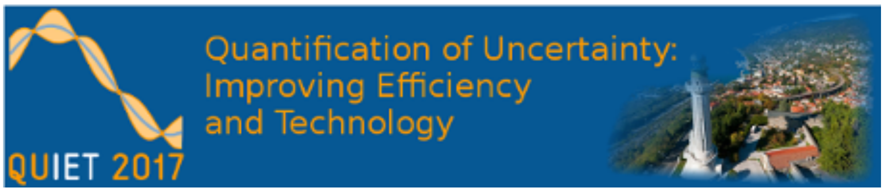

Sparse approximation of high-dimensional functions via convex and nonconvex regularizations

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In this talk, we present and analyze a novel compressed sensing approach for optimal polynomial recovery of both high-dimensional complex-valued and Hilbert-valued signals. The latter typically comes from the solution of parameter PDEs, where the target function is smooth, characterized by a rapidly decaying orthonormal expansion, whose most important terms are captured by a lower (or downward closed) set. By exploiting this fact, we develop a novel weighted minimization procedure with a precise choice of weights, and a modification of the iterative hard thresholding method, for imposing the downward closed preference. We will also present theoretical results that reveal our new computational approaches possess a provably reduced sample complexity compared to existing compressed sensing, least squares, and interpolation techniques. In addition, the recovery of the corresponding best approximation using our methods is established through an improved bound for the restricted isometry property. Finally, we will present an entirely new theory for compressed sensing that reveals that nonconvex minimizations are at least as good as ℓ_1 minimization in exact recovery of sparse signals. Our theoretical recovery guarantees are developed through a unified null space property based-condition that encompasses all currently proposed nonconvex functionals in literature. Several nonconvex functionals will be explored and the specific conditions in order to guarantee improved recovery will be given. Numerical examples, related to polynomial approximation of several functions in high dimensions, will be provided to support the new theory and demonstrate the computational efficiency of both weighted ℓ_1 and nonconvex regularizations.