An Ensemble-Proper Orthogonal Decomposition Method for the Incompressible Navier Stokes Equation

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- We consider J Navier-Stokes equations on a bounded domain, each subject to the no-slip boundary condition, and driven by J different initial conditions $u^{j,0}(x)$, viscosities ν_j and body force densities $f^j(x,t)$, i.e., for $j = 1, \dots, J$, we have:

$$\begin{cases} u_t^j + u^j \cdot \nabla u^j - \nu_j \triangle u^j + \nabla p^j = f^j(x,t) & \forall x \in \Omega \times (0,T] \\ \nabla \cdot u^j = 0 & \forall x \in \Omega \times (0,T] \\ u^j = 0 & \forall x \in \partial\Omega \times (0,T] \\ u^j(x,0) = u^{j,0}(x) & \forall x \in \Omega, \end{cases}$$

- We would like to be able to solve our model for a number of different parameters J.
- After discretizing our model this would be the equivalent of at each time step solving:

$$A_j x_j = b_j \quad j = 1, \ldots J$$

- Very expensive, especially when the size of A is large.

- In recent work a scheme was devised so at each time step instead one has to solve:

$$Ax_j = b_j \quad j = 1, \ldots J$$

- Much cheaper (A is now independent of j) to solve using methods such as block CG.
- Our goal is to make this even cheaper by reducing the size of the system A we need to solve via the incorporation of a reduced basis where the new system is much smaller than the original ensemble system i.e.

$$A_R x_j = b_j \quad j = 1, \ldots J$$

- We also incorporate a differential filter into the model to improve performance for high reynolds number flows.