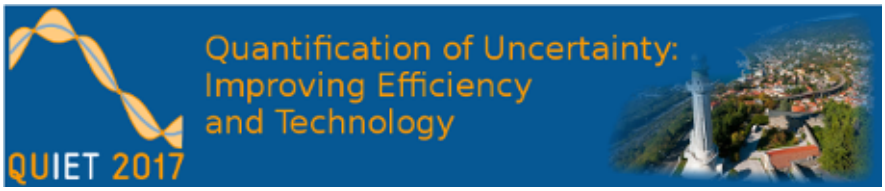

Conditional Expectation as the Basis of Bayesian Updating

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Introducing new information into a probabilistic description of knowledge is typically performed via some kind of application of Bayes's by now classical theorem. To avoid ambiguities (which did arise historically), the mathematically precise description of conditional probabilities in Bayes's theorem, especially when conditioning on events of vanishing probability, is formulated via conditional expectations, and is due to Kolmogorov. Nevertheless, most sampling approaches to Bayesian updating typically start from the classical formulation involving conditional measures and densities. These are usually the distributions of some random variables describing the prior knowledge. Here an alternative track is taken, in that the notion of conditional expectation is also taken computationally as the prime object. Being able to numerically approximate conditional expectations, one has a complete description of the posterior probability. A further task is to construct a new – transformed, or filtered – random variable which has a distribution as required by the (posterior) conditional expectations. In the talk, the abstract task and its solution will be presented first, and then different computational approximations will be sketched, as well as different ways of stochastic discretisations, adding another level of approximation. It is also possible – although not necessary – to formulate these concepts in a more algebraic/functional analytic setting. Here the fundamental notions are algebras of random variables and a distinguished linear functional called expectations. These connections will be shortly sketched, and show a possible joint theoretical and computational basis.