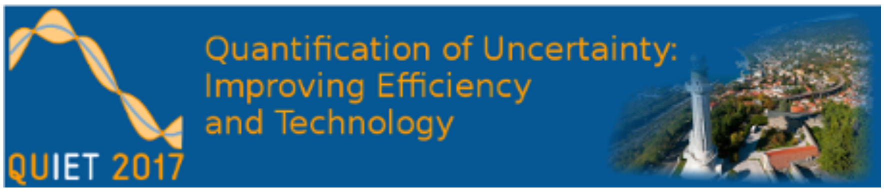

Dictionary measurement selection for state estimation with reduced basis

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Parametric PDEs of the general form

$$\mathcal{P}(u,a) = 0$$

are commonly used to describe many physical processes, where $\mathcal{P}$ is a differential operator, a is a high-dimensional vector of parameters and u is the unknown solution belonging to some Hilbert space V. A typical scenario in state estimation is the following: for an unknown parameter a, one observes m independent linear measurements of $u(a)$ of the form $\ell_i(u) = (w_i, u)$, $i = 1, ..., m$, where $\ell_i \in V'$ and w_i are the Riesz representers, and we write $W_m = \text{span}\{w_1, ..., w_m\}$. The goal is to recover an approximation u^* of u from the measurements. Due to the dependence on a the solutions of the PDE lie in a manifold and the particular PDE structure often allows to derive good approximations of it by linear spaces V_n of moderate dimension n. In this setting, the observed measurements and V_n can be combined to produce an approximation u^* of u up to accuracy

$$\|u - u^*\| < \beta^{-1}(V_n, W_m) dist(u, V_n)$$

where

$$\beta(V_n, W_m) := \inf_{v \in V_n} \frac{\|P_{W_m} v\|}{\|v\|}$$

plays the role of a stability constant. For a given V_n, one relevant objective is to guarantee that $\beta(V_n, W_m) > \gamma > 0$ with a number of measurements $m > n$ as small as possible. We present results in this direction when the measurement functionals ℓ_i belong to a complete dictionary.