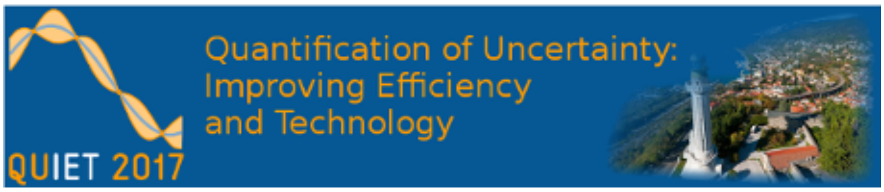

Hessian-based sampling for goal-oriented model reduction

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Model reduction techniques for parametric partial differential equations have been well developed to reduce the computational cost in many-query or real-time applications, such as optimal design/control, parameter calibration, and uncertainty quantification. However, it remains a great challenge to construct an efficient and accurate reduced order model (ROM) for high-dimensional parametric problems. One reason is that sampling in the high-dimensional parameter space for the construction of the ROM often faces curse of dimensionality, i.e., the computational complexity grows exponentially with respect to the number of parameter dimensions. The other is that the parametric solution manifold may be essentially high-dimensional such that a very large number of reduced basis functions have to be used in order to achieve certain required accuracy, which limits the efficacy of the computational reduction.

In this talk, we present a Hessian-based sampling method for goal-oriented model reduction to effectively construct a ROM that has good approximation property for some given quantity of interest (QoI) as a function of the parametric solution [1]. The rationale is that even the dimension of the solution manifold is high, the dimension of the quantity of interest, such as an average of the solution in a particular physical domain, is relatively low. To capture this low-dimensionality, we explore the curvature of the QoI in the parameter space informed by its Hessian [2, 3]. More specifically, take a (infinite-dimensional) parameter field m with Gaussian measure $N(m_0, C)$ for example, where m_0 is the mean and C is the covariance. A QoI Q that depends (implicitly through the solution) on the parameter m can be approximated by a truncated Taylor expansion up to the quadratic term as

$$Q_{\text{quad}}(m) = Q(m_0) + \langle Q_m, m - m_0 \rangle + \frac{1}{2}\langle Q_{mm}(m - m_0), m - m_0 \rangle.$$

Then the expectation of Q can be approximated by

$$\mathbb{E}[Q_{\text{quad}}] = Q(m_0) + \frac{1}{2}\text{tr}(H),$$

where $\text{tr}(\cdot)$ is a trace operator, and $H = C^{1/2} Q_{mm} C^{1/2}$ is the (covariance-preconditioned) Hessian. Thus, the variation of Q_{quad} is captured by the trace of the Hessian and the most sensitive directions of the parameter for the QoI are the eigen-directions of the Hessian corresponding to its leading eigenvalues. Therefore, we project the parameter m to the subspace spanned by the eigen-directions of the Hessian and sampling from this subspace for the construction of the ROM. We demonstrate by several numerical experiments that this Hessian-based sampling gives much smaller ROM approximation error for the QoI than that by a random sampling method.

References

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[3] P. Chen, U. Villa, and O. Ghattas. Taylor approximation and variance reduction for PDE-constrained optimal control under uncertainty. *preprint*, 2016.