

Reduced Order Methods for Environmental Marine Problems by Optimal Flow Control



**M. Strazzullo, F. Ballarin,
R. Mosetti and G. Rozza**

MathLab, Mathematics Area, SISSA International
School for Advanced Studies, Trieste, Italy
OGS, National Institute of Oceanography and
Experimental Geophysics, Trieste, Italy

QUIET 2017, Trieste, July 19, 2017

Motivations

- OFCP(μ) are a useful mathematical model since they are suited for **data assimilation**, **inverse problems**, as well as **uncertainty quantification** and **parameter estimation** problems.
They have a drawback: they are very **demanding**.
- Reduced Order Methods (**ROMs**) are **fast** and **reliable** tools in order to solve those problems in a **low-dimensional** framework.

Methodology

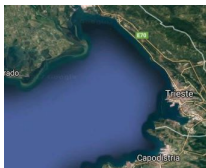
In order to manage the OFCP(μ):

- we cast it into a **saddle-point structure**,
- solved by a **Partitioned POD-Galerkin** approach,
- with **aggregated space** and relying on **affinity assumption**.

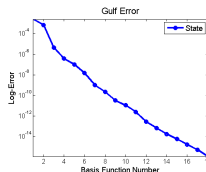
ROMs applied to a Loss of Pollutant in the Gulf of Trieste

Results of the simulated **loss of pollutant** in the **Gulf of Trieste**.

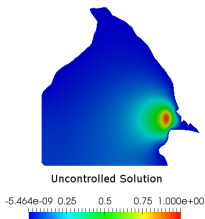
Data: collected in loco.



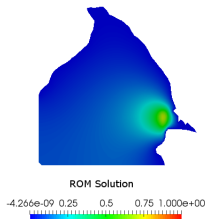
(a) Satellite image: Gulf of Trieste.



(b) Convergence error: FE vs ROMs.



(c) Uncontrolled concentration.

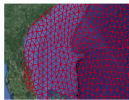


(d) Controlled concentration (ROMs).

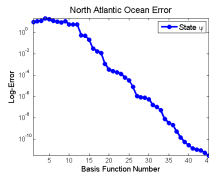
ROMs applied to a Solution Tracking on the North Atlantic Ocean

Results of the **solution tracking** of **North Atlantic Ocean** current.

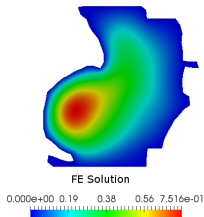
Data: collected in loco and from simulation.



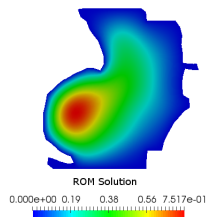
(e) Satellite image and mesh: Florida peninsula.



(f) Convergence error: FE vs ROMs.






(g) FE Stream-function.



(h) RB Stream-function.

References

-  F. Ballarin, A. Manzoni, A. Quarteroni, and G. Rozza.
Supremizer stabilization of POD–Galerkin approximation of parametrized steady incompressible Navier–Stokes equations.
International Journal for Numerical Methods in Engineering, 102(5):1136–1161, 2015.
-  J. S. Hesthaven, G. Rozza, and B. Stamm.
Certified reduced basis methods for parametrized partial differential equations.
SpringerBriefs in Mathematics, 2015, Springer, Milano.
-  F. Negri, A. Manzoni, and G. Rozza.
Reduced basis approximation of parametrized optimal flow control problems for the Stokes equations.
Computers & Mathematics with Applications, 69(4):319–336, 2015.

Acknowledgements

We acknowledge the support by European Union Funding for Research and Innovation – Horizon 2020 Program – in the framework of European Research Council Executive Agency: H2020 ERC CoG 2015 AROMA-CFD project 681447 “Advanced Reduced Order Methods with Applications in Computational Fluid Dynamics” (PI G. Rozza). We also acknowledge the INDAM-GNCS project “Metodi numerici avanzati combinati con tecniche di riduzione computazionale per PDEs parametrizzate e applicazioni”.