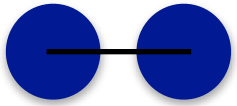
Covariance: PDF bin correlation

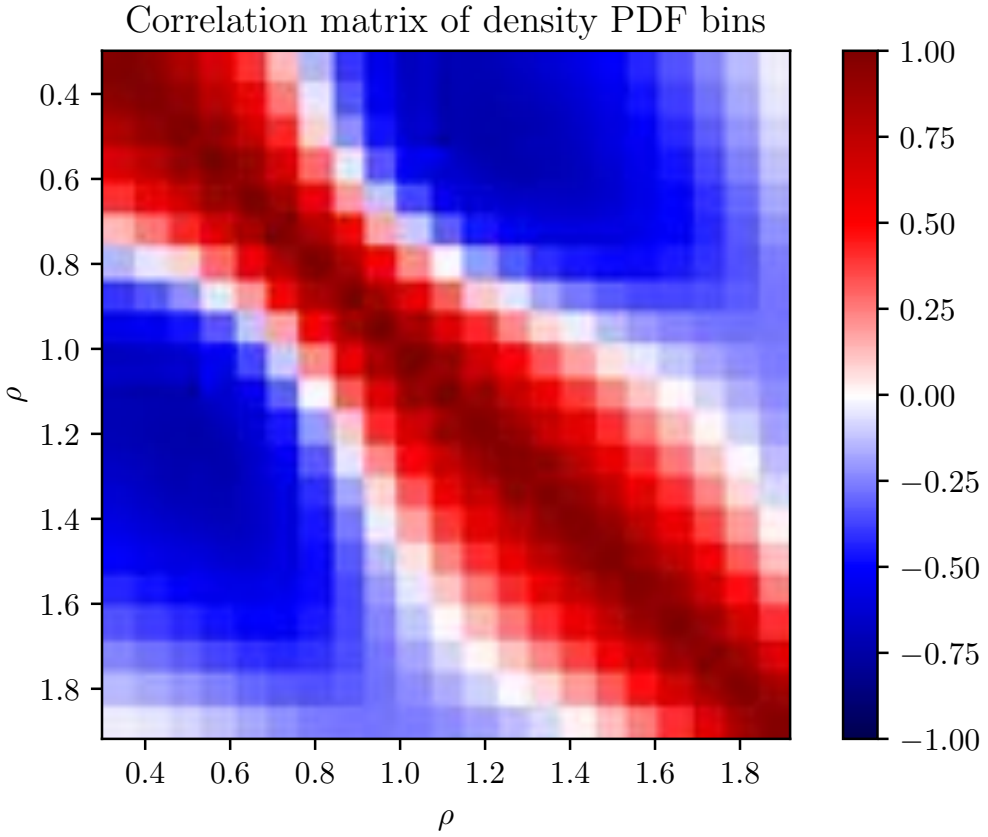
determined by density-dependent clustering

diagonal

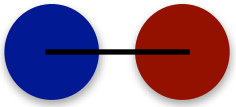


extra clustered

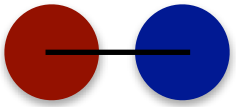




corners

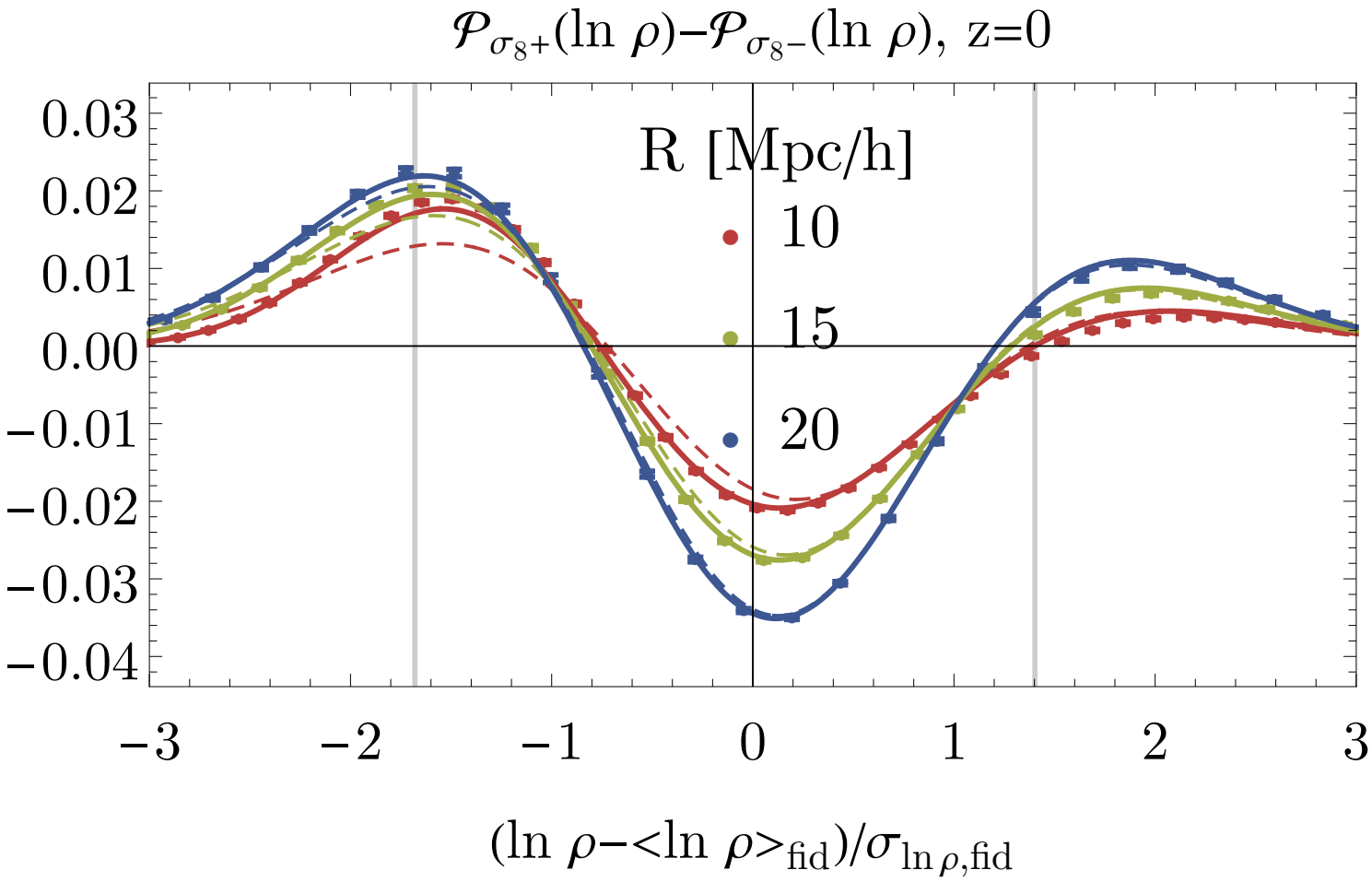


anti-clustered



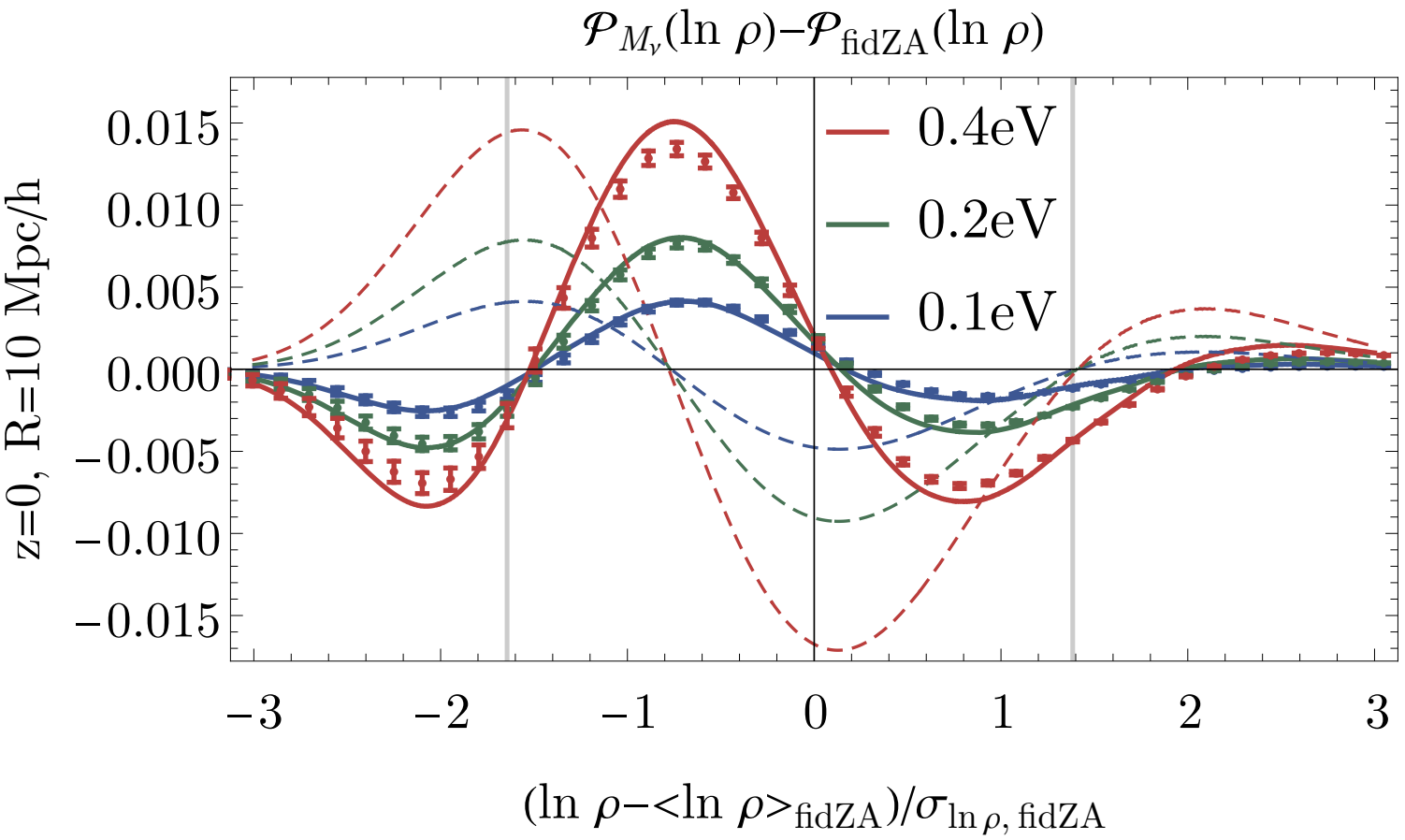
Fisher for Complements

width: clustering amplitude σ_8

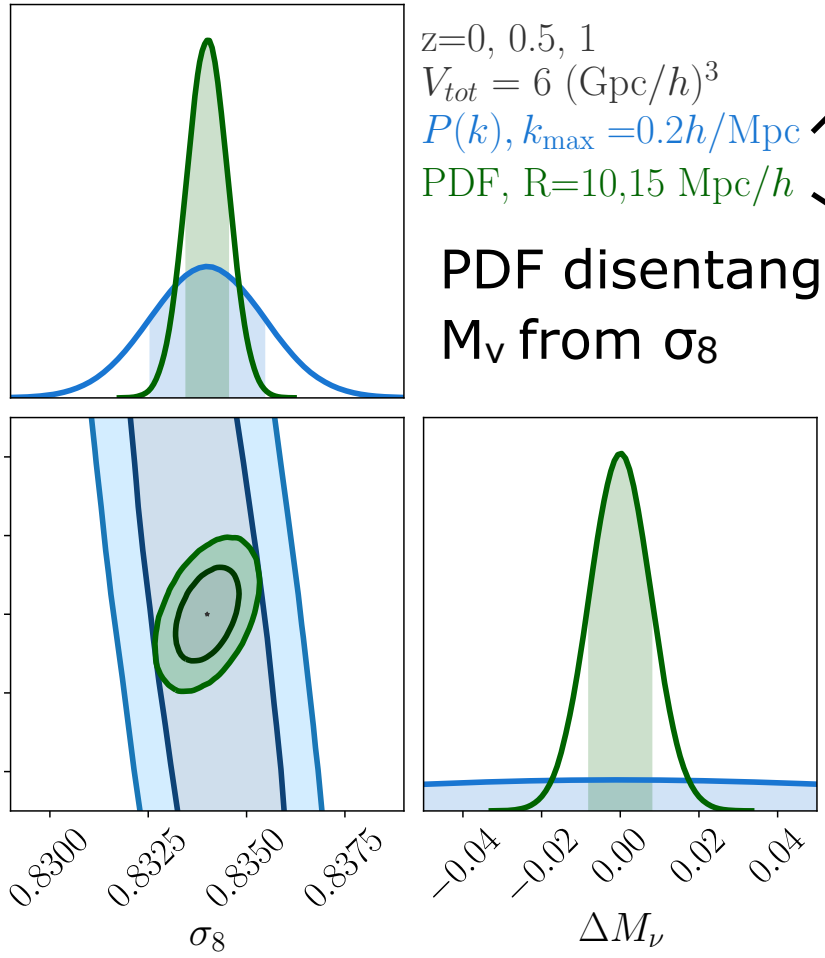


Fisher for Complements

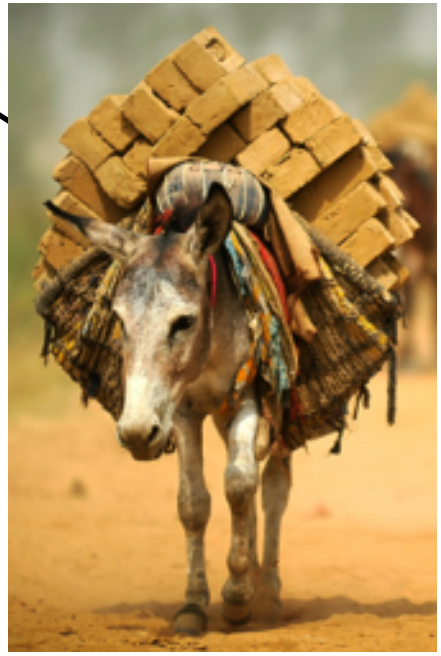
environment-dependence: M_ν



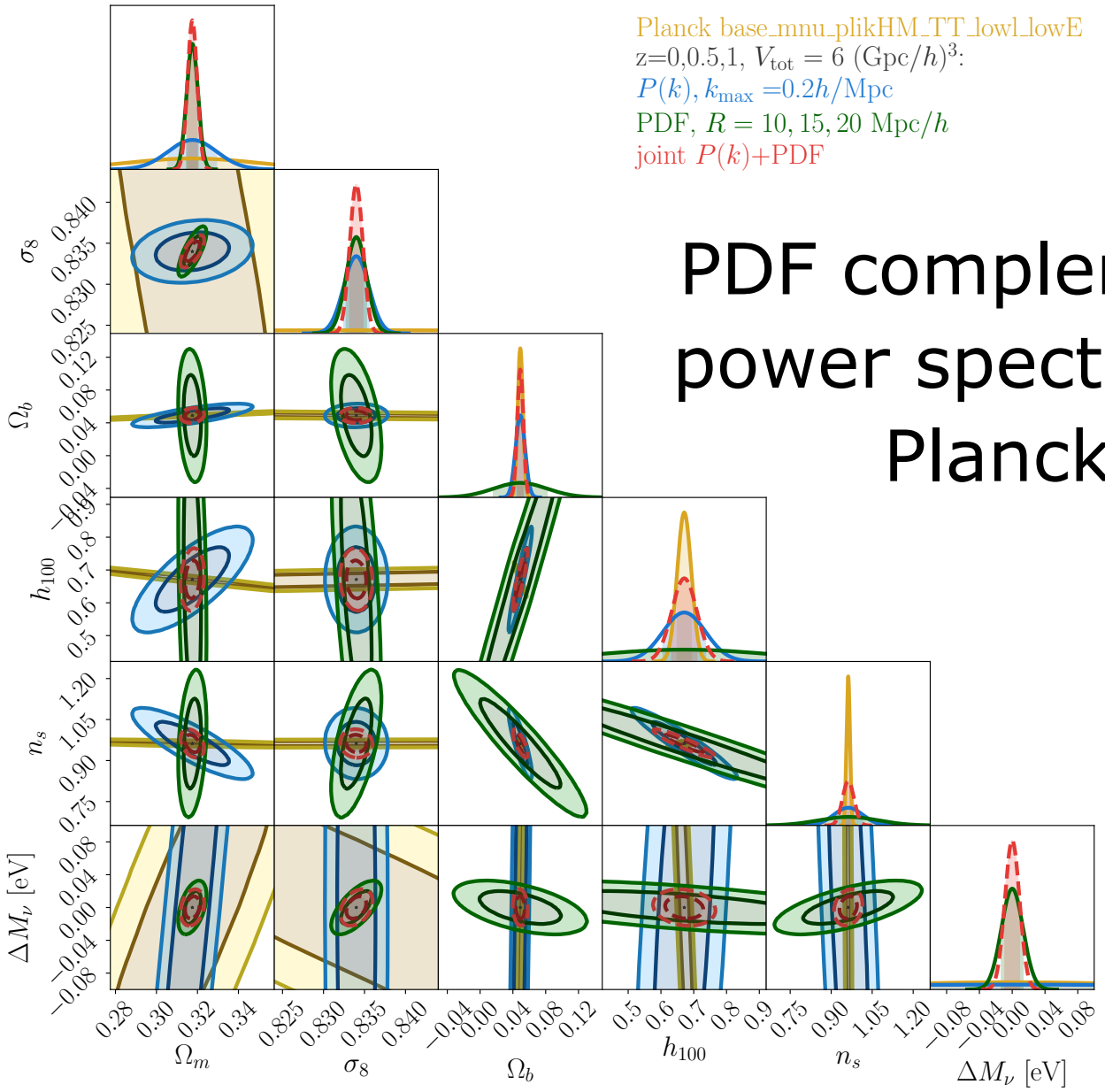
Fisher for Complements



PDF disentangles M_ν from σ_8



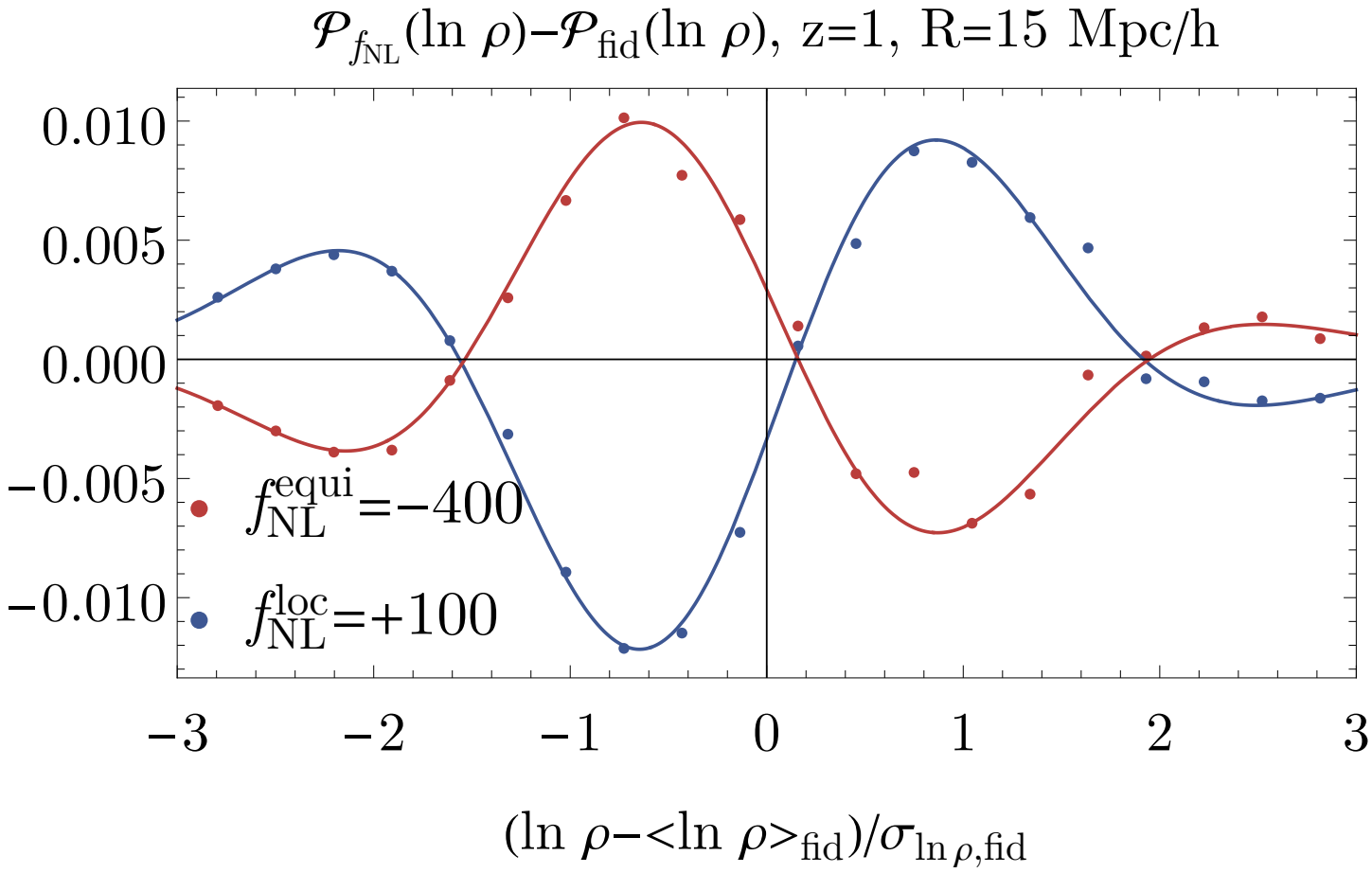
Fisher for Complements



PDF complements power spectrum & Planck

Fisher for Complements

prim. non-Gaussianity f_{NL}: extra tilt



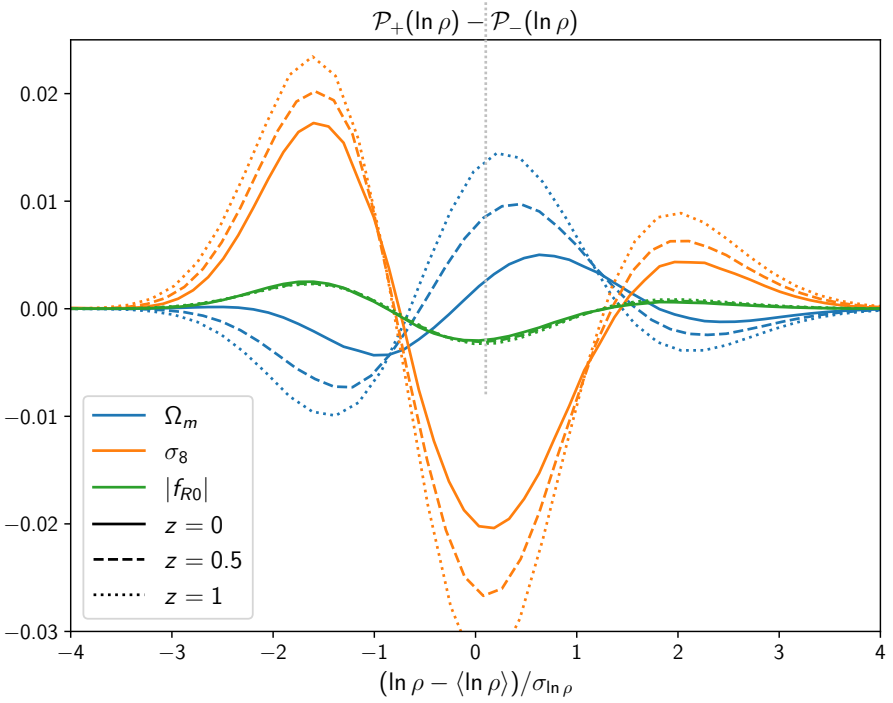
sims: Oriana (LasDamas)

Friedrich, **CU** ++ 19

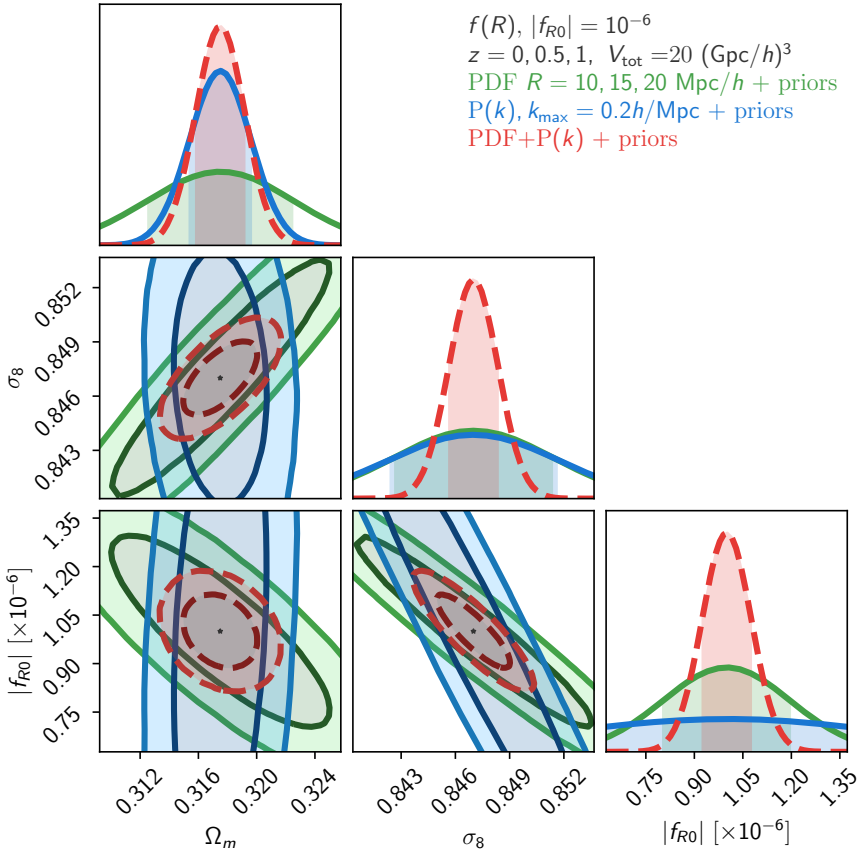
Fisher for Complements

modified gravity: growth & skewness

forecast by Alex Gough



PDF & P(k) stronger together



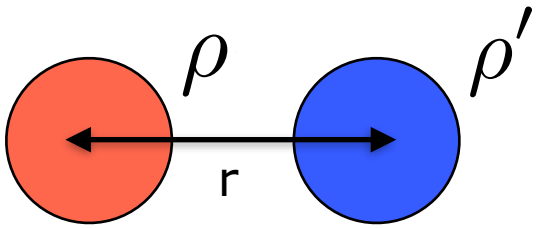
Cataneo,**CU**,Arnold**, AG**++ 21

Matter 1-Point Statistics

extensions

Density-Split Clustering

$$\frac{P(\rho(x), \rho'(x+r))}{P(\rho)P(\rho')} = 1 + \xi(r)b(\rho)b(\rho')$$



average density at separation r

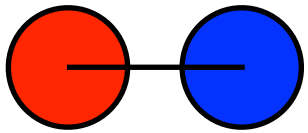
$$b_R(\rho) = \frac{\langle \rho'_R(r)|\rho_R\rangle - 1}{\xi_R(r)}$$

separation independent

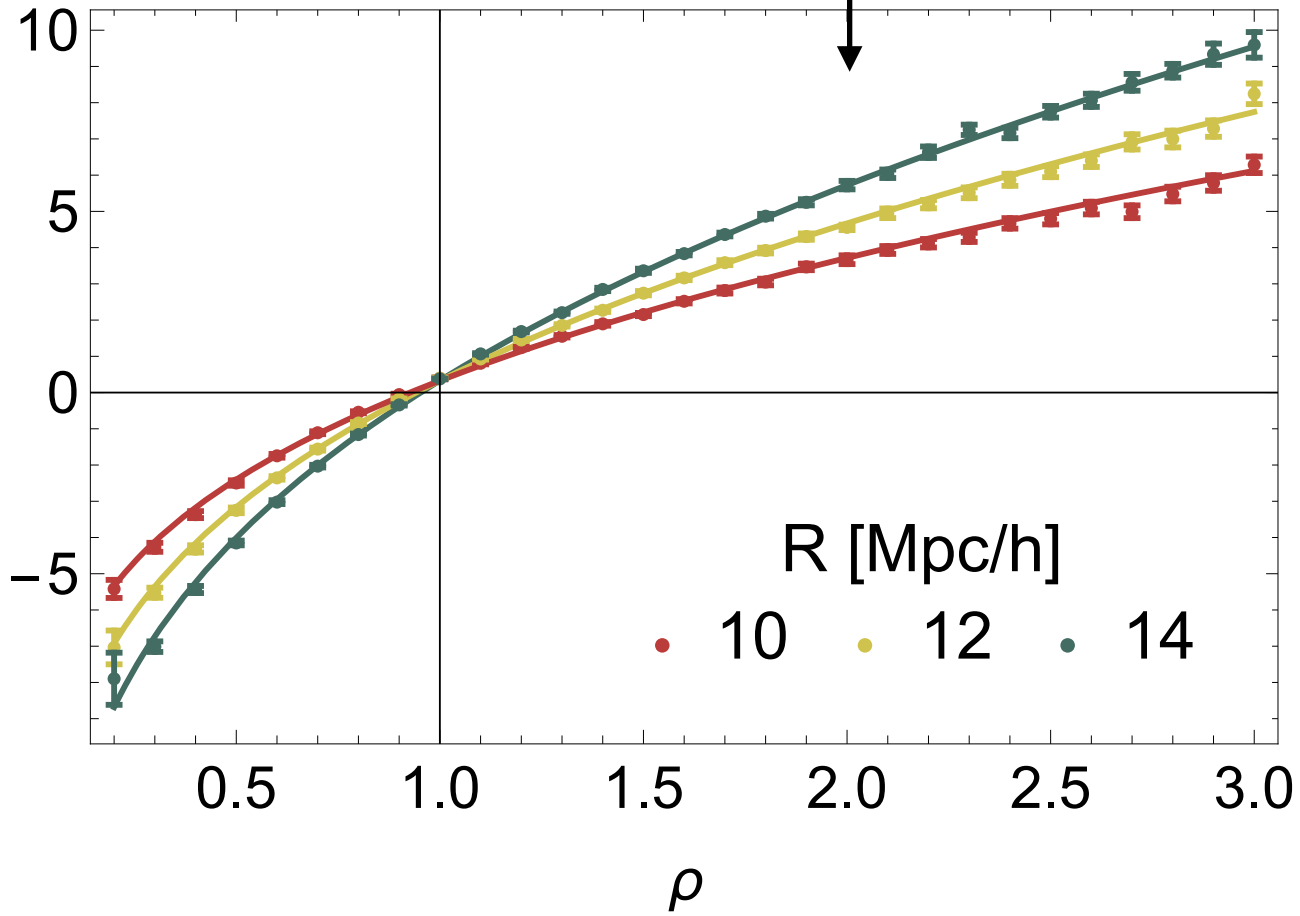
Gaussian ICs + spherical collapse

Density-Split Clustering

$$\frac{P(\rho(x), \rho'(x+r))}{P(\rho)P(\rho')} = 1 + \xi(r)b(\rho)b(\rho')$$



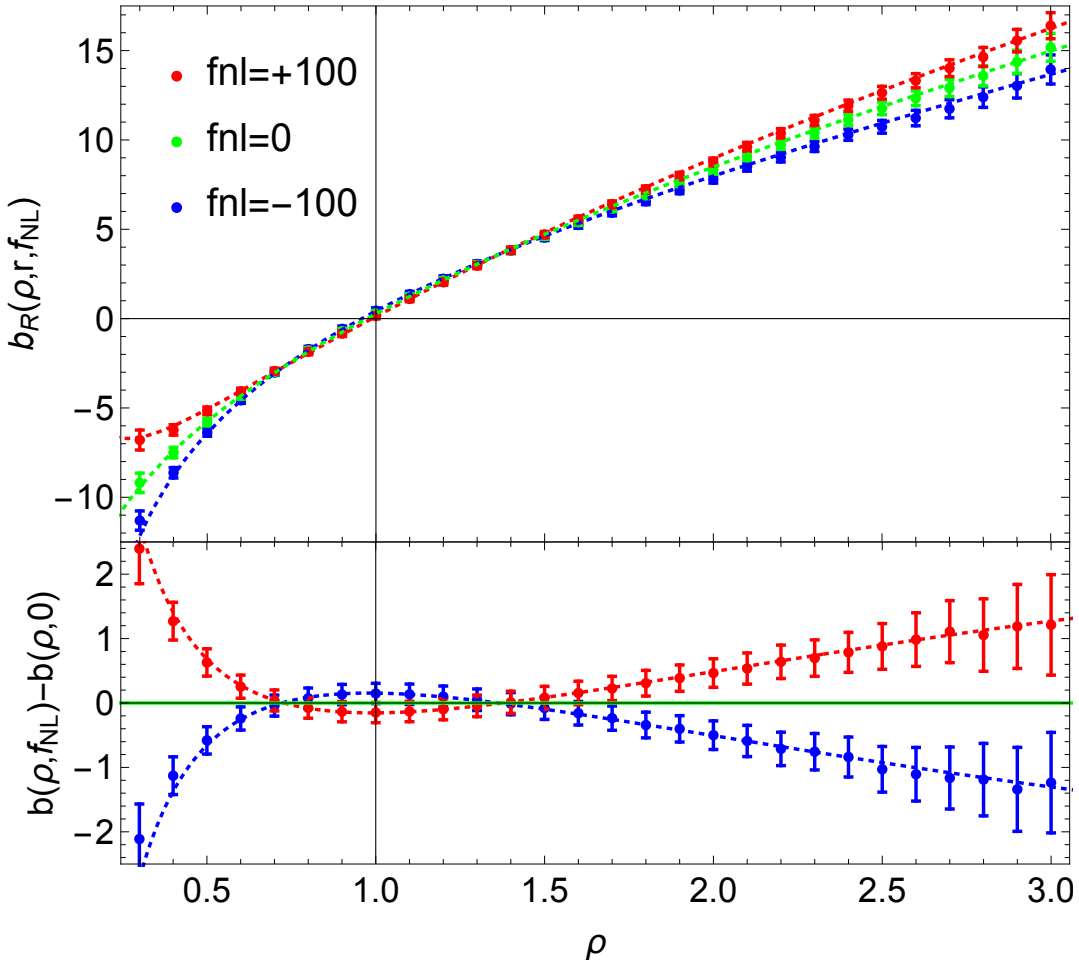
CU, Codis ++ 17



Density-Split Clustering

local primordial non-Gaussianity

analogue of
scale-dependent
halo & void bias



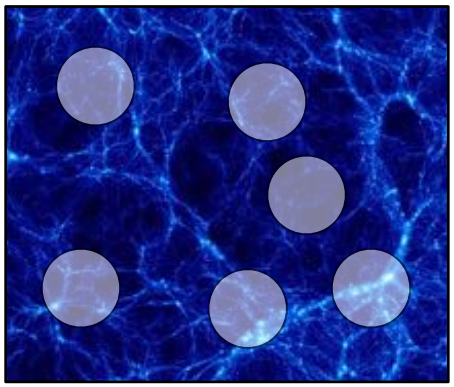
CU++ `18c

Cosmo with 1-pt Statistics

Powerful statistics

non-Gaussian, beyond PT

robust & accurate predictions



Cosmology & fundamental physics

Ω_m, σ_8, M_ν, f_{NL} **CU**, Friedrich ++ 19 Friedrich, **CU** ++ 19

DE: $w_{0,a}$, MG: Ω_{rc}, f_{R0} Cataneo, **CU**, Arnold, **AG**++ 21

Reality: no 3D matter field (see Oliver's talk)

weak lensing: projected matter

galaxy clustering: bias & stochasticity