Stabilization techniques for pressure recovery applied to POD-Galerkin methods for the incompressible Navier-Stokes equations

G. Stabile, G. Rozza

SISSA, International School for Advanced Studies,
MathLab, Mathematics Area, Trieste, Italy

Quantification of Uncertainty
Improving Efficiency and Technology
QUIET 2017 – Trieste
18-21/07/2017

Framework and motivations

- In order to efficiently apply **Uncertainty Quantification** in computational fluid dynamics problems one needs inexpensive computational models to solve the **forward problem**. In this direction the development of efficient and reliable reduced order models (ROMs) would be a great advantage.
- It is well known that Galerkin based ROMs of the incompressible Navier-Stokes equations suffer from **stability issues** for what concern the **pressure term**.

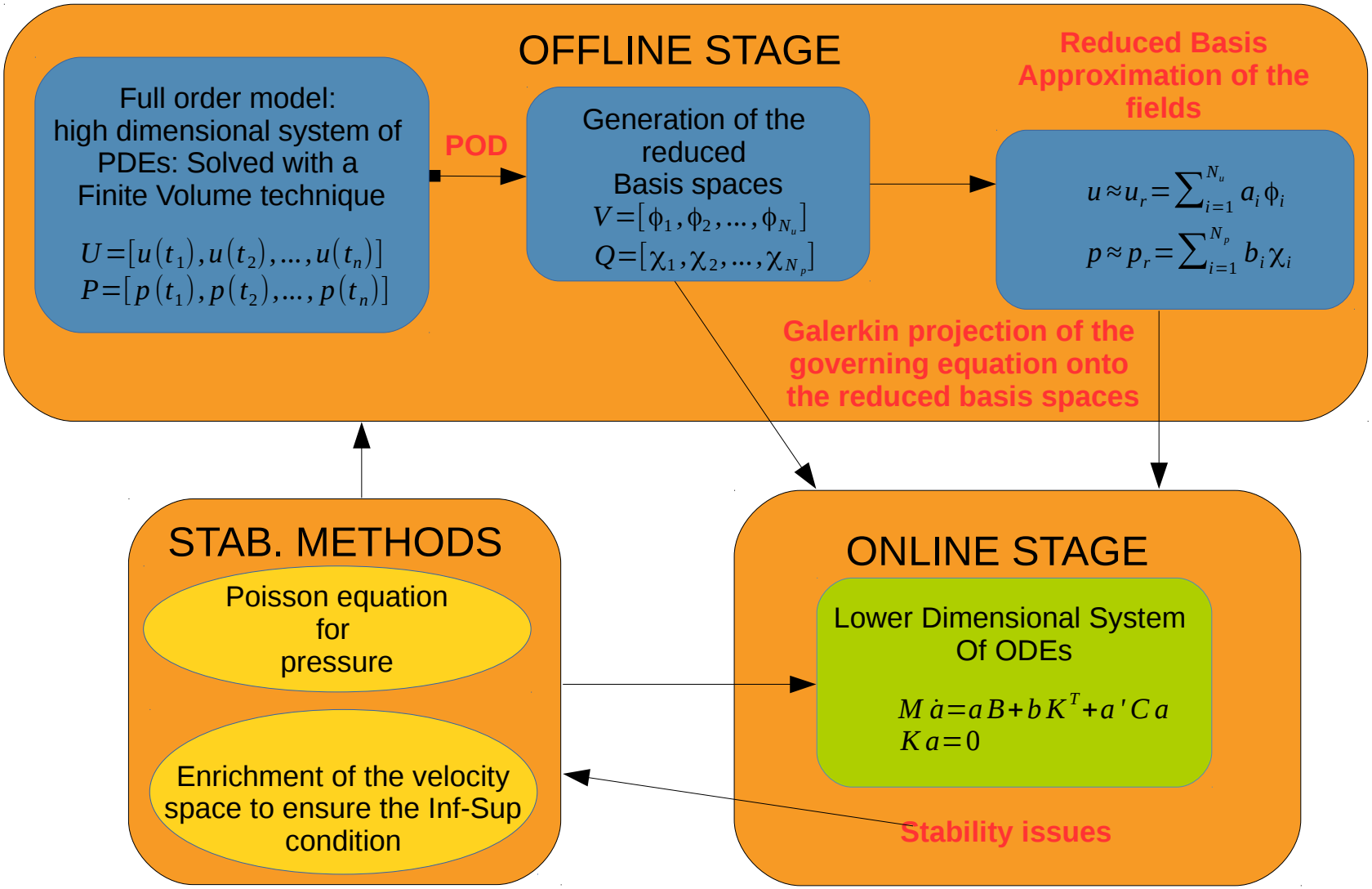
The considered system of PDEs consists in the **unsteady parametrized incompressible Navier Stokes Equations.**

$$\begin{cases} \frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \boldsymbol{\nabla})\boldsymbol{u} - \boldsymbol{\nabla} \cdot \nu \boldsymbol{\nabla} \boldsymbol{u} = -\boldsymbol{\nabla} p & \text{in } \Omega \\ \boldsymbol{\nabla} \cdot \boldsymbol{u} = \boldsymbol{0} & \text{in } \Omega \\ \boldsymbol{u} = \overline{\boldsymbol{u}}(\mu) & \text{on } \partial\Omega_{,in} \\ \boldsymbol{u} = \boldsymbol{0} & \text{on } \partial\Omega_{,0} \\ (\mu \nabla \boldsymbol{u} - p\boldsymbol{I})\boldsymbol{n} = 0 & \text{on } \partial\Omega_{,out} \end{cases} \tag{1}$$

The offline stage is performed using a **Finite Volume Method** (OpenFOAM) while the projection and online stage are based on the in-house package **ITHACA-FV**.

Reduced Order Modelling

Most of the problems require high dimensional parametrized simulations.



References

[1] G. Stabile, S. Hijazi, A. Mola, S. Lorenzi, and G. Rozza. POD-Galerkin reduced order methods for CFD using Finite Volume Discretisation: vortex shedding around a circular cylinder. In Press, 2017

[2] G. Stabile and G. Rozza, Stabilized Reduced order POD-Galerkin techniques for finite volume approximation of the parametrized Navier–Stokes equations, submitted, 2017.

[3] F. Ballarin, A. Manzoni, A. Quarteroni, and G. Rozza, Supremizer stabilization of POD-Galerkin approximation of parametrized steady incompressible Navier–Stokes equations, International Journal for Numerical Methods in Engineering, vol. 102, no. 5, pp. 1136–1161, 2015.