Spectral stochastic finite elements for two nonlinear problems

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We study applications of spectral stochastic finite element methods (SSFEM) in two nonlinear problems: inverse subspace iteration for eigenvalue computations and Navier-Stokes equations.

In the first part we focus on random eigenvalue problems in the context of spectral stochastic finite elements [1]. In particular, given a parameter-dependent, symmetric positive-definite matrix operator, we explore the performance of algorithms for computing its eigenvalues and eigenvectors represented using polynomial chaos expansions. We formulate a version of stochastic inverse subspace iteration, which is based on the stochastic Galerkin finite element method, and we compare its accuracy with that of Monte Carlo and stochastic collocation methods. The coefficients of the eigenvalue expansions are computed from a stochastic Rayleigh quotient. Our approach allows the computation of interior eigenvalues by deflation methods, and we can also compute the coefficients of multiple eigenvectors using a stochastic variant of the modified Gram-Schmidt process. The effectiveness of the methods is illustrated by numerical experiments on benchmark problems arising from vibration analysis.

In the second part we study the steady-state Navier-Stokes equations in the context of stochastic finite element discretizations [2]. Specifically, we assume that the viscosity is a random field given in the form of a generalized polynomial chaos expansion. For the resulting stochastic problem, we formulate the model and linearization schemes using Picard and Newton iterations in the framework of the stochastic Galerkin method, and we explore properties of the resulting stochastic solutions. We also propose a preconditioner for solving the linear systems of equations arising at each step of the stochastic (Galerkin) nonlinear iteration and demonstrate its effectiveness for solving a set of benchmark problems.

References

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