Statistically optimal weights for distributed Tikhonov-regularization

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Weighted L^2-Tikhonov regularization for linear source identification

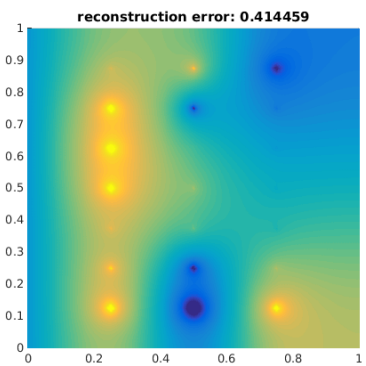
$$\hat{u}_{w,\alpha}(z) = \arg\min_{u} \frac{1}{2}\sum_{k=1}^{K}|y(x_k)-z_k|^2+\frac{\alpha}{2}\int_D w(\cdot)u^2\,\mathrm{d}x \quad \text{s. t.} \quad \begin{cases} -\Delta y = u & \text{in } D, \\ \partial_\nu y + \gamma\chi_{\Gamma_R} y = 0 & \text{on } \partial D \end{cases}$$

- "training data" u sampled from a random field $U \in L^2(\Omega, \mathrm{d}P, L^\infty(D))$
  - extends dictionary learning approaches (u sampled from a dictionary)
- selection of optimal weight by variance minimization

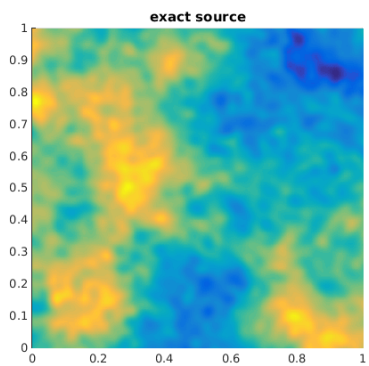
$$\min_{w\in W_{\mathrm{ad}}} \mathbb{E}\left[\|U - \hat{u}_{w,\alpha}(KU)\|^2_{L^2(D)}\right] = \|U - T(w)U\|^2_{L^2(\Omega,\mathrm{d}P,L^2(D))}$$

Stochastic optimization problem for the optimal weight

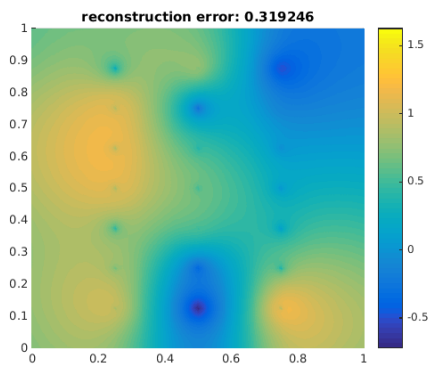
- analysis of the stochastic control problem
- discretization with KLE-expansion and numerical solution



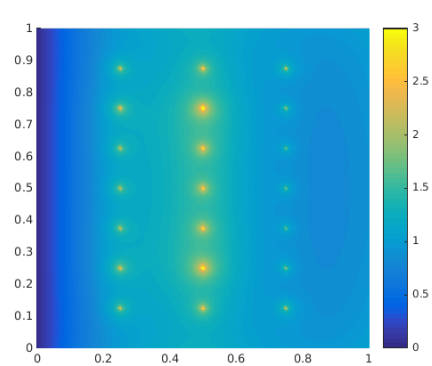
with $w \equiv 1$



exact source



with optimal $\bar{w}$



optimal weight $\hat{w}$