Technische Universität München

Lehrstuhl für
Numerische Mathematik

MLMC for transmission problems with geometric uncertainties

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Model transmission problem

$$\begin{cases} -\nabla\cdot(\alpha(\Gamma(\boldsymbol{y}))\nabla u) - \kappa^2(\Gamma(\boldsymbol{y}))u = 0 \quad \text{in } D_{in}(\boldsymbol{y}) \cup D_{out,R_{out}}(\boldsymbol{y}), \\ [\![u]\!]_{\Gamma(\boldsymbol{y})} = 0, \quad [\![\alpha(\Gamma(\boldsymbol{y}))\nabla u \cdot \boldsymbol{n}]\!]_{\Gamma(\boldsymbol{y})} = 0, \\ \dfrac{\partial}{\partial \boldsymbol{n}_{out}}(u - u_i) = \mathrm{DtN}(u) - \mathrm{DtN}(u_i) \quad \text{on } \partial D_{R_{out}}, \\ \text{for every } \boldsymbol{y} \in \mathcal{P}_J,\ J \in \mathbb{N}, \end{cases}$$

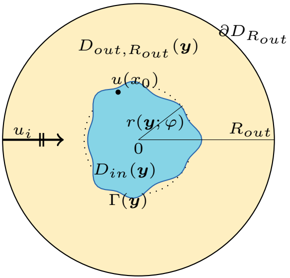
Quantity of interest:

$\boldsymbol{u} = \{u(\boldsymbol{x}_i)\}_{i=0}^{N-1}$,

$\boldsymbol{x}_i$ close to stochastic interface.

Goal:

Computing $\mathbb{E}[\boldsymbol{u}]$.



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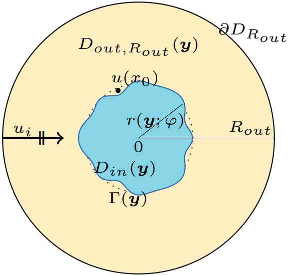

Non-smooth parameter dependence

1. $u(\boldsymbol{x}_0)$ not smooth across

$$\mathcal{P}_J^{\Gamma}(\boldsymbol{x}_0) := \{\boldsymbol{y} \in \mathcal{P}_J : \boldsymbol{x}_0 \in \Gamma(\boldsymbol{y})\}.$$

2. Discontinuities hard to locate.

$$\Rightarrow \quad \text{multilevel Monte Carlo.}$$

To select the sequence $(M_l)_{l=0}^{L-1}$:

- J-independent space regularity for point evaluation
- J-independent finite element convergence of point evaluation.