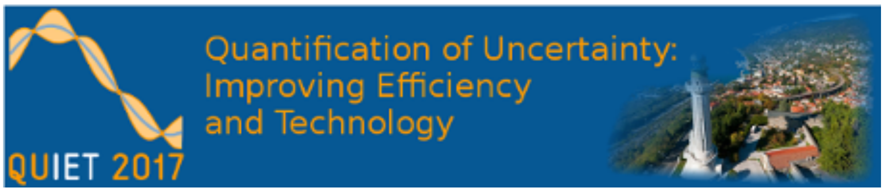

Multi-Index polynomial chaos methods for random PDEs

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In this talk we consider the problem of computing statistics of the solution of a partial differential equation with random data, where the random coefficient is parametrized by means of a finite or countable sequence of terms in a suitable expansion. We focus in particular on polynomial chaos type approximations with respect to the random parameters combined with hierarchical discretizations in the physical space.

When the polynomial chaos approximation is computed by Stochastic Collocation, this gives rise to a Multi-Level or more generally a Multi-Index Stochastic Collocation (MISC) method [2, 1, 4]. MISC is a combination technique based on mixed differences of spatial approximations and quadratures over the space of random data. Provided enough mixed regularity is available, MISC can achieve better complexity than a single level Stochastic Collocation method. Moreover, we show that in the optimal case the convergence rate of MISC is only dictated by the convergence of the deterministic solver applied to a one-dimensional spatial problem. We propose optimization procedures to select the most effective mixed differences to include in MISC. Such optimization is a crucial step that allows us to make MISC computationally effective.

Alternative to Stochastic Collocation methods, we present also Multi-level / Multi-index versions of discrete (weighted) least squares approximations on polynomial spaces, based on evaluations in random points and with different accuracy levels [3]. In particular, we show rigorous results on the minimum number of evaluations to acquire on each level to have a stable approximation and optimal complexity.

We show the effectiveness of multi-level / multi-index Stochastic Collocation and least squares methods on some computational tests, including tests with a infinite countable number of random parameters.

References

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