An efficient reduced basis method for the stochastic Darcy flow model

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Goal: **Efficient** numerical methods for **PDEs with uncertain data**.

In **groundwater flow** modelling, the permeability coefficient is often **uncertain**: model the coefficient as $a_M^{-1}(\mathbf{x}, \mathbf{y})$.

Given $\mathbf{y} \in \Gamma$, find $p(\cdot, \mathbf{y}) : \mathcal{D} \to \mathbb{R}$ and $\vec{u}(\cdot, \mathbf{y}) : \mathcal{D} \to \mathbb{R}^2$ such that

$$\begin{aligned} a_M^{-1}(\mathbf{x}, \mathbf{y})\vec{u}(\mathbf{x}, \mathbf{y}) + \nabla p(\mathbf{x}, \mathbf{y}) &= 0, && \mathbf{x} \in \mathcal{D}, \\ \nabla \cdot \vec{u}(\mathbf{x}, \mathbf{y}) &= f(\mathbf{x}), && \mathbf{x} \in \partial \mathcal{D}, \\ p(\mathbf{x}, \mathbf{y}) &= g(\mathbf{x}), && \mathbf{x} \in \partial \mathcal{D}_{\mathrm{D}}, \\ \vec{u}(\mathbf{x}, \mathbf{y}) \cdot \vec{n} &= 0, && \mathbf{x} \in \partial \mathcal{D}_{\mathrm{N}}. \end{aligned}$$

Approximations to $p(\cdot, \mathbf{y})$ and $\vec{u}(\cdot, \mathbf{y})$ for each $\mathbf{y} \in \Gamma$ can be obtained using **mixed finite element methods**, however, this can be expensive.

Using reduced basis methods we can approximate $p(\cdot, \mathbf{y})$ and $\vec{u}(\cdot, \mathbf{y})$ for any $\mathbf{y} \in \Gamma$ at a significantly cheaper cost.

We develop an **efficient** reduced basis method that we combine with a sparse grid stochastic collocation method.

This allows us to cheaply perform forward UQ.

We demonstrate **significant** computational savings over standard finite element methods.

Please come and visit our poster!

See also our **preprint:**

Craig J. Newsum and **Catherine E. Powell**, "Efficient reduced basis methods for saddle point problems with applications in groundwater flow." (2016). **MIMS EPrint: 2016.60**