Amplitudes meet the Swampland

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Intepretable and higher-order statistics for late-time cosmology

June 27, 2022 to July 1, 2022
Institute for Fundamental Physics of the Universe
Europe/Rome timezone

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Contact

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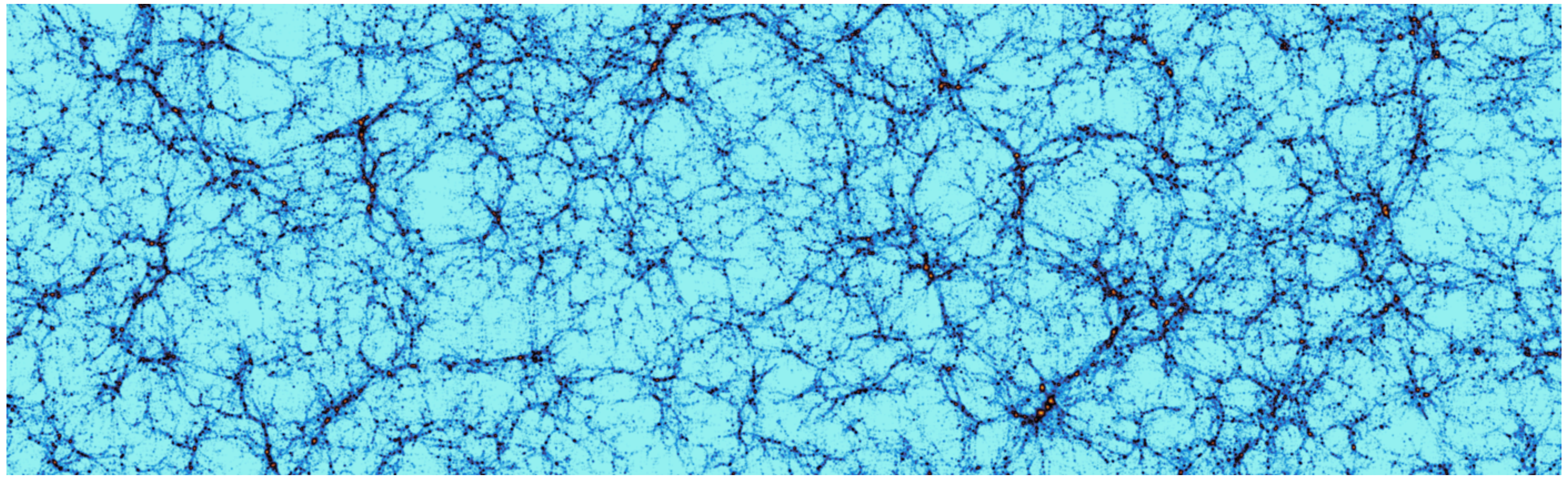

In the last decade, the study of the large-scale structures of the universe has undergone exponential progress thanks to increasingly powerful experiments. Galaxy surveys mapping the entire sky, observing millions of bright galaxies up to high redshifts, are transforming cosmology into a data-driven, precision science.

The goal of the focus week will be to discuss higher-order summary statistics as a promising method to extract cosmological information from galaxy surveys, providing complementary information to conventional methods based on low-order correlation functions.

Starts Jun 27, 2022, 9:00 AM
Ends Jul 1, 2022, 5:30 PM
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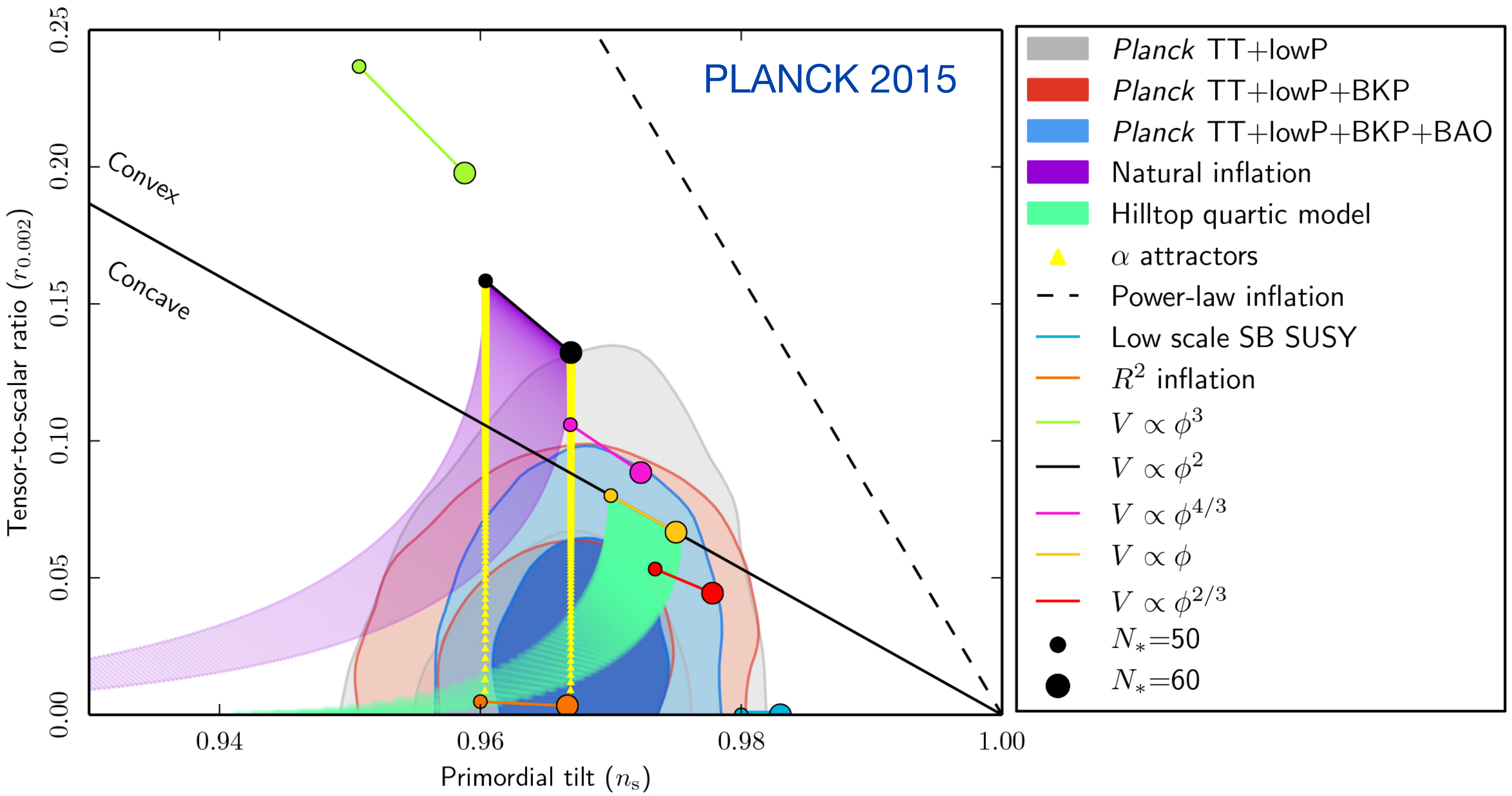
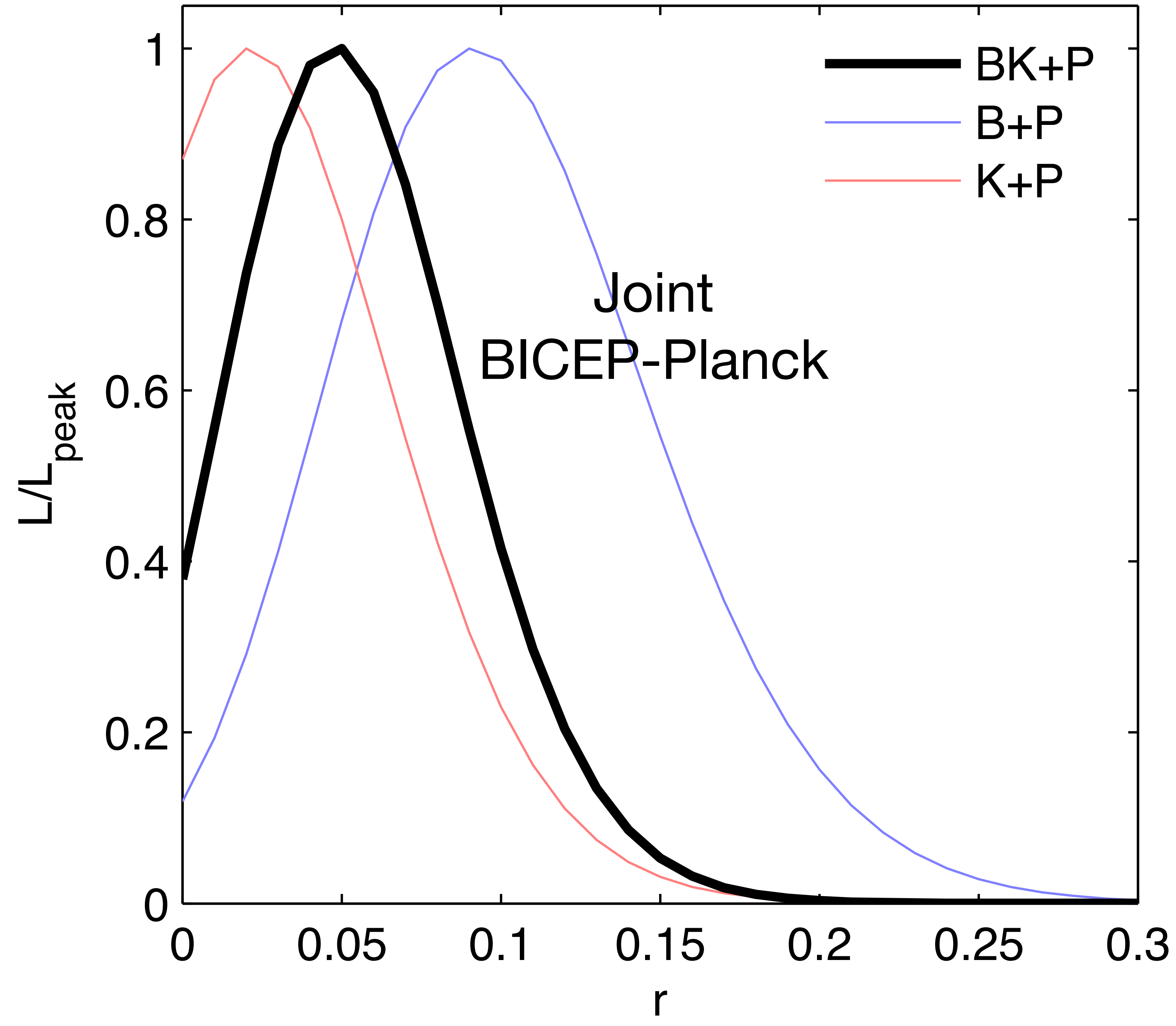
Primordial Gravitational Waves





Ongoing experiments can potentially detect primordial B-mode with a tensor-to-scalar ratio r as small as ~10^{-2}.

Further experiments, such as CMB-S4 and LiteBIRD, .. may improve further the sensitivity to r as small as ~ 10^{-3}.

Sign Matters

- The goal of the swampland program is to delineate the landscape from the swamp. Swampland constraints often take the form of an inequality, e.g., the WGC:

$$\exists \text{ state with } \frac{q}{m} \geq \lim_{M \to \infty} \frac{Q}{M}\Big|_{\text{ext}}$$

[Arkani-Hamed, Motl, Nicolis, Vafa '06]

- Analyticity constraints on the S-matrix similarly give rise to inequalities (**positivity bounds**) that constrains the EFT coefficients.
- Natural to put 2+2 together. Indeed, some remarks were already made in [Arkani-Hamed, Motl, Nicolis, Vafa '06];[Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi, '06].
- Leading irrelevant operators shift the extremality bound of RN black hole:

$$-F_{\mu\nu}^2 + a(F^2)^2 + b(F\tilde{F})^2 + \cdots \xrightarrow[\text{(causality)}]{\text{analyticity}} M^2_{\text{extremal electric}} = Q^2 - \frac{2a}{5} < Q^2$$

Gravity Matters

- In the presence of dynamical gravity, the analyticity and boundedness properties of the S-matrix are less understood.

- An added assumption of **Regge boundedness**:

$$\lim_{s\to\infty,\ t<0:\text{fixed}} M(s,t)/s^2 = 0$$

- Graviton exchange in the t-channel: $M(s,t) \sim -\frac{1}{M_P^2}\frac{s^2}{t}$

- One can argue that the amplitude cannot grow faster than s^2 using the chaos bound [Maldacena, Shenker, Stanford, '15] (or heuristically, using the `signal model' [Camanho, Edelstein, Maldacena, Zhiboedov, '14]).

- A more careful treatment can be found in [Chandorkar, Chowdhury, Kundu, Minwalla, '21]. Establishing this behavior is relevant for the Classical Regge Growth Conjecture.

Gravity Matters

- More recently, gravitational Regge boundedness for $D \geq 5$ was shown in [Hairing, Zhiboedov, '22] to hold under some assumptions, including **subexponentiality**:

$$|M(s,t)| < e^{C|s|^{\beta}}, \quad \beta < 1 \quad \text{for fixed } t < 0$$

  everywhere in the region of analyticity in the upper half-plane $\arg(s) \in (0,\infty)$.

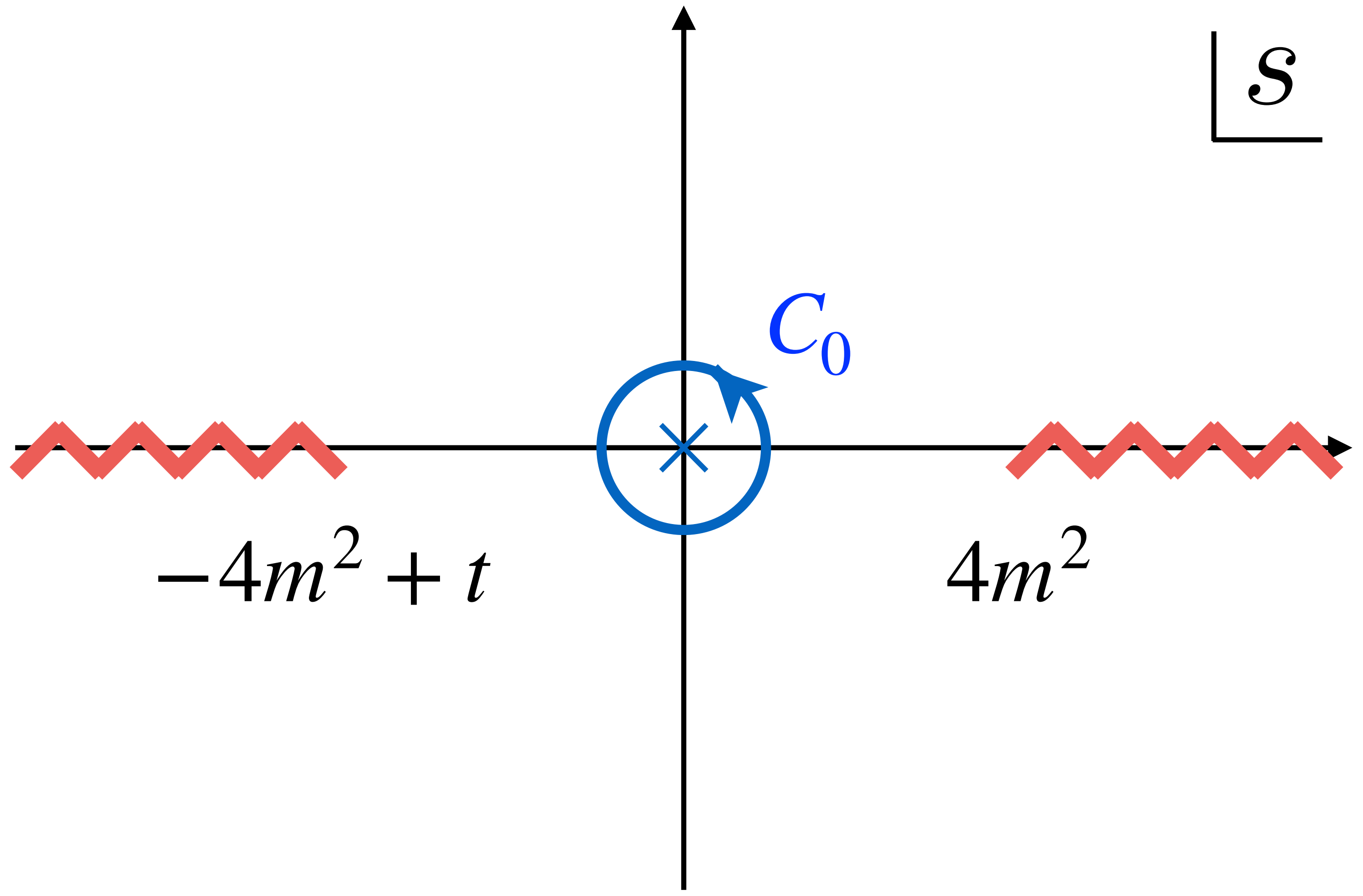
- Interestingly, arguments using large IR logs to show swampland constraints are 4d specific [Arkani-Hamed, Huang, Liu, Remmen, '21].

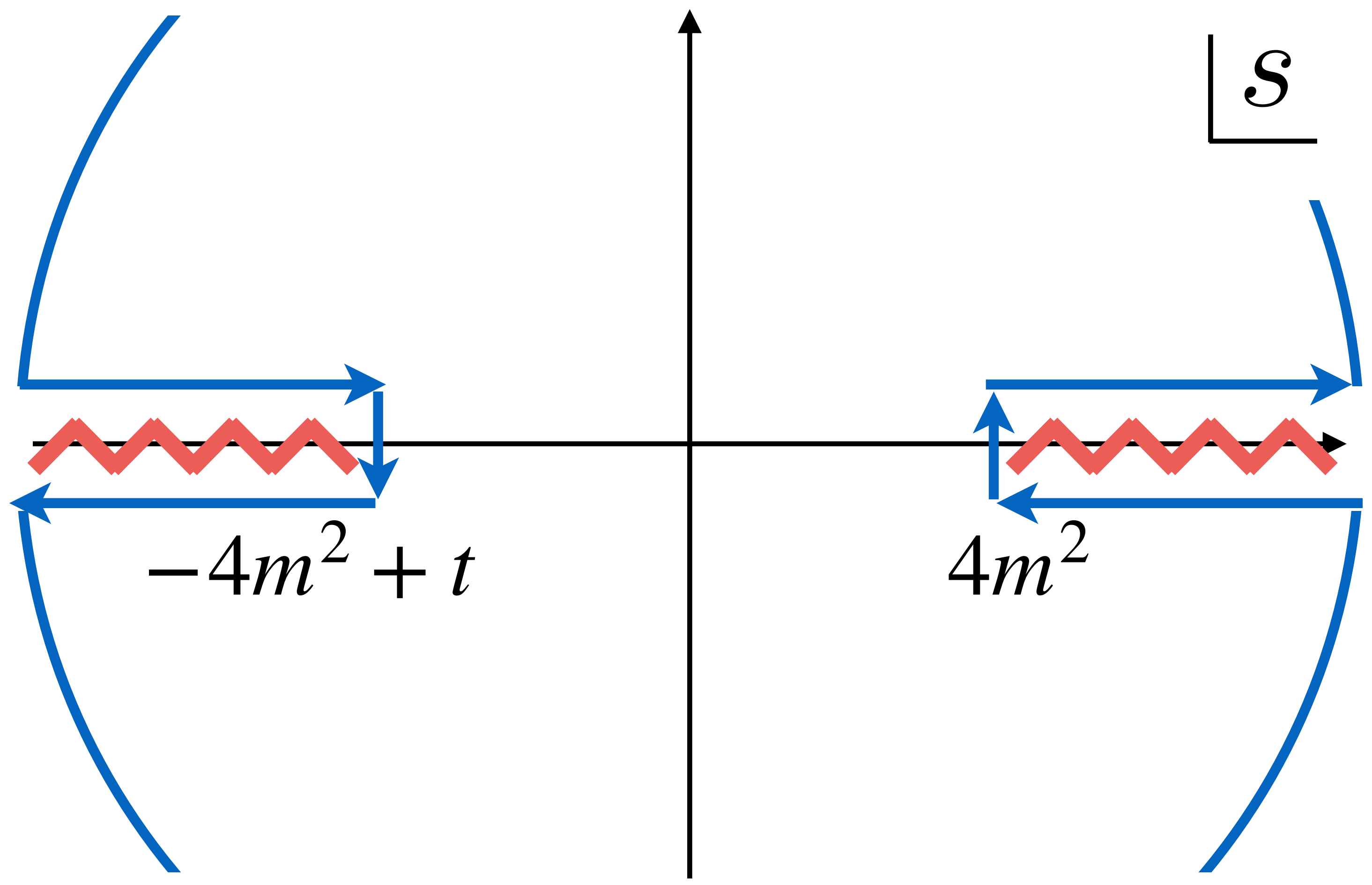
- Even though the Regge limit does not probe strong gravity (black hole exchange), understanding whether this is true for all UV completion may teach us lessons about the UV.

- **Swampland constraint?** Evidence: 1) perturbative string amplitude in flat space, 2) CFT argument [Caron-Huot, '17] for AdS scattering (leading $1/N$ gives $M \sim s^2$, to all orders $M \sim s$).

- The gravitational positivity bounds may be **only approximately positive** [Hamada, Noumi, GS, '18];[Alberte, de Rham, Jaity, Tolley, '20]; [Tokuda, Aoki, Hirano, '20];[Caron-Huot, Mazac, Rastelli, Simmons-Duffin, '21].

Gravitational Positivity Bounds





- How the graviton t-channel pole gets canceled depends on UV completion.

$$\mathcal{M}(s,t) = -\frac{4su}{M_{\rm Pl}^2 t} - \frac{4tu}{M_{\rm Pl}^2 s} - \frac{4ts}{M_{\rm Pl}^2 u} + \sum_{n=0}^{\infty} \frac{c_n(t)}{n!}\left(s+\frac{t}{2}\right)^n \qquad c_2(t) - \frac{8}{M_{\rm Pl}^2 t} = \frac{4}{\pi}\int_{4m^2}^{\infty} ds \frac{\mathrm{Im}\mathcal{M}(s,t)}{(s+t/2)^3}$$

- The leftover t^0 piece can be positive or negative, modifying the “positivity bounds”.

[Hamada, Noumi, GS, ’18]

Weak Gravity Conjecture

WGC from Unitarity and Causality

- With these caveats, one can "prove" the WGC from unitarity and causality [Hamada, Noumi, GS '18], [Cheung, Liu, Remmen, '18];[Bellanzini, Lewandowski, Serra ,'19], [Arkani-Hamed, Huang, Liu, Remmen, '21]

$$\mathscr{L} = \frac{1}{2}R - \frac{1}{4}F^2 + \frac{\alpha_1}{4}(F^2)^2 + \frac{\alpha_2}{4}(F\tilde{F})^2 + \frac{\alpha_3}{2}FFW$$

$$\frac{\sqrt{2(Q^2+P^2)}}{M} \leq 1 + \frac{32\pi^2}{5(Q^2+P^2)^3}\left[2\alpha_1(Q^2-P^2)^2 + 2\alpha_2(2QP)^2 - \alpha_3(Q^4-P^4)\right]$$

- Such arguments have been extended to more complicated charged black holes [Loges, Noumi, GS, '19];[Loges, Noumi, GS, '20].

- In some cases, positivity bounds alone do not imply the WGC; additional symmetries of the EFT (well motivated from UV completions like $SL(2,\mathbb{R})$ and $O(d,d,\mathbb{R})$) are needed to enforce the WGC. [Loges, Noumi, GS, '20].

Sketch of the Proof: Step 1

[Hamada, Noumi, GS, '18]

- We first show that under the assumption of weakly coupled UV completion, **causality** implies

$$|\alpha_1| \gg |\alpha_3|$$

because α_3 leads to causality violation and an infinite tower of massive higher spin states is required to UV complete the EFT at tree-level [Camanho, Edelstein, Maldacena, Zhiboedov, '14].

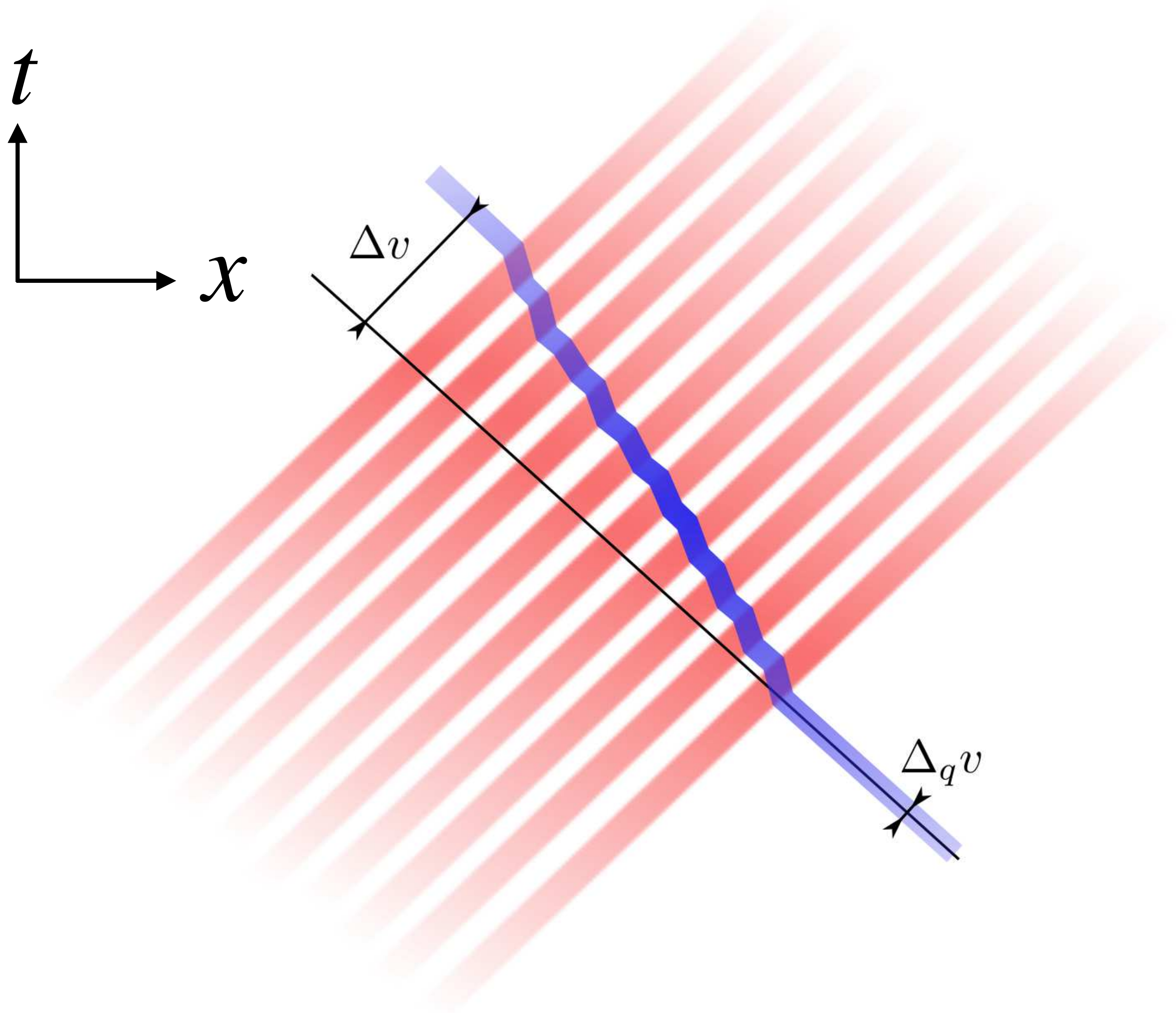


fig: Camanho et al '14

phase shift of photon propagation:

$$\delta \sim s\left(\ln(L_{\rm IR}/b) \pm \frac{|\alpha_3|}{b^2} + \ldots\right)$$

time delay in GR

helicity dependent phase shit

b : impact parameter $\quad L_{\rm IR}$: IR cutoff

Sketch of the Proof: Step 1

[Hamada, Noumi, GS, '18]

- **Time advancement** if $b^2 \ln(L/b) \ll |\alpha_3|$

- Phase shift generated by spin J is $\delta \sim s^{J-1}$. A finite # of higher spin particles does not help → infinite tower of higher spin states.

- **Causality violation** unless the scale $M_P/\alpha_3^{1/2}$ is above Λ_{QFT}.

- Integrating out light neutral scalars doesn't give significant contributions to α_3 so $|\alpha_1| \gg |\alpha_3|$

- If there are different Regge towers as in theories with open strings:

$$\alpha_{1,2,3}^{\text{closed}} \sim \frac{M_{\text{Pl}}^2}{M_s^2} \quad \ll \quad \alpha_{1,2}^{\text{open}} \sim \frac{M_{\text{Pl}}^2}{g_s M_s^2}\,, \qquad g_{\text{open}} \sim \sqrt{g_s} \gg g_s$$

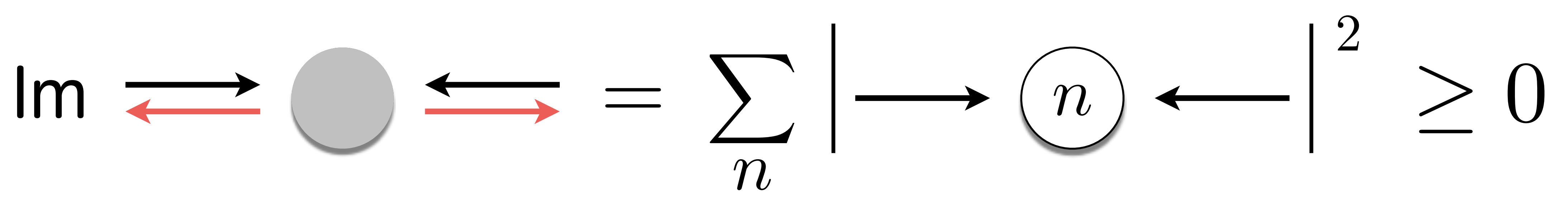
- If there are light fields or different Regge towers, α_3 is **subdominant** compared with the causality preserving terms α_1 and α_2.

Sketch of the Proof: Step 2

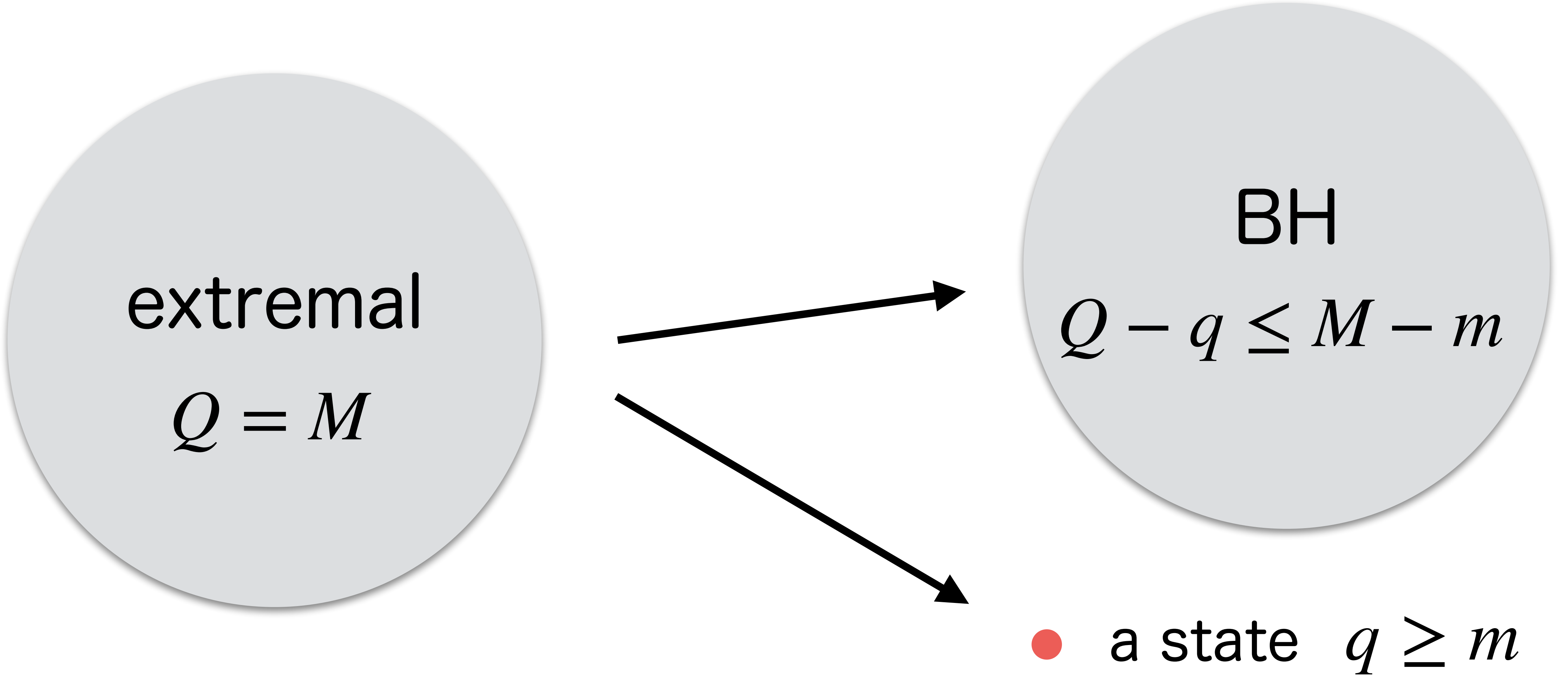
[Hamada, Noumi, GS, '18]

- Consider the forward limit $t \to 0$ of $\gamma\gamma$ scattering, Regge boundedness implies that the UV contour does not give $\mathcal{O}(s^2)$ contribution to $\mathcal{M}$.

- The higher derivative operator parametrized by α_1 leads to: $\alpha_1(F_{\mu\nu}F^{\mu\mu})^2 \quad \Rightarrow \quad \mathcal{M} \sim \alpha_1 s^2$



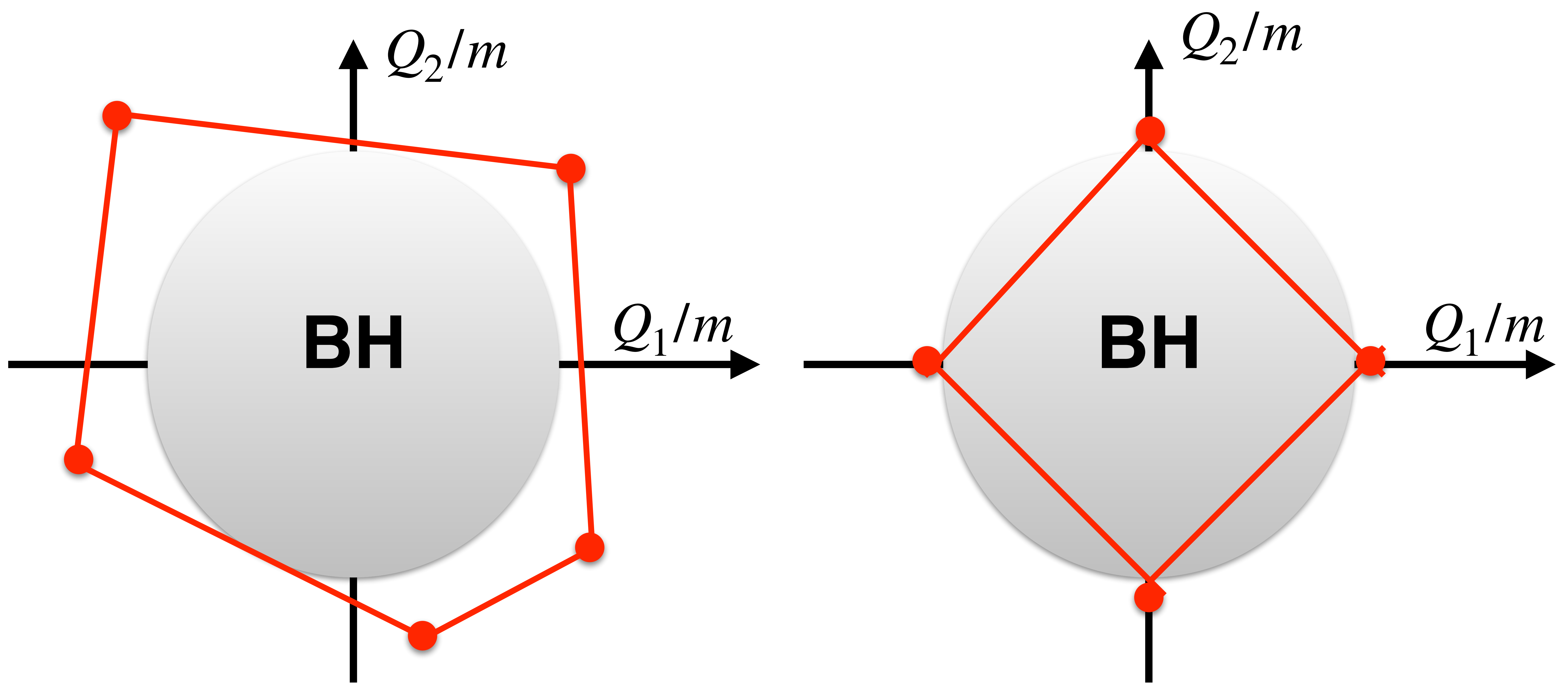
Unitarity $\Rightarrow$ $\alpha_1 > 0$



can be an extremal BH!

Stronger forms of the WGC

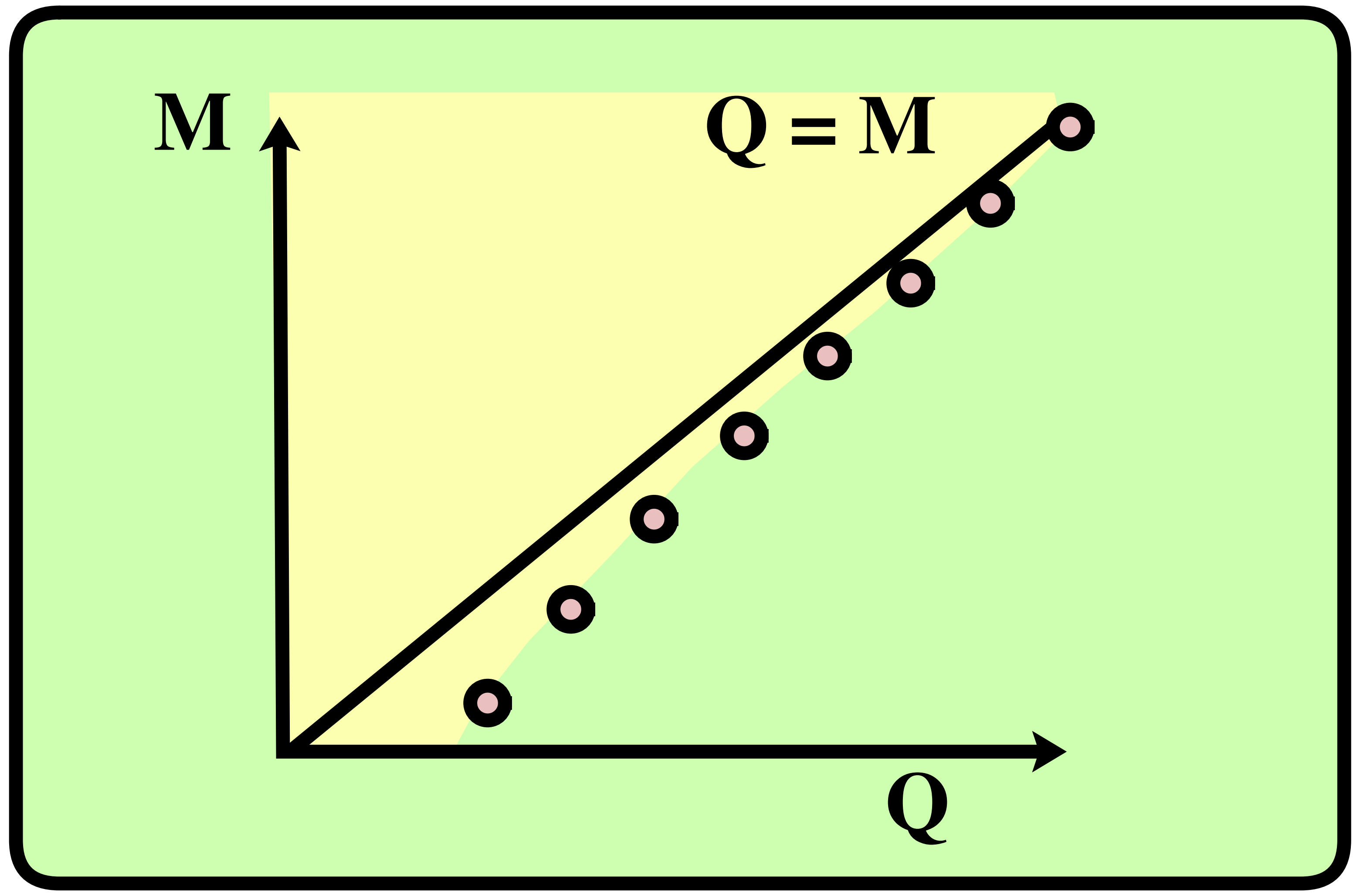
- Consistency with dimensional reduction and duality suggests stronger versions of the WGC known as the sub-lattice WGC [Montero, GS, Soler '16], [Heidenreich, Reece, Rudelius '16] and tower WGC [Andriolo, Junghans, Noumi, GS '18].



- Applying the Convex Hull Condition [Cheung, Remmen,'14];[Brown, Cottrell, GS, Soler, '15] to BHs carrying KK U(1)'s leads to tower/sublattice WGC.

Strong forms of the WGC

- The strongest evidence comes from string theory, suggesting a monotonic behavior.



Can we upgrade the scattering positivity bound arguments to show this monotonicity?

- The correspondence principle [Horowitz, Polchinski '96] states that $S_{\text{string}} \sim \mathcal{O}(1) S_{\text{BH}}$ but does not guarantee that the extremal curve stays on one side.
- For BHs with near-horizon BTZ geometry, entropy matching is exact implying that superextemality curve stays superextremal upon turning on g_s. [Aalsma, Cole, GS '19].

Spinning WGC?

Rotating BHs

- Could there be similar constraints for rotating BHs? [Aalsma, GS, '22]. **Maybe not:**
  - Rotating BHs can lose energy via **superradiance**.
  - For pure gravity in $D \geq 6$, BHs w/ a given mass can have arbitrarily large J [Myers, Perry, 86].
- **But ….**
  - Spinning WGC for BTZ BH follows from c-theorem of the dual CFT [Aalsma, Cole, Loges, GS, '20]
  - In string theory, spin can sometimes be mapped to charge, e.g.,

$$\begin{tikzcd}
 & \textbf{5d Pure Gravity} \arrow[dl] \arrow[dr] & \\
\textbf{4D KK BH} & & \textbf{5D Myers-Perry BH} \\
M_4, Q, P, J_4 \arrow[rr, leftrightarrow] & & M_5, J_1, J_2
\end{tikzcd}$$

Rotating BHs

- However, in string theory, spin can sometimes be mapped to charge, e.g.,

5d Pure Gravity

4D KK BH | **5D Myers-Perry BH**

$$M_4, Q, P, J_4 \longleftrightarrow M_5, J_1, J_2$$

- The leading correction to Einstein gravity is the Gauss-Bonnet term:

$$L_{5D} = \frac{1}{2}R + \lambda\left(R_{abcd}R^{abcd} - 4R_{ab}R^{ab} + R^2\right)$$

Can consider KK BH with $J_4 = 0$
to fix the sign of λ using the WGC

[Aalsma, GS, ’22]

Mapping Spins to Charges

- Consider a **rotating dyonic** KK BH with (M_4, Q, P, J_4)

  $\rightarrow$ BH sitting at the tip of Taub-NUT $(r_{BH} \ll R_5)$

- Taking a **decompactification** limit, this becomes a 5D MP BH with (M_5, J_1, J_2).

- Map between parameters:

$$M_5 \sim M_4 - M_{\text{monopole}}$$

$$QP \sim \frac{1}{2}\left(J_1 + J_2\right)$$

$$J_4 \sim \frac{1}{2}\left(J_1 - J_2\right)$$

Can map 5d rotations to pure charges when $J_1 = J_2$!

- Calculate the leading higher derivative corrections to the extremality bound for the KK BH, either in 5D or its reduction to 4D, $R_{GB} \rightarrow 4 - \text{derivative terms involving } R, F, \phi$.

KK Black Hole

- **KK black hole:**

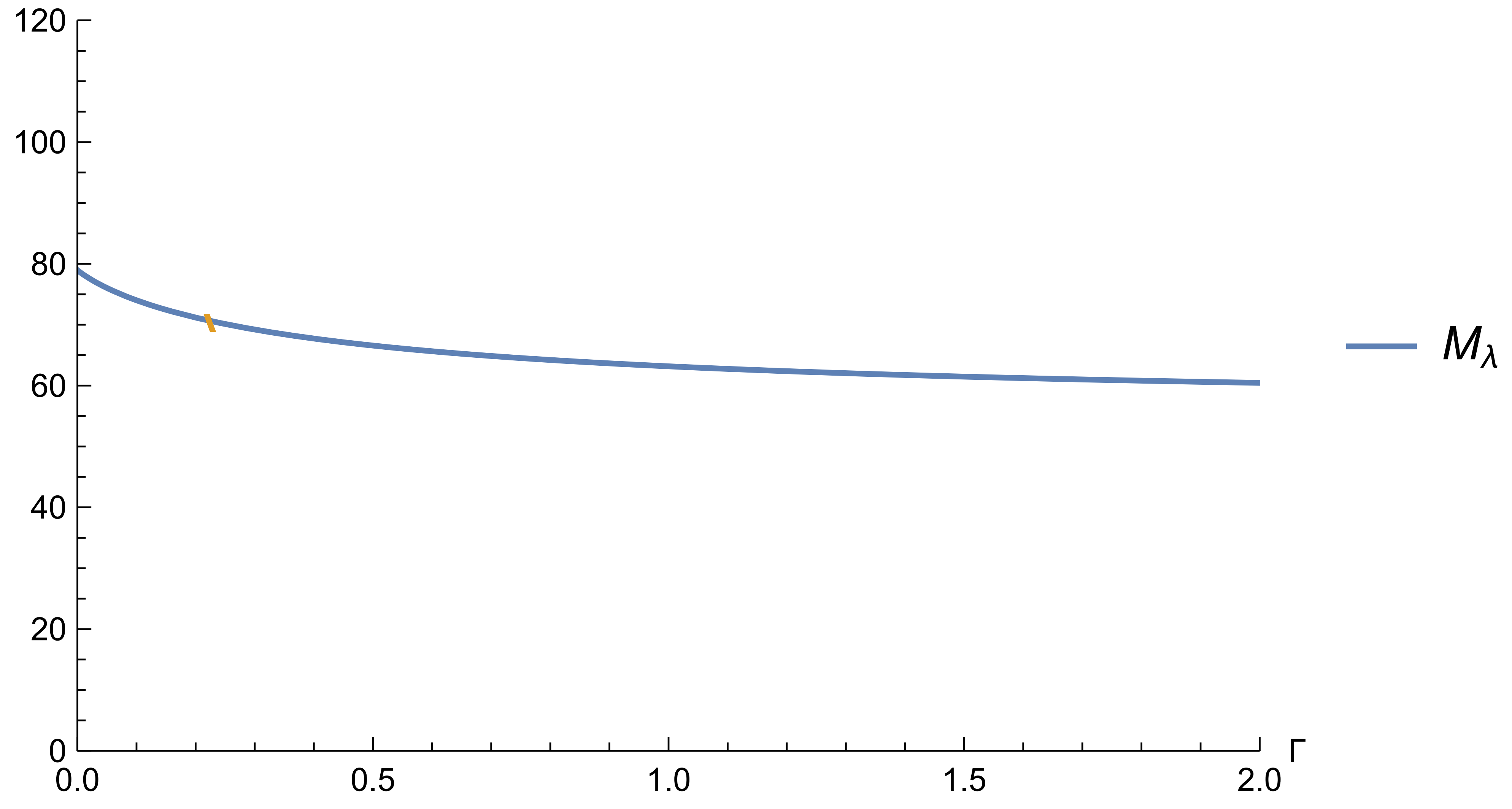
$$\delta M_4 \sim \int d^5x\sqrt{-g}\lambda R_{GB} = -\frac{8\pi^2\lambda R}{p}\frac{(1+\Gamma)}{(1-\Gamma)^2\sqrt{\Gamma^2-1}}\left(3\pi\Gamma^2\mathrm{sgn}(\Gamma-1)+(1-4\Gamma)\sqrt{\Gamma^2-1}+6\Gamma^2\arctan\left[\sqrt{\frac{\Gamma+1}{\Gamma-1}}\right]\right)$$

where $\Gamma = q/p$ $\quad Q = 4\pi\sqrt{\frac{q(q^2-4m^2)}{p+q}}\,, \quad P = 4\pi\sqrt{\frac{p(p^2-4m^2)}{p+q}}\,.$ Extremal BH: $m = 0$

- For $Q = P$, the expression simplifies:

$$\delta M_4^{\mathrm{KK}}\Big|_{p=q} = -\frac{32\pi^2 R\lambda}{5qL}$$

- **δM_4 does not change sign for a fixed λ.**
- The WGC $\Rightarrow \lambda \geq 0.$



Myers-Perry Black Hole

- **MP black hole**:

$$\delta M_5 \sim \int d^5x\sqrt{-g}\,\lambda R_{GB} = -\frac{4\pi^2\lambda}{L}\left(\frac{J_1^2 + J_2^2 - 6|J_1J_2|}{|J_1J_2|}\right)$$

indefinite sign!

- However, for $J_1 = \pm J_2$

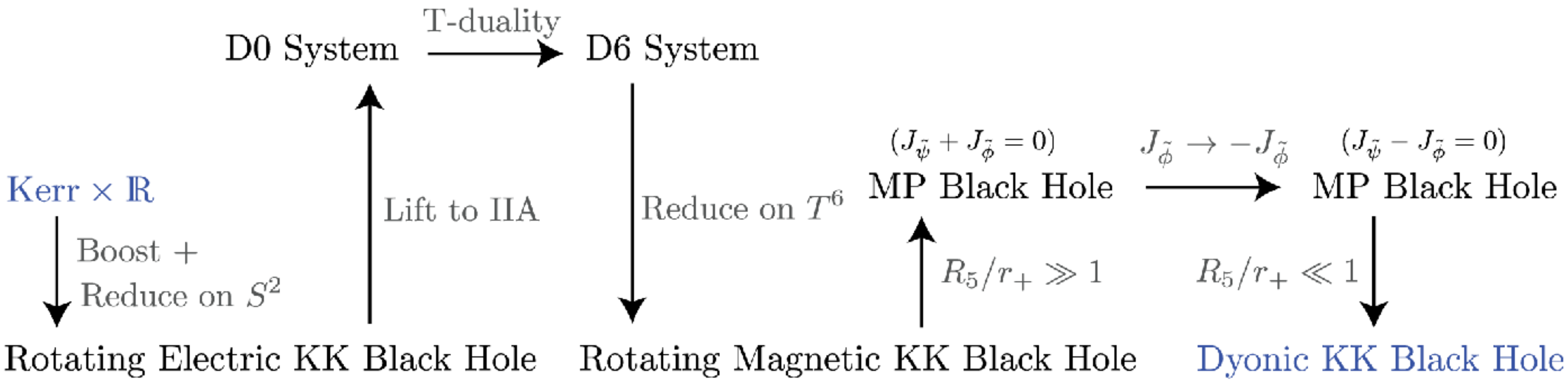
$$\delta M_5 = +\frac{16\pi^2\lambda}{L} \geq 0$$

where we used the Charge WGC $\Rightarrow \lambda \geq 0$

- The extremality bound for rotating BH is **shifted negatively**.

Kerr BH

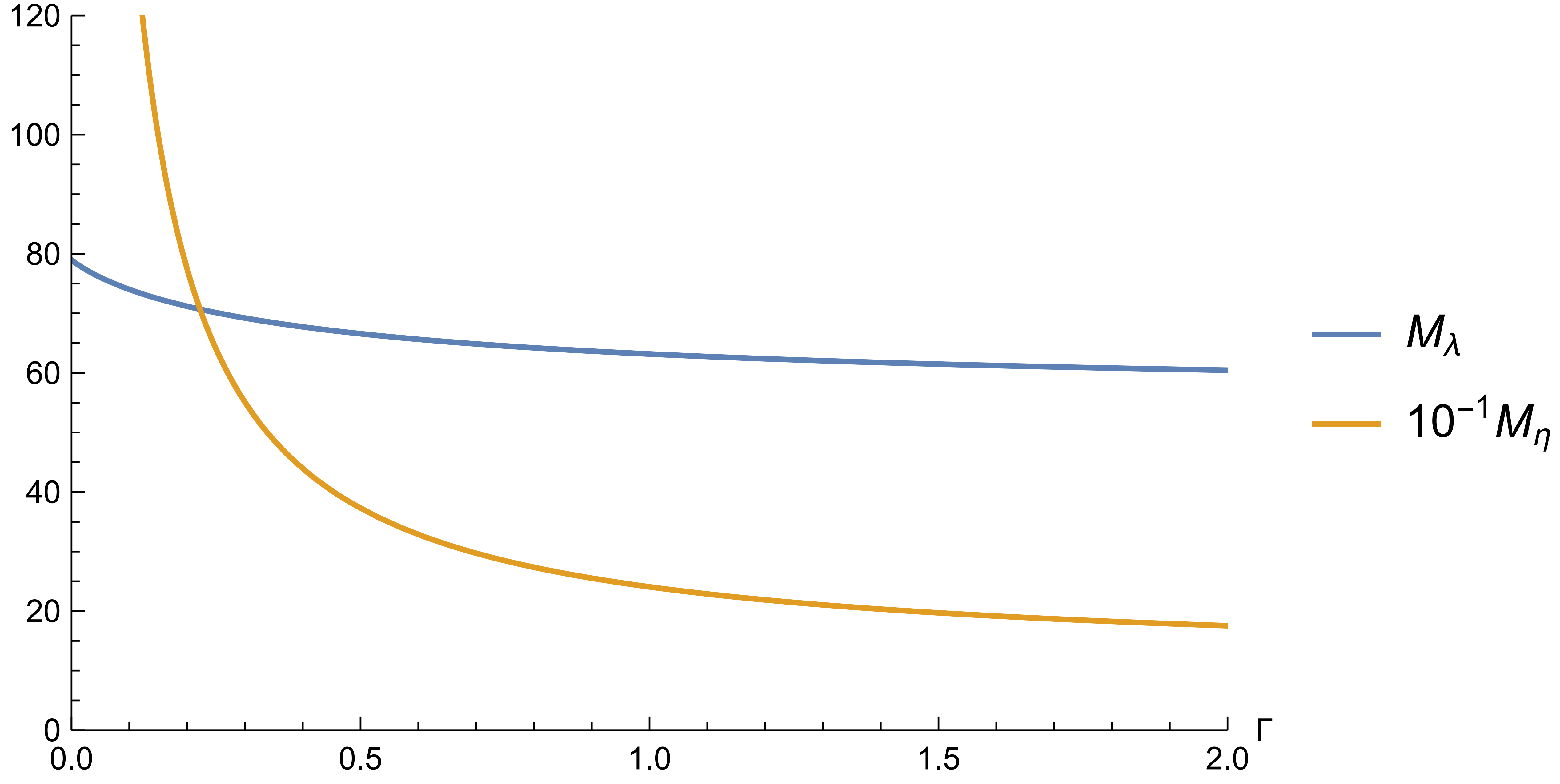
- A chain of dualities maps a Kerr BH to a non-rotating charged dyonic BH:



- Similar logic can fix corrections to Kerr BH. However, the Gauss-Bonnet term is topological in 4D, and so the leading correction is the 6-derivative operator:

$$\delta L = \frac{\lambda}{L} R_{abcd} R^{abcd} + \eta L \, R_{ab}{}^{cd} R_{cd}{}^{ef} R_{ef}{}^{ab}$$

Corrections to Extremality Bounds

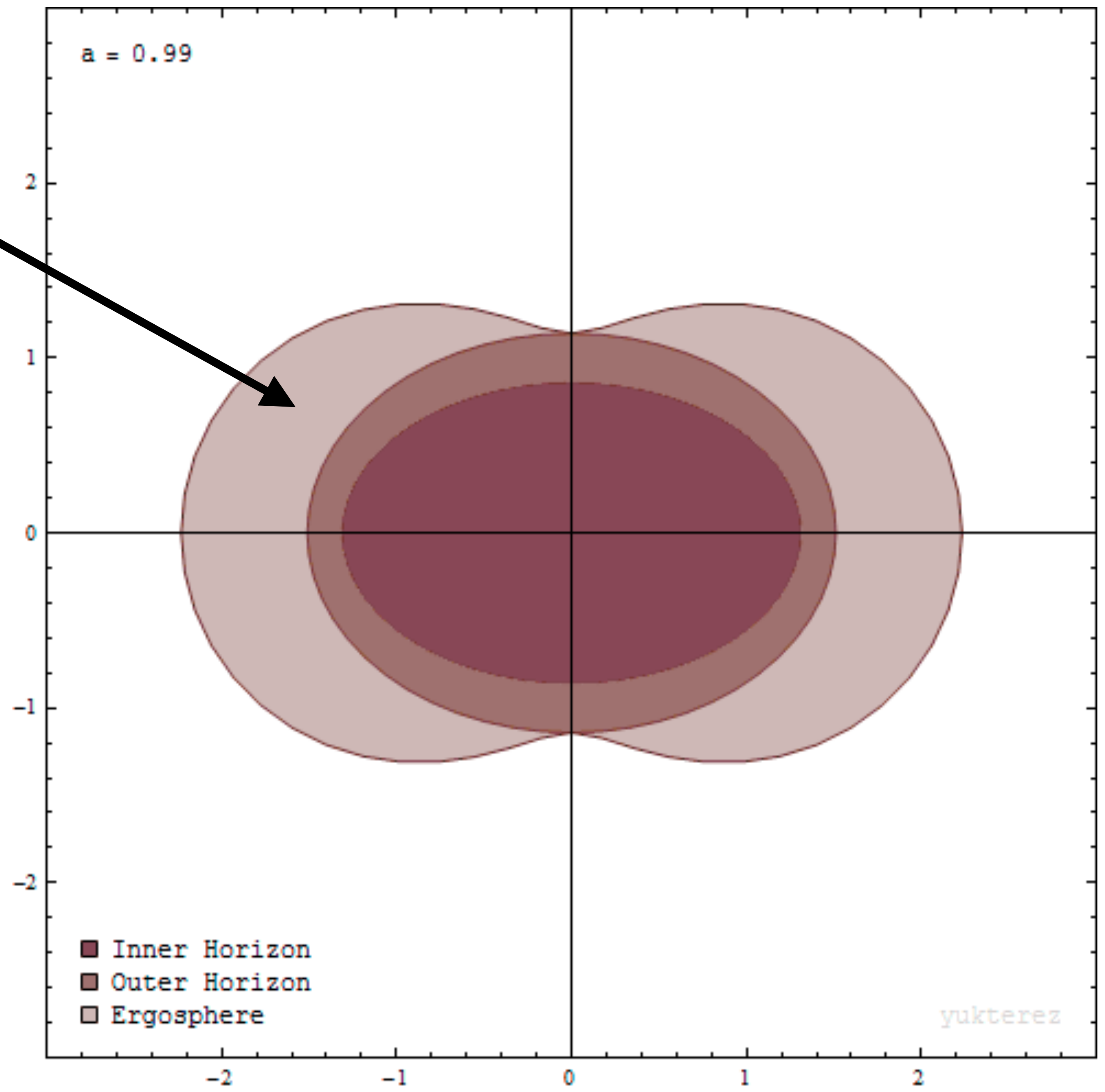


	$\frac{\lambda}{L}\mathcal{R}^2$	$\eta L \mathcal{R}^3$
δM_4^{KK}	$-\frac{\lambda}{L}\mathcal{M}_\lambda$	$\eta L \mathcal{M}_\eta$
WGC:	$\lambda \geq 0$	$\eta \leq 0$
δM_5^{MP}	$\frac{\lambda}{L} 16\pi^2$	$\eta L \frac{192\pi^2}{7a^2}$
Sign:	$+$	-
$\delta M_4^{\mathrm{Kerr}}$	0	$\eta L \frac{8\pi}{7\hat{a}^3}$
Sign:	n.a.	-

Superradiance

- Rotating BHs are unstable due to superradiance which occurs when there is an **ergosphere**:

$\omega < 0$



Extract energy ω and angular momentum j, BH can lose its mass if

$$dM = TdS + \Omega_i dJ^i \quad = TdS \frac{\omega}{\omega - j^i \Omega_i} \quad \leq 0$$

$$\Rightarrow \quad \omega \leq j^i \Omega_i$$

guaranteed if $\exists$ ergosphere!

- **How does the superradiant instability of rotating BHs manifest in the charged BH?**

Superradiance vs WGC

- Charged BHs have no ergosphere, but can lose energy in a similar sense if

$$\mathrm{d}M = T\mathrm{d}S + \Psi_q \mathrm{d}Q + \Psi_p \mathrm{d}P \quad \leq 0$$

- If the particle extracting energy ω and electric, magnetic charges (k_q, k_p) from the BH:

$$\frac{16\pi G_4 \omega}{k_q\sqrt{1+(P/Q)^{2/3}} + k_p\sqrt{1+(Q/P)^{2/3}}} \leq 1$$

charged superradiance

- This stronger charged superradiance condition **implies the WGC**: $$\frac{16\pi G_4 \omega}{(k_q^{2/3}+k_p^{2/3})^{3/2}} \leq 1$$

- The superradiance condition and the WGC coincide when $k_q/k_p = Q/P$.

- Phrased in term of superradiance, rotating and charged BHs are treated in unified manner.

Axion WGC

WGC for p-form Symmetry

- One can generalize the WGC for 1-form gauge fields to the WGC for (p+1)-form gauge fields which couple to p-branes:

$$\frac{Q_p}{T_p} \geq \left(\frac{Q_p}{T_p}\right)_{\mathrm{Ext}}$$

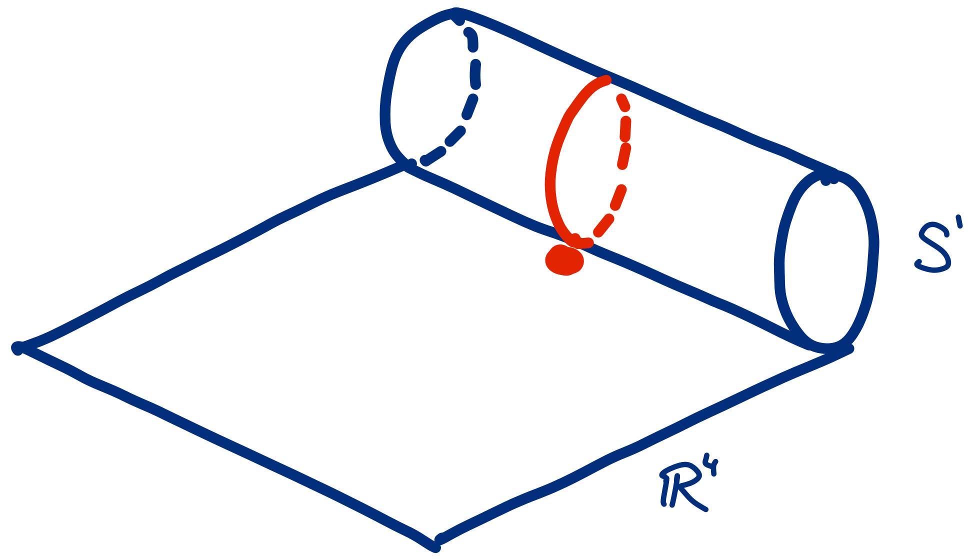
- The 0-form gauge field (axion) case **(-1 form symmetry)** is most interesting (axion inflation) but subtle as the "branes" that couple to it are **instantons**.

- Obtaining an axion by duality [Brown, Cottrell, GS, Soler, '15] or dimensional reduction [Heidenreich, Reece, Rudelius, '16] suggests that the above inequality can indeed be extrapolated to:

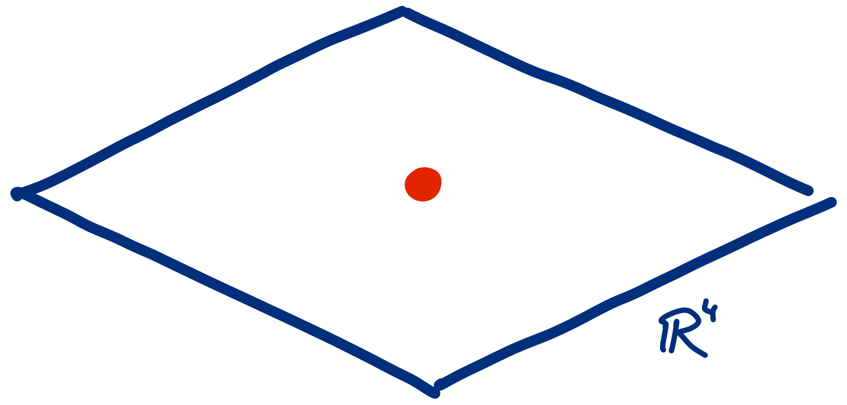
$$f \cdot S_{\mathrm{inst}} \leq \mathcal{O}(1) M_P$$

Axionic WGC

- Consider a 5d particle with mass m_5 and charge q_5 whose (Euclidean) worldline wraps the compact dimension



$$
\begin{aligned}
S_\psi &= m_\psi \oint d\ell + i \oint A \\
&= 2\pi R\, m_\psi + i\phi(x)
\end{aligned}
$$



- This particle sources the axion and is localized to a point in 4d spacetime, i.e. it is an **instanton**:

$$V(\phi) \sim e^{-S_{inst}} \cos\left(\frac{\phi}{f}\right)$$

$$
\begin{aligned}
S_{inst} &= 2\pi R m_\psi \\
f^{-1} &= g_5 \sqrt{2\pi R}
\end{aligned}
$$

- The 5d WGC for charged particles $m_5 < g_5 M_{Pl,5d}$ translates into:

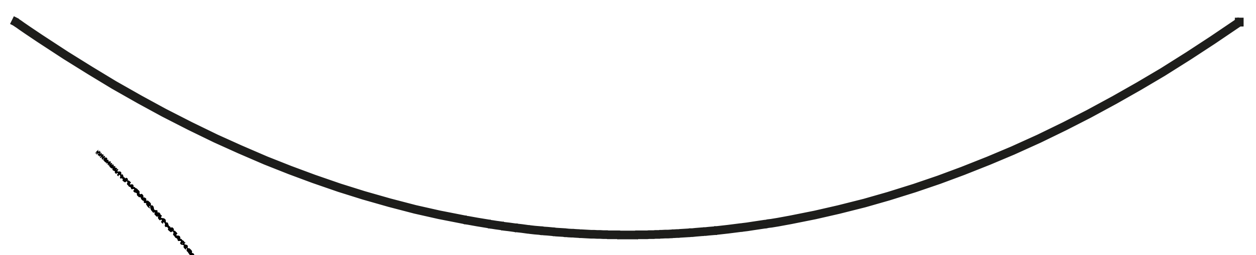
$$\boxed{f \cdot S_{inst} \leq M_p}$$

Gravity Wave and UV Sensitivity

A detection of CMB B-mode at the targeted level implies that the inflaton potential is nearly flat over a **super-Planckian** field range. For single-field inflation:

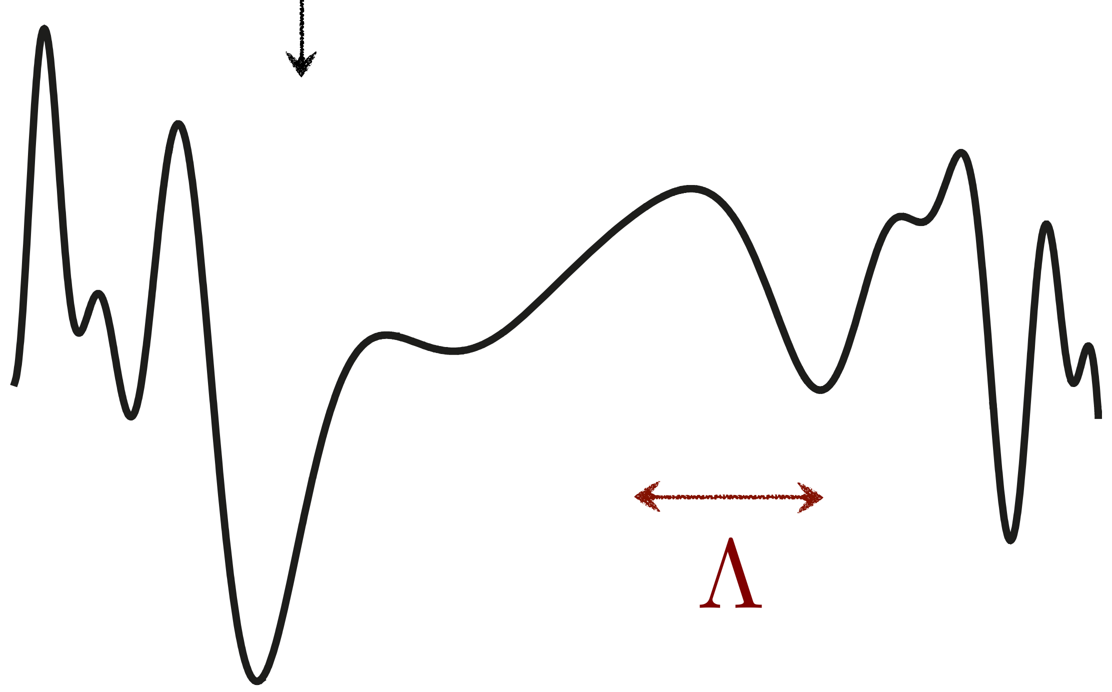
$$\Delta\phi \gtrsim \left(\frac{r}{0.01}\right)^{1/2} M_{\rm Pl}$$

Lyth '96



$$\mathcal{L}_{\rm eff}[\phi] = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2\left(1 + \sum_{i=1}^{\infty} c_i \frac{\phi^{2i}}{\Lambda^{2i}} + \cdots\right)$$

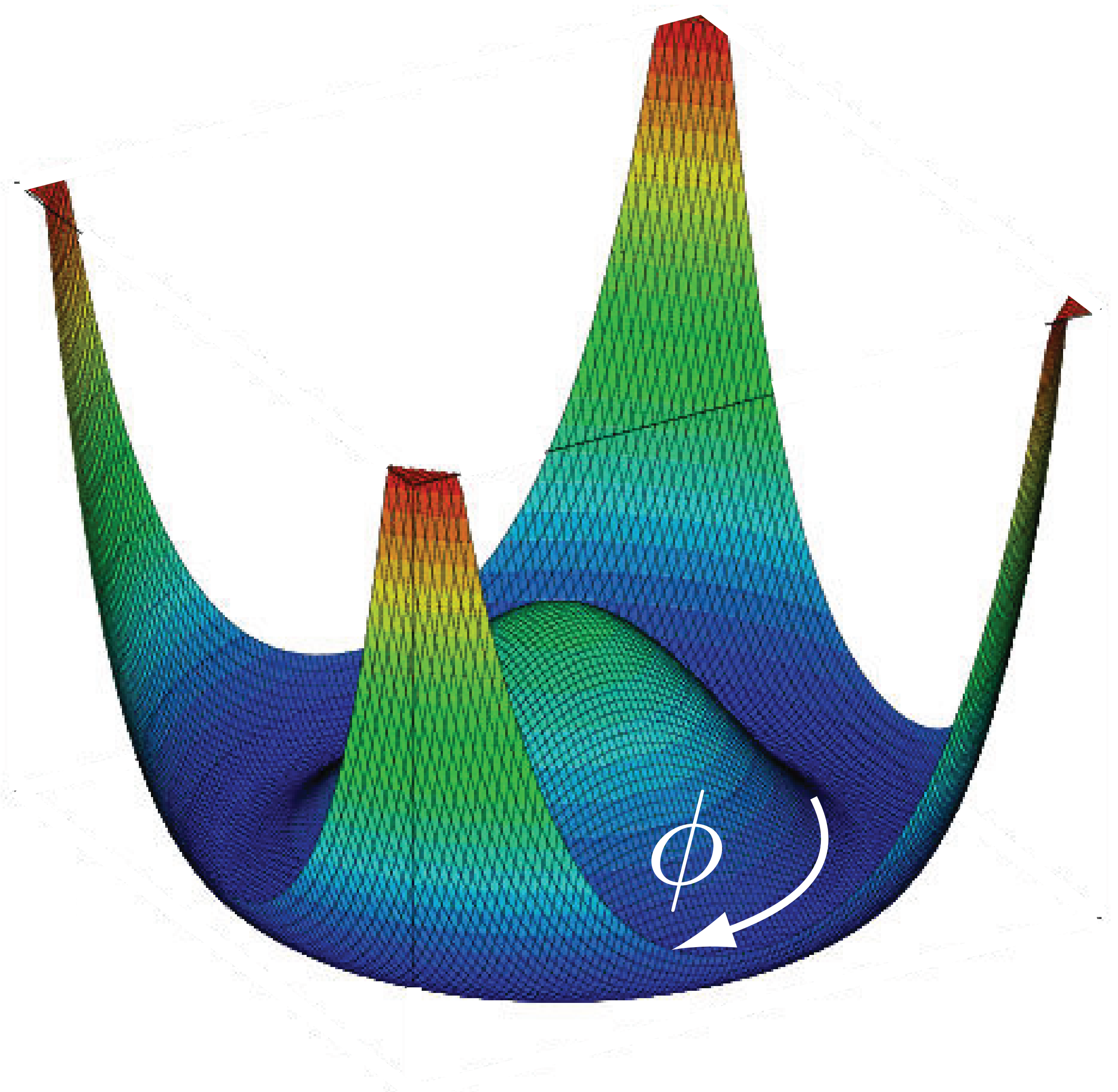
$$c_i \sim \mathcal{O}(1)$$



"Large field inflation" are highly **sensitive to UV physics**

Axions and Large Field Inflation

Pseudo-Nambu-Goldstone bosons are natural inflaton candidates. Natural Inflation [Freese, Frieman, Olinto]



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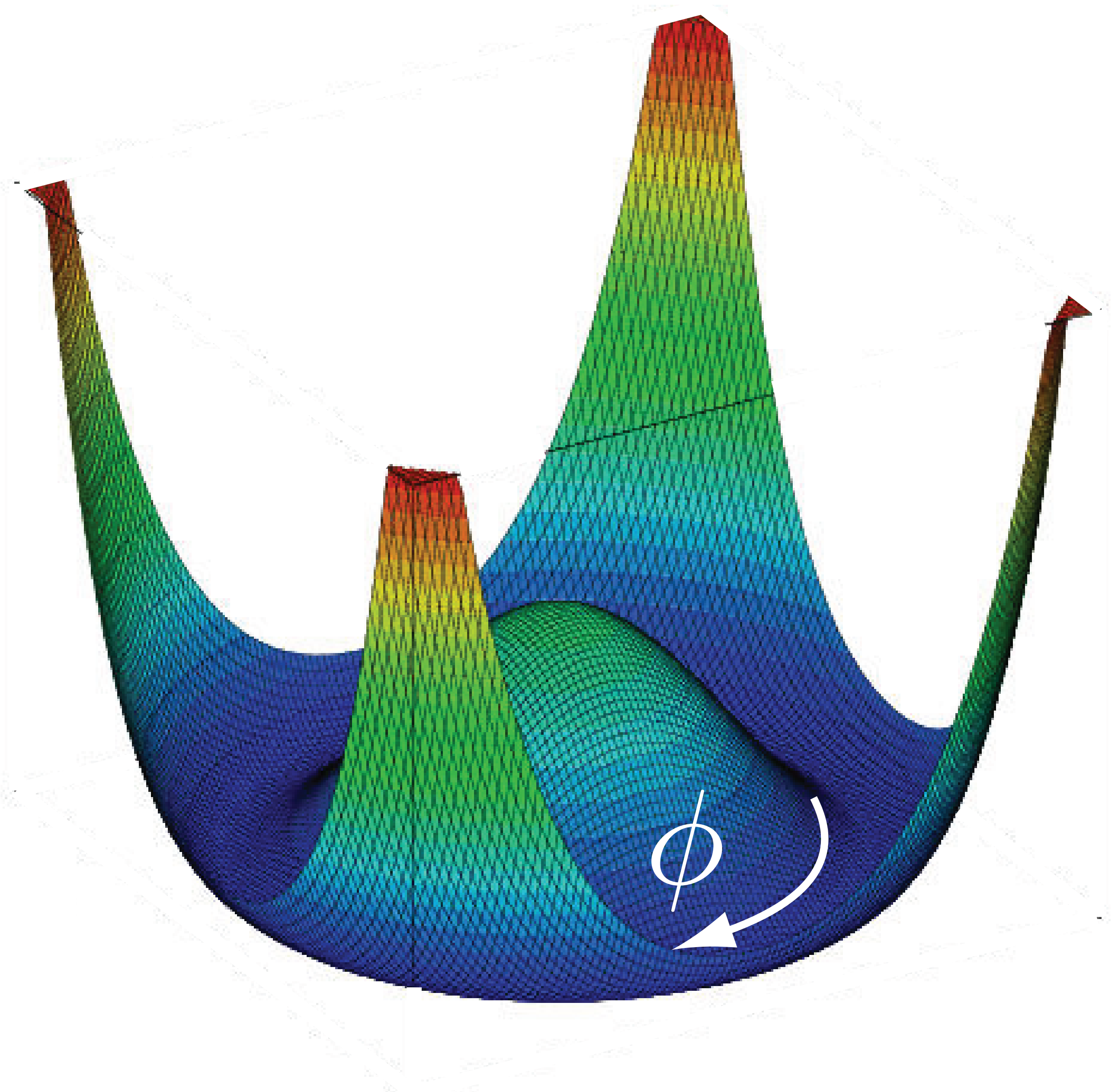
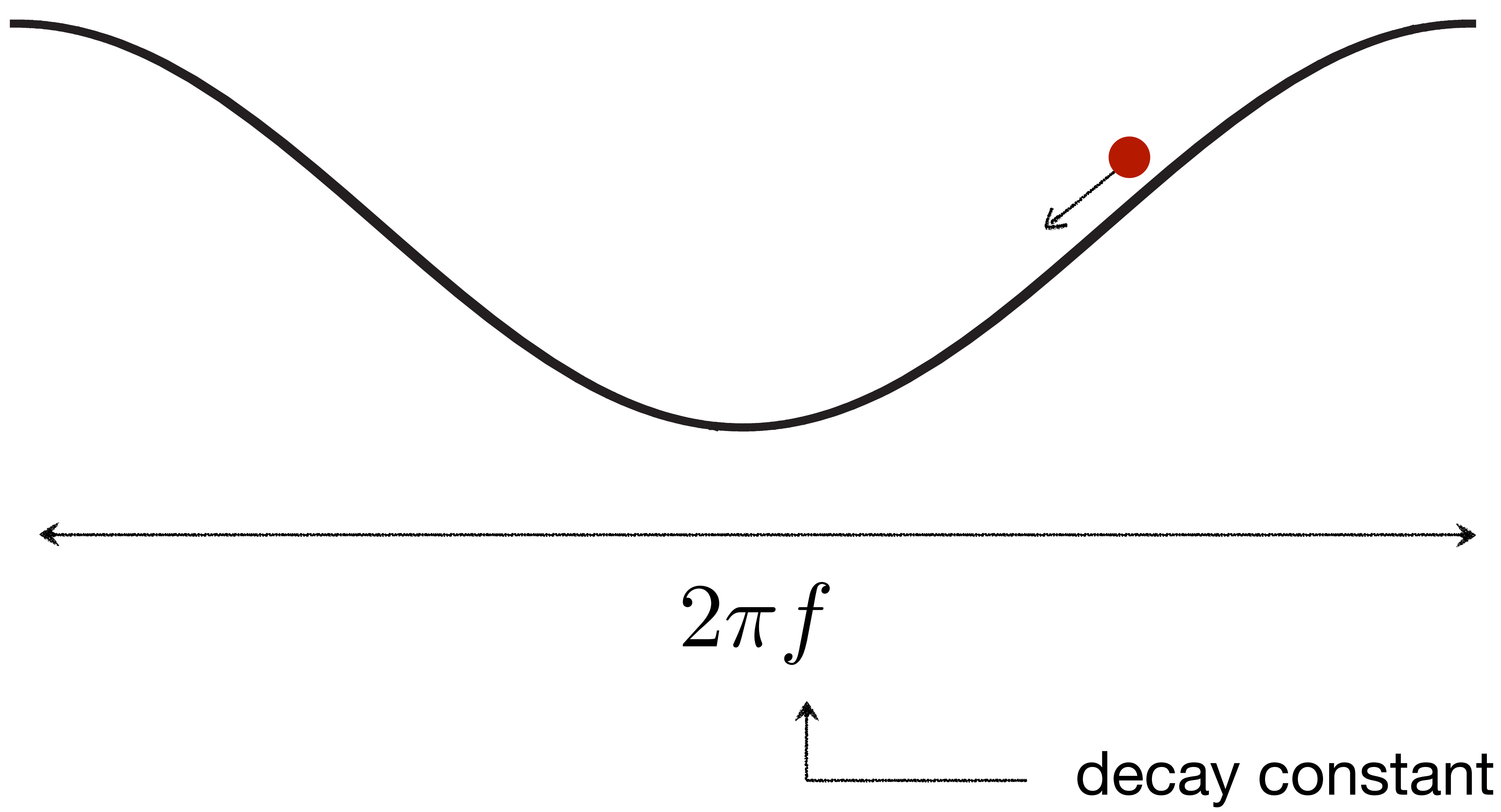


They satisfy a shift symmetry that is only broken by non-perturbative effects:



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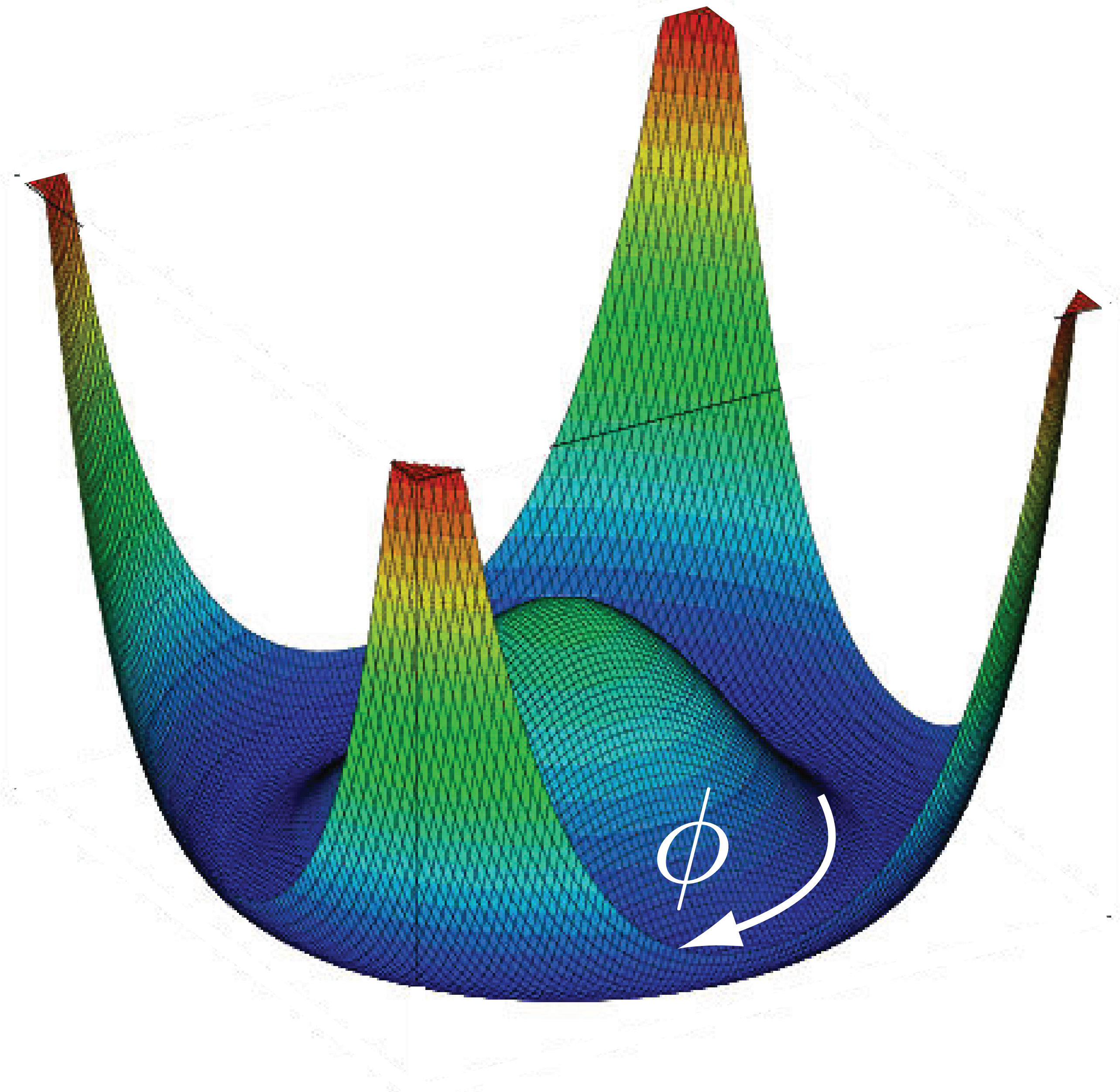

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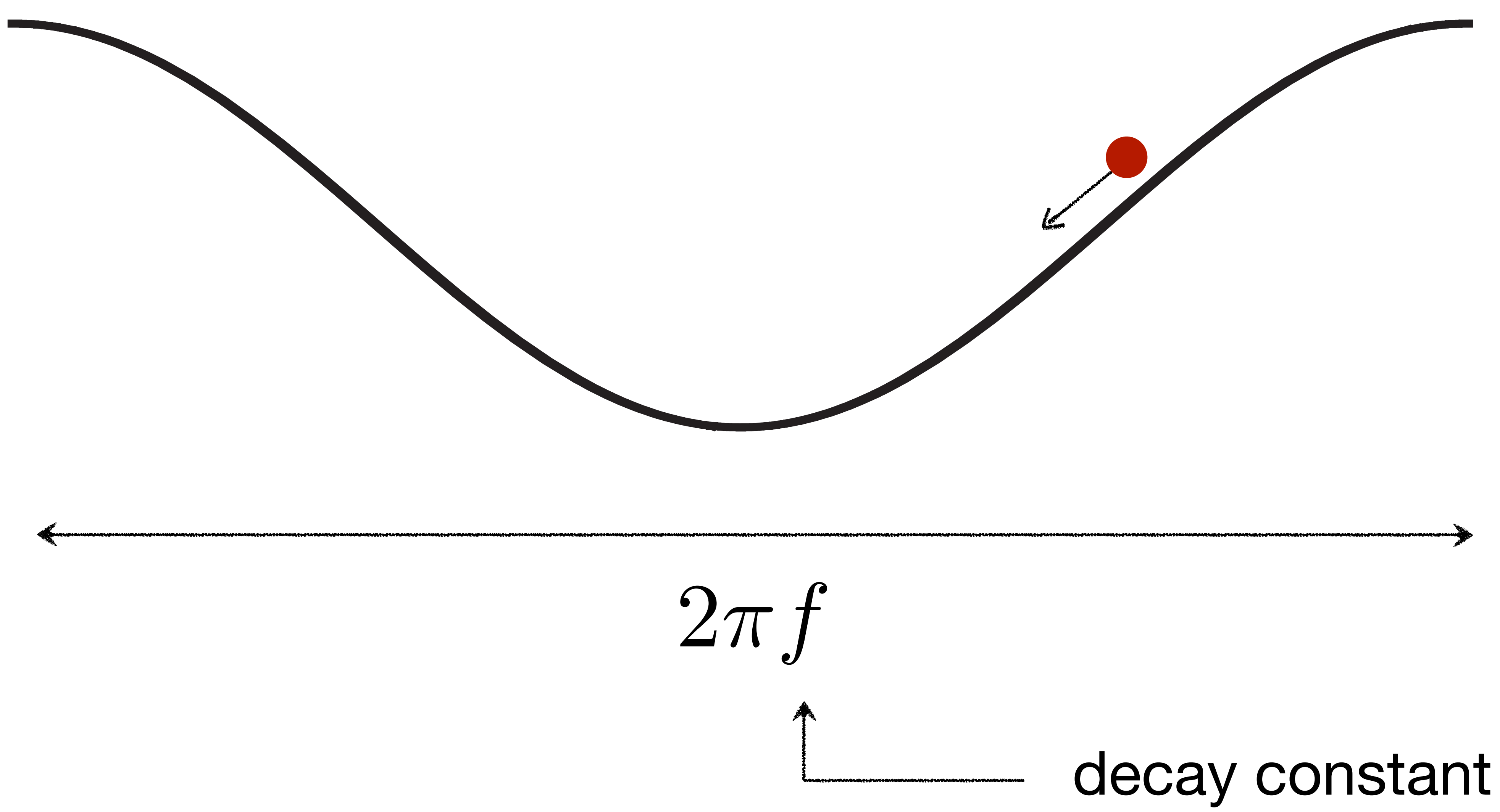

Slow roll: $f > M_P$

$$V(\phi) = 1 - \Lambda^{(1)} \cos\left(\frac{\phi}{f}\right) + \sum_{k>1} \cancel{\Lambda^{(k)} \left[1 - \cos\left(\frac{k\phi}{f}\right)\right]} \quad \text{if} \quad \frac{\Lambda^{(n+1)}}{\Lambda^{(n)}} \sim e^{-S_{\text{inst}}} << 1$$

Axions and Large Field Inflation

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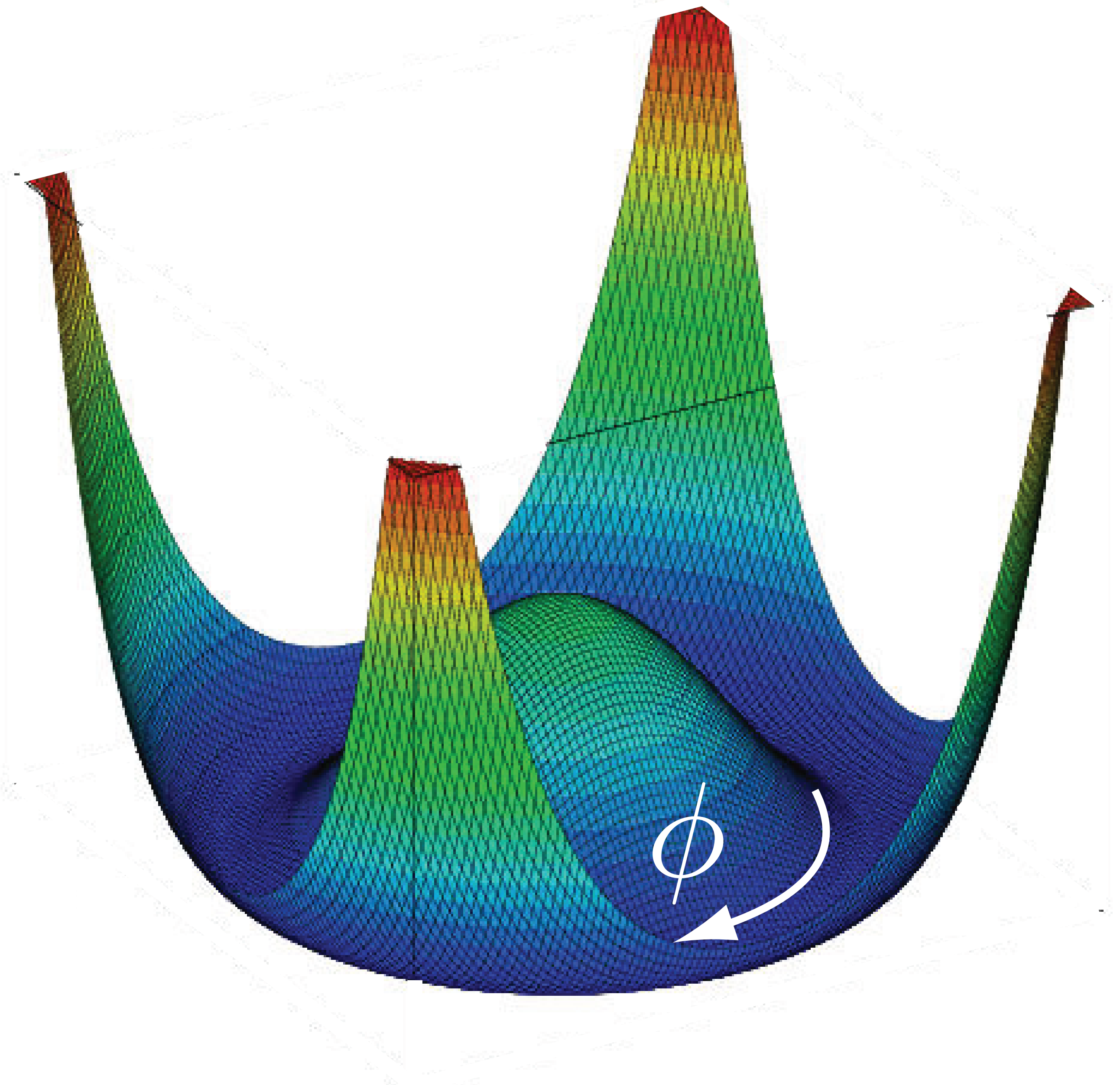
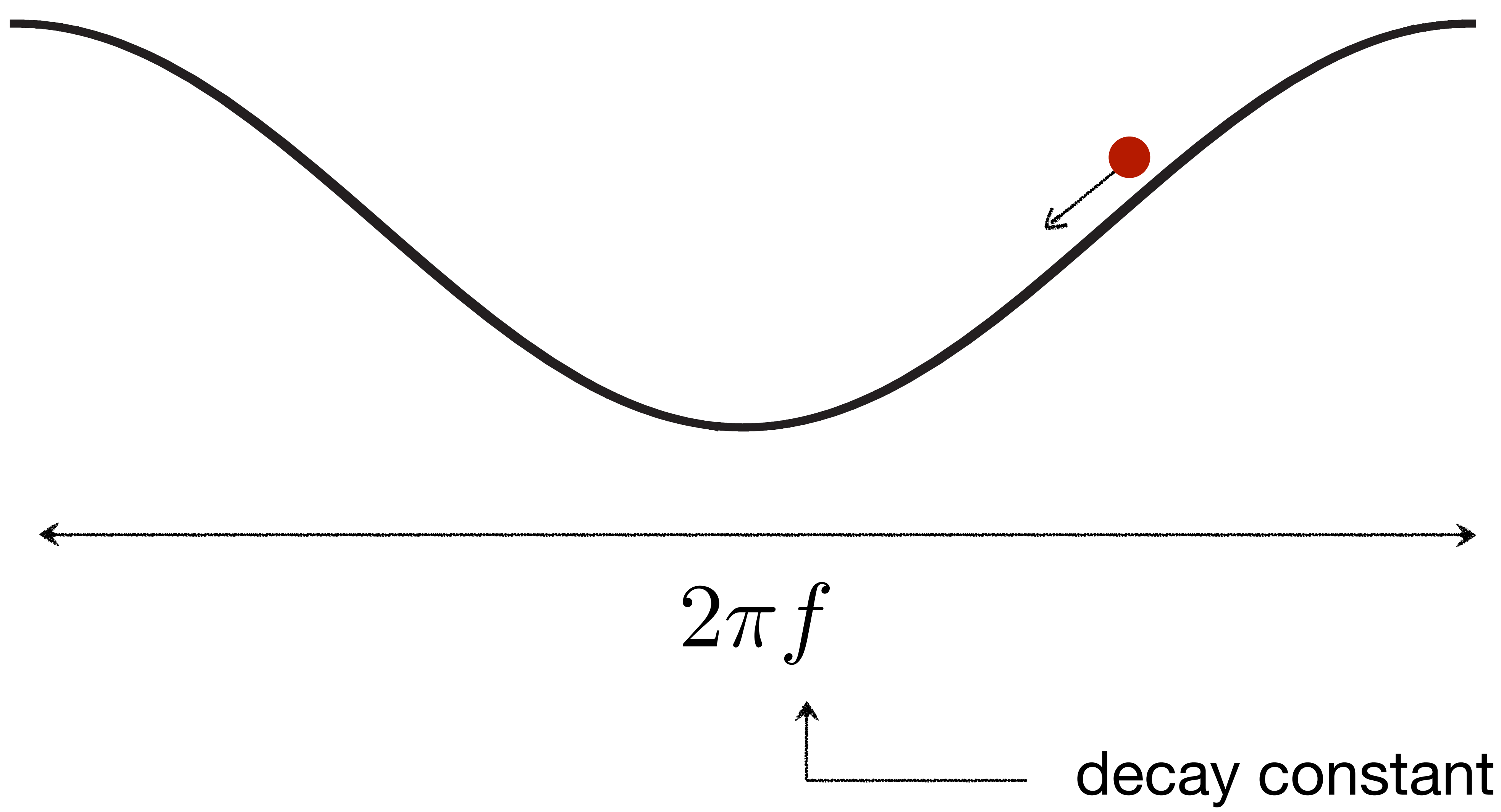


They satisfy a shift symmetry that is only broken by non-perturbative effects:



Slow roll: $f > M_P$

$$V(\phi) = 1 - \Lambda^{(1)} \cos\left(\frac{\phi}{f}\right) + \sum_{k>1} \cancel{\Lambda^{(k)} \left[1 - \cos\left(\frac{k\phi}{f}\right)\right]} \quad \text{if} \quad \frac{\Lambda^{(n+1)}}{\Lambda^{(n)}} \sim e^{-S_{\text{inst}}} << 1$$

The WGC implies that these conditions cannot be *simultaneously* satisfied.

Loopholes

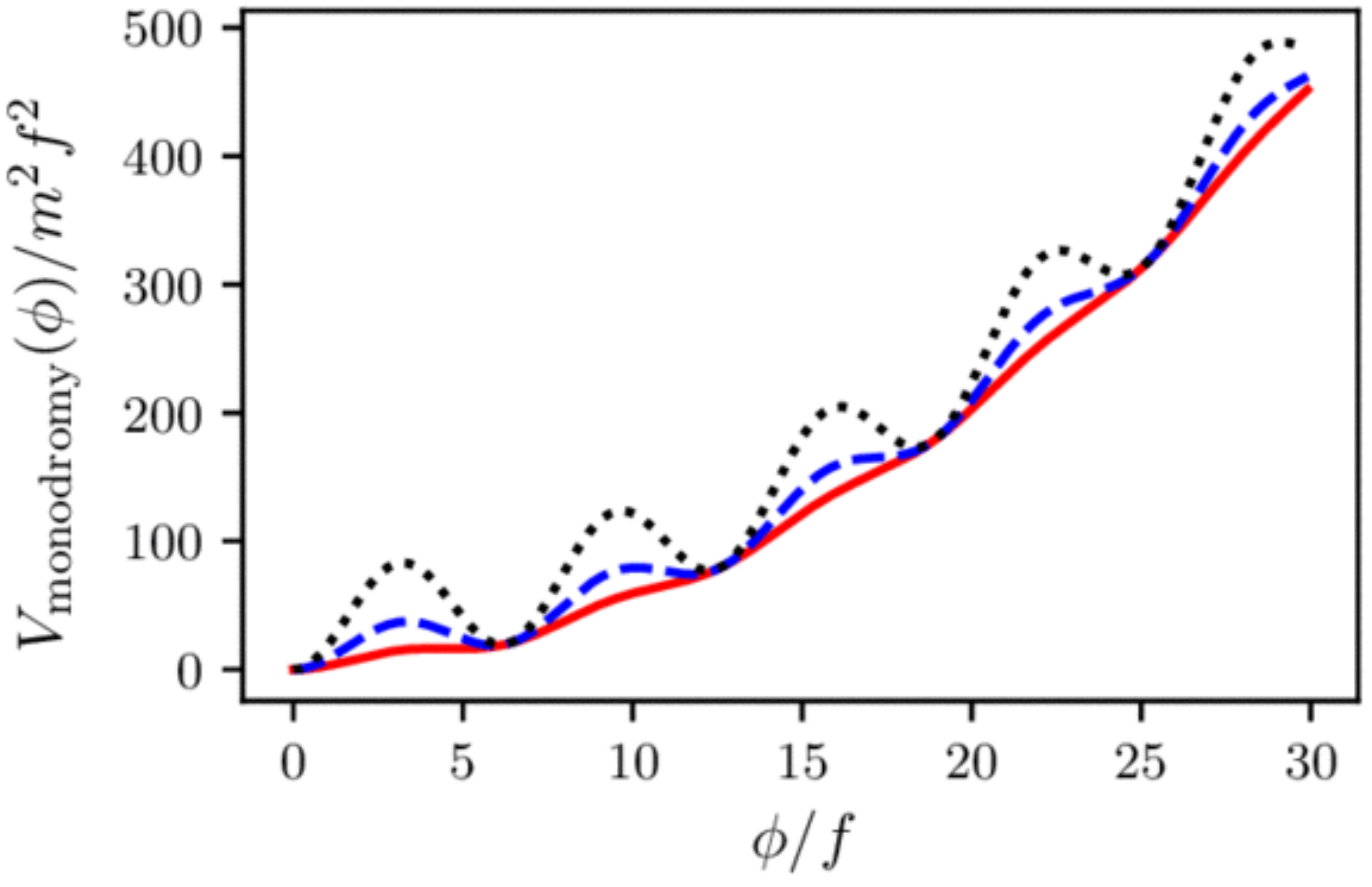
- Whether the WGC can rule out natural inflation depends on whether it takes a strong form. The weak form can be satisfied by a spectator instanton [Brown, Cottrell, GS, Soler, '18];[Rudelius, '18]:

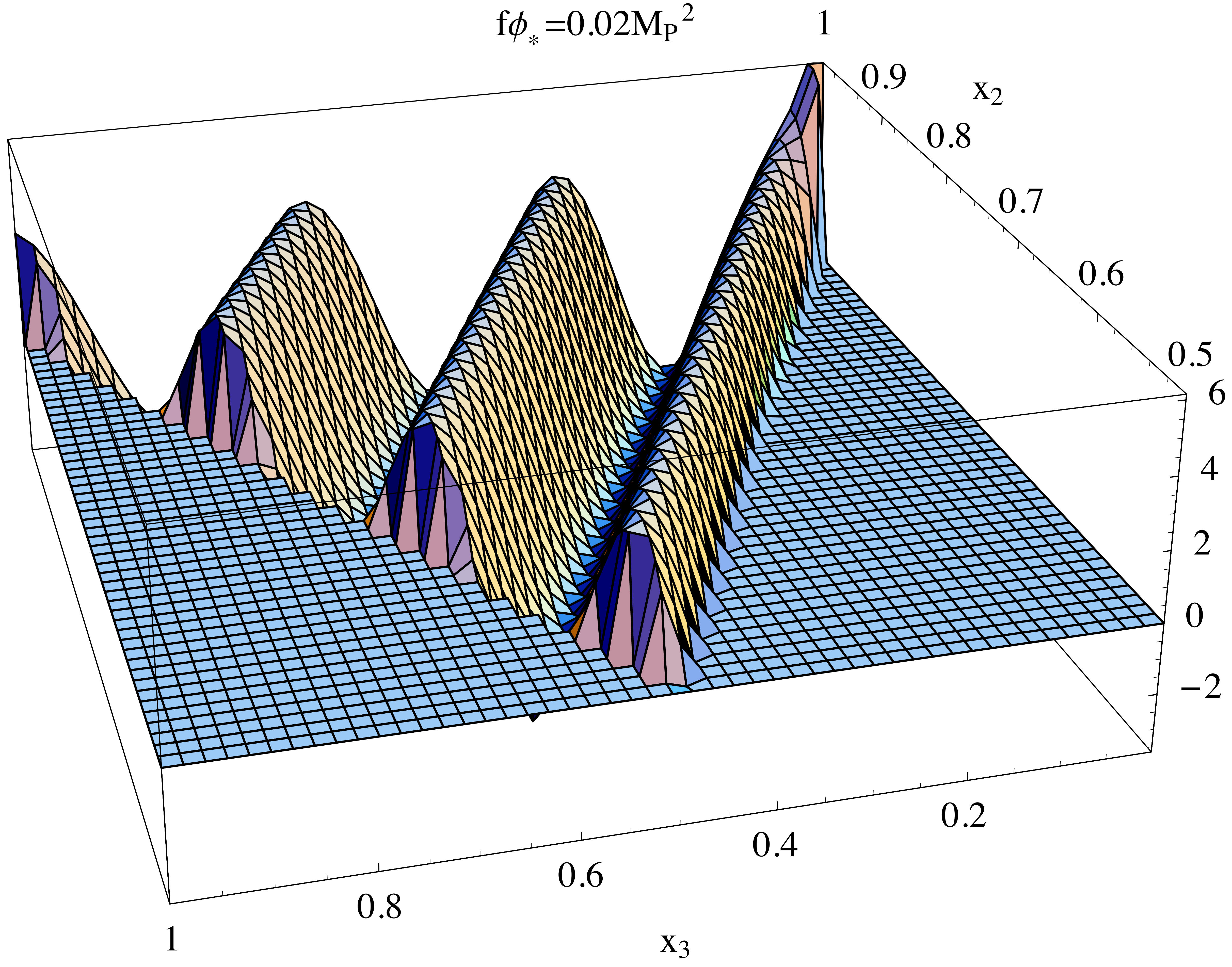
$$V = e^{-m}\left[1-\cos\left(\frac{\Phi}{F}\right)\right] + \quad e^{-M}\left[1-\cos\left(\frac{\Phi}{f}\right)\right]$$

with $1 < m \ll M, \qquad F \gg M_P > f, \qquad M \times f \ll 1$

- Another loophole is inflation with non-periodic axions (aka axion monodromy [Silverstein, Westphal, '08];[McAllister, Silverstein, Westphal, '08];[Marchesano, GS, Uranga, '14];[Hebecker, Kraus, Witkowski, '14]; [Blumenhagen, Plauschinn, '14]; ...) as they are not mapped to long-range gauge fields.

Resonant Non-Gaussianity

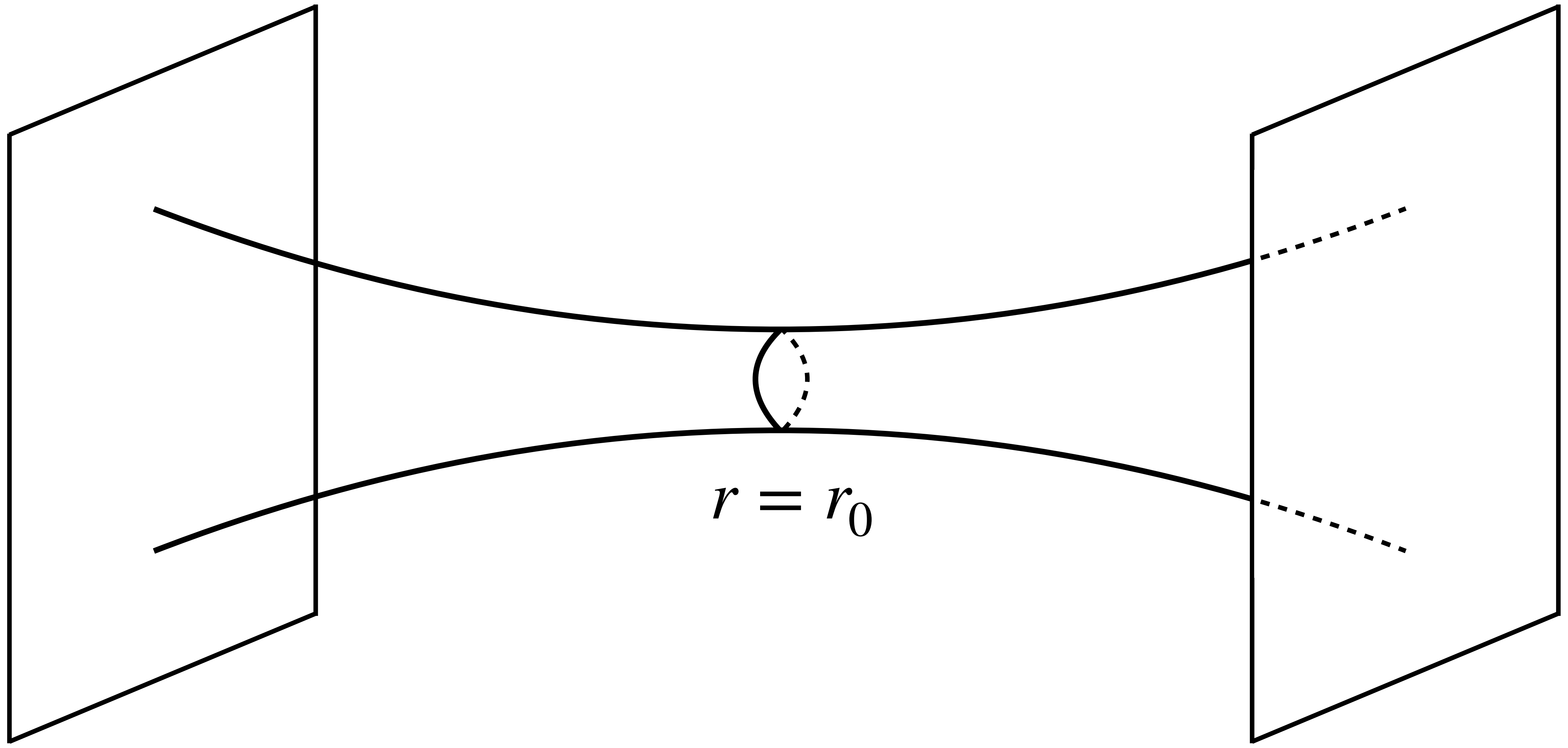




Axionic WGC and Wormholes

- Without a clear notion of extremality for -1 form symmetries, wormholes have been used to set the WGC $f \cdot S_{inst} < \mathcal{O}(1) M_P$ [Andriolo, Huang, Noumi, Ooguri, GS ’20]; [Andriolo, GS, Soler, Van Riet ’22].
- The Giddings-Strominger wormhole is a solution to the Euclidean eoms for axion gravity:

$$\mathrm{d}s^2 = \left(1 - \frac{r_0^4}{r^4}\right)^{-1} \mathrm{d}r^2 + r^2\, \mathrm{d}\Omega_3^2 \qquad H = \frac{n}{2\pi^2}\mathrm{vol}_3 \qquad 24\pi^4 r_0^4 = n^2$$



Two questions:

1) wormhole action decreases with charge?

2) wormholes perturbatively stable ?

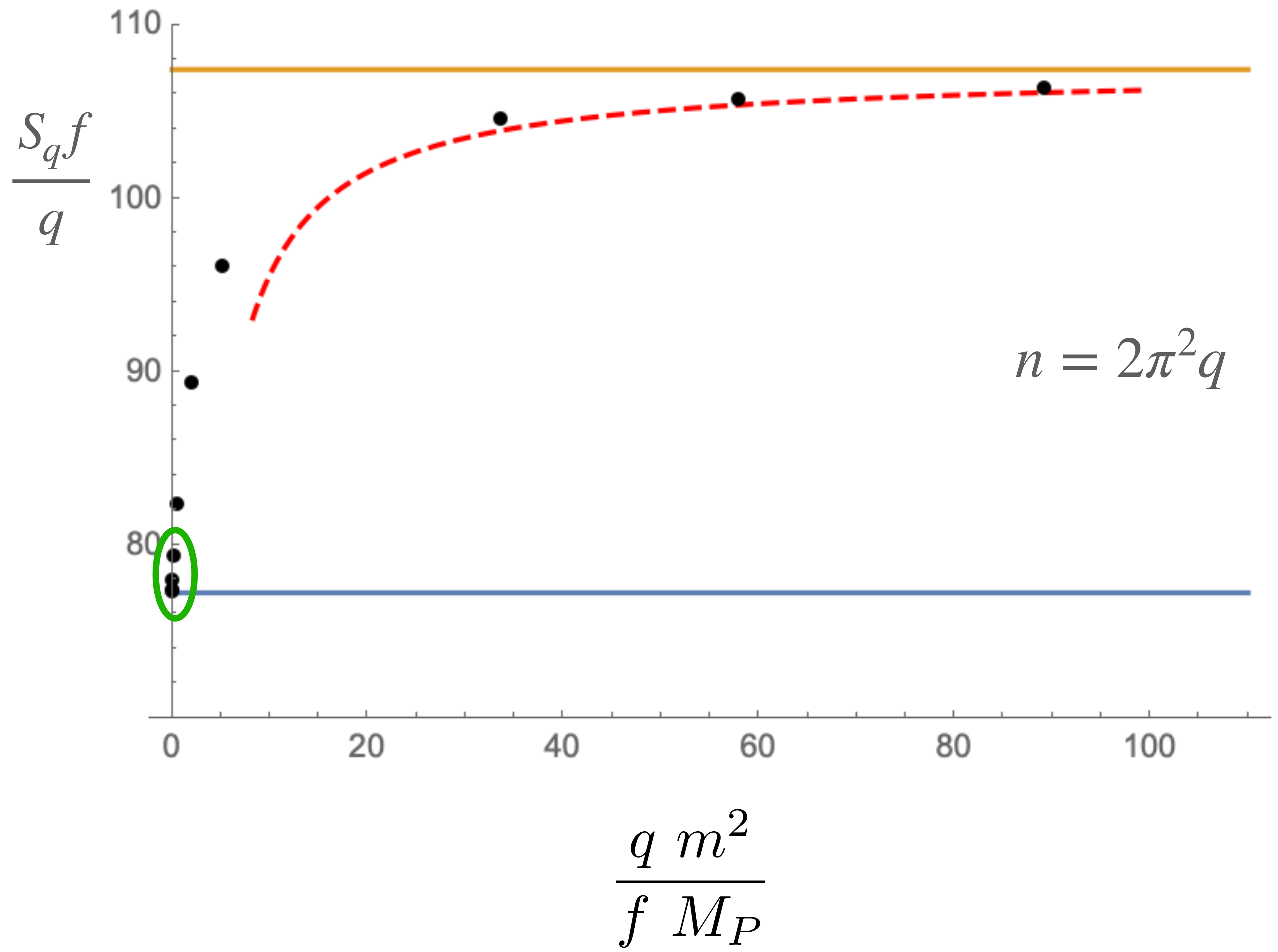
Evidence for Axionic WGC

- The WGC is set by the action-to-charge ratio of a macroscopic semi-wormhole (considering axion-gravity and axion-dilaton-gravity) [Andriolo, Huang, Noumi, Ooguri, GS '20]; [Andriolo, GS, Soler, Van Riet '22].

$$\frac{S_{instanton}}{n} \lesssim \frac{S_{wormhole}}{n} = \frac{\sqrt{6\pi}}{4} \cdot \frac{M_P}{f}$$

axion gravity

- Action-to-charge ratio was shown to decrease with charge by considering leading irrelevant operators with signs fixed by unitarity/causality [Andriolo, Huang, Noumi, Ooguri, GS '20] and further by numerically solving wormhole solutions with general dilaton mass [Andriolo, GS, Soler, Van Riet '22]



Wormhole Stability

- Previous works (25+ years) on perturbative stability of axion wormholes have led to contradictory claims, casting doubts on their contributions to the Euclidean path integral.

	Frame	Stable	Gauge-inv	j=0,1	B.C.
Rubakov, Shvedov, '96	axion	No	No	physical	✗
Alonso, Urbano, '17	axion	Yes	Yes	physical	✗
Hertog, Truijen, Van Riet, '18	axion	No	Yes	pure gauge	✗
Loges, GS, Sudhir, '22	3-form	Yes	Yes	pure gauge	✓

Boundary Conditions and Gauge Invariance

- Under diffeomorphism, metric and axion/3-form perturbations are mixed. Physically meaningful conclusions can only be drawn on gauge-invariant perturbations.

- In analyzing scalar perturbations around the GS wormhole, the boundary conditions in the 3-form picture can be imposed more straightforwardly. Finite energy perturbations:

$$\int \delta H \wedge \star \delta H < \infty \,,$$

  which corresponds to:

$$\int \mathrm{d}\delta\theta \wedge \star \mathrm{d}\delta\theta < \infty \,,$$

- Metric perturbations vanish at the boundaries. Gauge invariant perturbations are Dirichlet in the H_3 picture, while in the θ picture, gauge invariant perturbations involve mixed b.c.

Wormhole Stability

[Loges, GS, Sudhir, ’22]

- We determine the stability of GS wormhole by carrying out the following steps:
  - Parametrization of scalar perturbations and their boundary conditions.
  - Diffeomorphisms and physical degrees of freedom.
  - Quadratic action.
  - Integrate out non-dynamical and unphysical, gauge-dependent modes.
  - Analyze spectrum of remaining physical modes.

Steps akin to analyzing gauge invariant perturbations in inflationary cosmology.

But as we shall show, not only is the spectrum of perturbations

but **on-shell value of the quadratic action** is important for determining stability.

Scalar Perturbations

$$\mathrm{d}s^2 = a(\eta)^2\Big[-(1+2\phi)\,\mathrm{d}\eta^2 + 2\partial_i B\,\mathrm{d}\eta\mathrm{d}x^i + \big((1-2\psi)\gamma_{ij} + 2\nabla_i\partial_j E\big)\,\mathrm{d}x^i\mathrm{d}x^j\Big]$$

$$H = \frac{n}{2\pi^2}\Big[(1+s)\mathrm{vol}_3 + \mathrm{d}\eta\wedge\big(\tfrac{1}{2}\sqrt{\gamma}\epsilon_{ijk}\partial^i w\,\mathrm{d}x^j\wedge\mathrm{d}x^k\big)\Big]$$

- 6 scalar perturbations: ϕ, ψ, E, B, s, w.
- **Dirichlet boundary conditions:** perturbations must go to zero.

Diffeomorphisms

- Some of these perturbations are unphysical and only represent the freedom to perform diffeomorphism.
- Under a diffeomorphism generated by $\xi = \zeta^0 \partial_0 + \gamma^{ij}(\partial_i \zeta)\partial_j$ parametrized by two scalar functions ζ^0, ζ, the perturbations transform:

$$\delta_\xi \phi = \dot{\zeta}^0 + \mathcal{H}\zeta^0 \qquad \delta_\xi B = -\zeta^0 + \dot{\zeta} \qquad \delta_\xi s = \Delta\zeta$$

$$\delta_\xi \psi = -\mathcal{H}\zeta^0 \qquad \delta_\xi E = \zeta \qquad \delta_\xi w = \dot{\zeta}$$

- **Only one physical scalar mode**. Convenient to pick:

$$\mathcal{S} = s - \Delta E \qquad \delta_\xi \mathcal{S} = 0$$

Quadratic Action for Physical Perturbations

- Integrating out non-dynamical and unphysical, gauge-dependent modes, we found that $j = 0,1$ modes are pure gauge while for $j \geq 2$, there is one physical dof $(\mathcal{S}_j = s_j + \lambda_j E_j)$

$$S_2 = \int \mathrm{d}\eta \sum_{j\geq 2} \frac{3a^2}{\lambda_j\left(\frac{9}{\lambda_j-3}+\frac{1}{1+\mathcal{H}^2}\right)}\left[\dot{S}_j^2 + \frac{6\lambda_j\left(1+\mathcal{H}^2\right)}{(\lambda_j-3)\mathcal{H}} S_j\dot{S}_j - \frac{\lambda_j}{\mathcal{H}^2}\left(\frac{\lambda_j-9}{\lambda_j-3}(1+\mathcal{H}^2)-1\right)S_j^2\right],$$

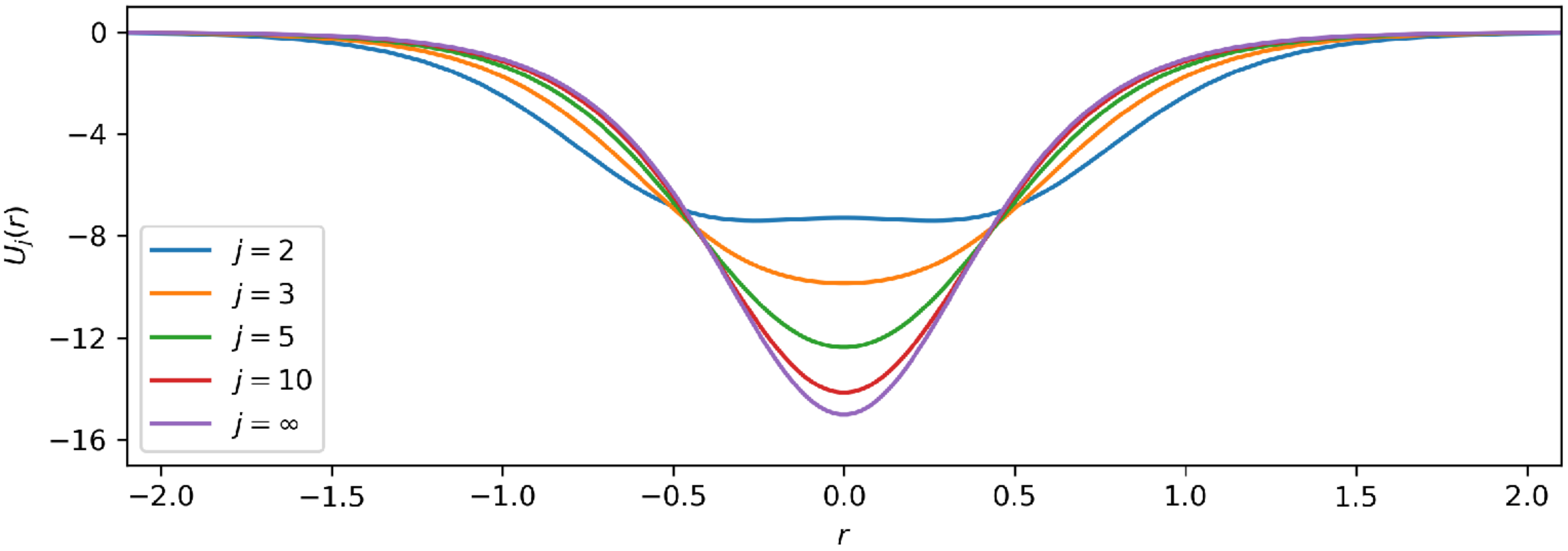
- Wick rotate $\eta \to -ir$ to GS wormhole and canonical normalize $\mathcal{Q}_j = (\cdots)\mathcal{S}_j$:

$$S_2^{\mathrm{E}} = \int \mathrm{d}r \sum_{j\geq 2}\left(\frac{1}{2}(\mathcal{Q}_j')^2 + \frac{1}{2}\underbrace{\left(U_j^{\mathrm{E}}+\lambda_j+1\right)}_{>0}\mathcal{Q}_j^2 + G_j^{\mathrm{E}}\right)$$

- S_2^E looks positive definite, but we will have to check the boundary term G_j^E.

Eigenvalue Problem

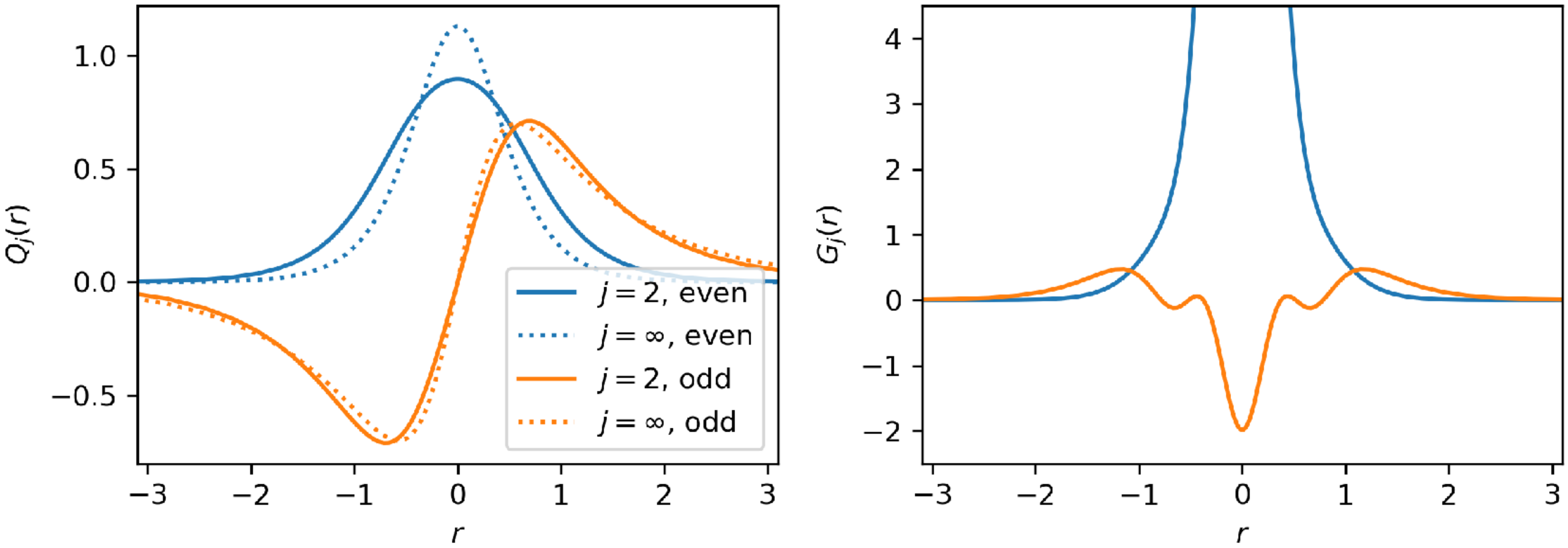
- Schrodinger-like problem: $$\mathcal{Q}_j^{(k)\prime\prime} + U_j^{\mathrm{E}}(r)\,\mathcal{Q}_j^{(k)} = \omega_j^{(k)}\,\mathcal{Q}_j^{(k)}$$



- $\mathcal{S} \to 0 \implies \mathcal{Q}_j^{(k)}$ must go to zero ***faster*** than $e^{-|r|}$. Look for bound states with $\omega_j^{(k)} < -1$.

Eigenfunctions

- There is exactly one even and one odd bound state for each $j \geq 2$:



- Total derivative term G_j^E is not integrable for the even eigenfunctions: $S_2[\mathcal{Q}^{(\mathrm{even})}] \to +\infty$

Spectrum and Stability

j	$\omega_j^{(\mathrm{odd})}$	$\omega_j^{(\mathrm{odd})} + \lambda_j + 1$	j	$\omega_j^{(\mathrm{odd})}$	$\omega_j^{(\mathrm{odd})} + \lambda_j + 1$
2	-1.5335	7.4665	6	-1.0921	47.9079
3	-1.2873	14.7127	7	-1.0705	62.9295
4	-1.1817	23.8183	8	-1.0556	79.9444
5	-1.1256	34.8744	9	-1.0450	98.9550

$$\mathcal{Q} = \sum_{j\geq 2} c_j \mathcal{Q}_j^{(\mathrm{odd})}(r) Y_j(\Omega) \quad \Longrightarrow \quad S_2 = \sum_{j\geq 2} \frac{1}{2}\left(\omega_j^{(\mathrm{odd})} + \lambda_j + 1\right) c_j^2 > 0$$

The Euclidean action only ever **increases** under scalar perturbations:

the GS wormhole is perturbatively stable.

Summary

- The S-matrix bootstrap program and the Swampland program both aim to make precise the boundaries between consistent and inconsistent theories.
- The Swampland program provides some clear targets for positivity bounds.
- Sharpening the gravitational positivity bounds is important for proving swampland constraints.
- No spinning WGC because of superradiance, but dualities mapping rotation to charges $\Rightarrow$ charged superradiance $\Rightarrow$ WGC.
- WGC on charged BHs $\Rightarrow \lambda_{GB} \geq 0, \eta_{R^3} \leq 0 \Rightarrow$ correction of extremality bound of MP/Kerr BH.
- Axionic WGC which constrains axion inflation is a statement about wormhole fragmentation.
- Swampland constraints (if established) can be used in combination with duality to obtain new positivity bounds which are otherwise difficult to prove directly with amplitude techniques.