An efficient reduced basis method for the stochastic Darcy flow model

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Goal: **Efficient** numerical methods for **PDEs with uncertain data**.

In **groundwater flow** modelling, the permeability coefficient is often **uncertain**: model the coefficient as \( a_M^{-1}(x, y) \).

Given \( y \in \Gamma \), find \( p(\cdot, y) : D \to \mathbb{R} \) and \( \vec{u}(\cdot, y) : D \to \mathbb{R}^2 \) such that

\[
\begin{align*}
    a_M^{-1}(x, y)\vec{u}(x, y) + \nabla p(x, y) &= 0, \quad x \in D, \\
    \nabla \cdot \vec{u}(x, y) &= f(x), \quad x \in \partial D, \\
    p(x, y) &= g(x), \quad x \in \partial D_D, \\
    \vec{u}(x, y) \cdot \vec{n} &= 0, \quad x \in \partial D_N.
\end{align*}
\]

Approximations to \( p(\cdot, y) \) and \( \vec{u}(\cdot, y) \) for each \( y \in \Gamma \) can be obtained using **mixed finite element methods**, however, this can be expensive.

Using reduced basis methods we can approximate \( p(\cdot, y) \) and \( \vec{u}(\cdot, y) \) for any \( y \in \Gamma \) at a significantly **cheaper** cost.
We develop an efficient reduced basis method that we combine with a sparse grid stochastic collocation method.

This allows us to cheaply perform forward UQ.

We demonstrate significant computational savings over standard finite element methods.

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