Reducing computational complexity of sparse grid stochastic collocation methods

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Joint work with **C. Webster, A. Teckentrup, M. Gunzburger, D. Galindo, G. Zhang**

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Improving Collocation Methods by Exploiting Structure

Uncertain input parameters:

$\boldsymbol{y} \in \Gamma \subset \mathbb{R}^d$

$\longrightarrow$

PDE model:

$\mathcal{P}(u, \boldsymbol{y}) = 0$

a.e. in $D \subset \mathbb{R}^n$

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Quantity of interest:

$Q[u(\cdot, \boldsymbol{y})]$

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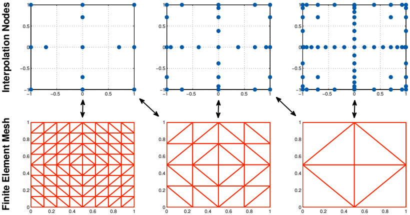
Method 1: Exploit the hierarchy in deterministic approximation. Multilevel methods reduce complexity by distributing computational costs among high and low fidelity approximations.



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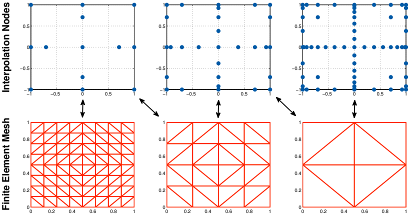
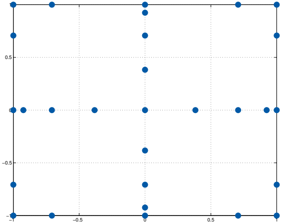


Key points:

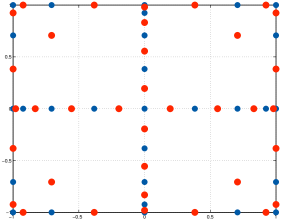
- Provably reduce the complexity of constructing collocation approximations by exploiting basic structure.
- Work practically even when we can't choose a sparse grid with the "optimal" number of points.

Improving Collocation Methods by Exploiting Structure

Method 2: Exploit the hierarchy in the polynomial approximation. Sparse grids with nested grid points provide a natural multilevel hierarchy which we can use to accelerate each PDE solve.



Solve $A_j \boldsymbol{c}_j = \boldsymbol{f}_j$
at all blue points
$\longrightarrow$
Interpolate to
improve
convergence



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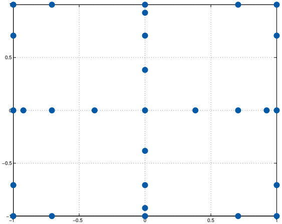

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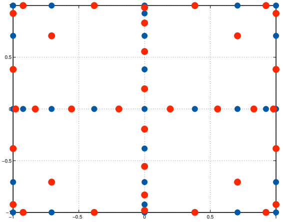

Key points:

- Acceleration works with preconditioning and initial solutions to speed up iterative solvers.
- Especially effective for non-linear iterative solvers
- Improves efficiency of iterative solvers even with the additional cost of interpolation.