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Sparse grid approximation of elliptic PDEs with lognormal diffusion coefficient

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This talk is concerned with sparse grid methodologies to efficiently approximate the solution, u, of an elliptic PDE whose diffusion coefficient is modeled as a lognormal random field. The presentation is divided in two parts.

In the first part, we build upon previous works available in the literature to establish a convergence result (in L^2 norm in probability) for the approximation of u by sparse collocation with Gauss–Hermite points, see [1]. More specifically, we first link the error to the size of the multi-index set defining the sparse collocation and then derive a bound on the number of points in the associated sparse grid. The result of the analysis is an algebraic convergence rate of the approximation error with respect to both the size of the multi-index set and the number of points in the sparse grid; interestingly, the analysis gives also an explicit "a-priori" estimate of the optimal multi-index set. We validate the results against numerical tests: in particular, we consider a family of random fields parameterized by a coefficient that sets the spatial smoothness of the field, in the spirit of the Matérn family.

As expected, the convergence rate for very rough fields turns out to be quite slow, even for optimized grids (be it the above-mentioned "a-priori" grids or the classical "a-posteriori"-adaptive grids). Thus, in the second part of the talk, we propose a remedy based on using the solution of the PDE on a smoothed versions of the random field as control variate for a Monte Carlo sampling of u, see [2].

References

[1] O. G. Ernst, B. Sprungk, and L. Tamellini. Convergence of sparse collocation for functions of countably many gaussian random variables (with application to lognormal elliptic diffusion problems). *ArXiv e-prints 1611.07239*, 2016.

[2] F. Nobile, L. Tamellini, F. Tesei, and R. Tempone. *An Adaptive Sparse Grid Algorithm for Elliptic PDEs with Lognormal Diffusion Coefficient*, volume 109 of *Lecture Notes in Computational Science and Engineering*, pages 191–220. Springer International Publishing Switzerland, 2016.