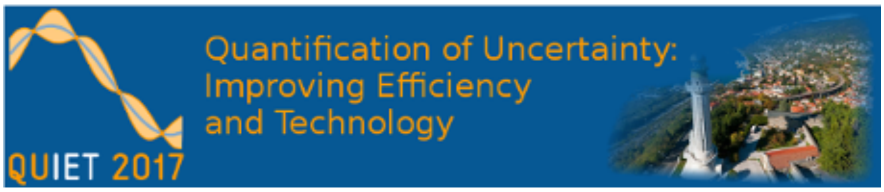

Recent advances in compressed sensing techniques for the numerical approximation of PDEs

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Compressed Sensing (CS) is a signal processing technique that allows to acquire a signal using far few measurement than those prescribed by the co-salled Nyquist-Shannon barrier. In particular, we can recover the best s-sparse approximation to an N-dimensional signal, where $s \ll N$, by performing $m \sim s \cdot \text{polylog}(N)$ linear randomized measurements. This approximation is recovered by means of computationally efficient strategies such as ℓ^1-minimization or greedy algorithms.

The aim of this talk is to present the main ideas that recently led to the application of CS to numerical methods for deterministic PDEs and to the uncertainty quantification of parametric PDEs with random inputs.

One the one hand, CS can be employed as a *dimension reduction* technique for the class of Petrov-Galerkin discretizations of PDEs in weak form. We will discuss the so-called CORSING technique, where the dimensionality of the stiffness matrix and of the load vector are reduced by exploiting the sparsity of the unknown solution with respect to a suitable basis of trial functions (e.g., wavelets or Fourier-like bases) [4, 5, 3].

On the other hand, in the case of the uncertainty quantification of high-dimensional parametric PDEs with random inputs, CS has been recently proved to be a useful tool for the construction of nonintrusive, highly parallelizable schemes that are able to alleviate the *curse of dimensionality*. These approaches are able to recover the best s-sparse approximation to a quantity of interest of the solution map with respect to a suitable universal sparsity basis (e.g., tensorized orthogonal polynomials) by means of a few random pointwise samples in the parametric space. In particular, we will present some recent results that show the robustness of this approach when the samples are subject to unknown error [2, 1].

In both cases, we will illustrate the benefits and the limits brought by CS from a numerical analyst's perspective and present some open challenges.

References

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