Bayesian Parameter Identification in Plasticity

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Overview

Plasticity Model

Chaboche Model

The resulting model for inelastic strain rate (Flow Rule), considering hardenings:

$$\dot{\boldsymbol{\epsilon}}_p(t) = \frac{\partial \sigma_{ex}}{\partial \boldsymbol{\sigma}} \dot{p}$$

where

- The over-stress: $\sigma_{ex} = \sigma_{eq} - \sigma_y - R(t)$
- The visco-plastic multiplier rate: $\dot{p} = \dot{\epsilon}_0 \left\langle \frac{\sigma_{ex}}{k} \right\rangle^n$
  - The McAuley bracket: $\langle \cdot \rangle = \max(0, x)$
- The equivalent stress: $\sigma_{eq} = \sqrt{\frac{3}{2} \mathrm{tr}((\boldsymbol{\sigma} - \boldsymbol{\chi})_D . (\boldsymbol{\sigma} - \boldsymbol{\chi})_D)}$
  - like $\boldsymbol{\sigma}_D = \boldsymbol{\sigma} - \frac{1}{3} \mathrm{tr} \boldsymbol{\sigma} \mathbf{1}$
- The isotropic hardening: $\dot{R} = b_R (H_R - R) \dot{p}$
- The kinematic hardening: $\dot{\boldsymbol{\chi}} = b_\chi (\frac{2}{3} H_\chi \frac{\partial \sigma_{eq}}{\partial \boldsymbol{\sigma}} - \boldsymbol{\chi}) \dot{p}$

Simulating the Model

- Employing Finite Element method

κ	G	σ_y	n	k	b_R	H_R	b_χ	H_χ
1.66e9	7.69e8	1.7e8	1	1.5e8	50	0.5e8	50	0.5e8

applying a cyclic load with the initial magnitude $2.5e8$ for 30 seconds

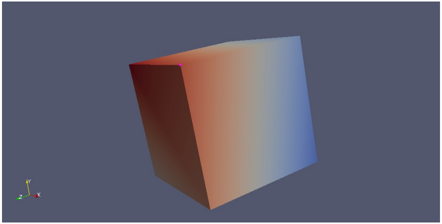


Figure : 3D element

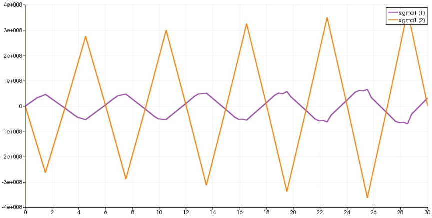


Figure : Decomposed applied force

Simulating the Model

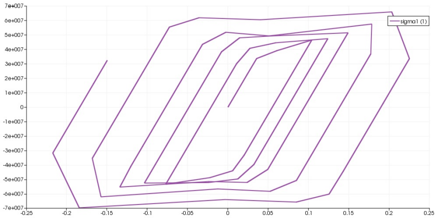


Figure : σ-ϵ in plane direction

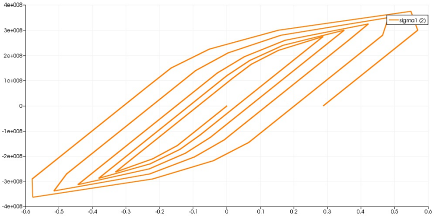
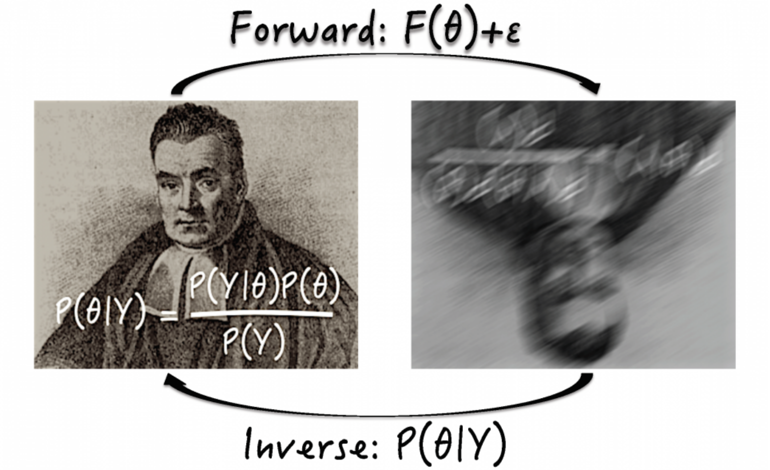


Figure : σ-ϵ in normal direction

- Vector of desired parameters $\boldsymbol{\theta} = [\kappa \ G \ b_R \ b_\chi \ \sigma_y]$

Forward and Inverse Problems

Forward and Inverse Problems



PCE based update

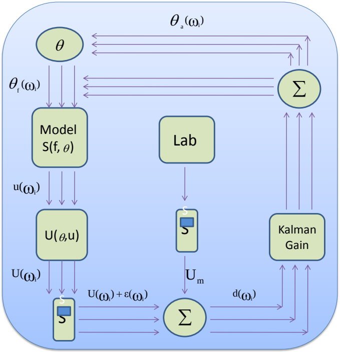
Polynomial Chaos Expansion filter:

$$\underbrace{\hat{\boldsymbol{\theta}}_a}_{\text{updated}} = \underbrace{\hat{\boldsymbol{\theta}}_f}_{\text{prior}} + \underbrace{\boldsymbol{k}}_{\text{Kalman gain}} \underbrace{(\hat{\boldsymbol{u}}_m - \hat{\boldsymbol{u}}_f)}_{\text{innovation}}$$

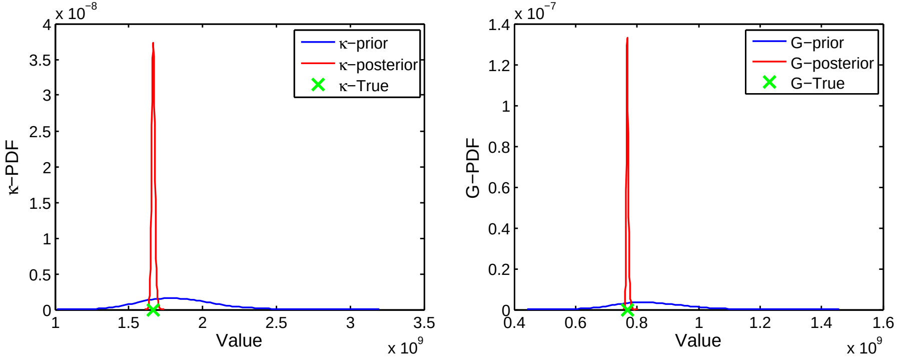
where

- $\boldsymbol{k} = C_{\boldsymbol{\theta}_f,\boldsymbol{u}_f}(C_{\boldsymbol{u}_f,\boldsymbol{u}_f} + C_{\boldsymbol{\epsilon},\boldsymbol{\epsilon}})^{-1}$
- $C_{\boldsymbol{\theta}_f,\boldsymbol{u}_f} := \sum_{\alpha>0} \alpha!\, \boldsymbol{\theta}_f^{\alpha} (\boldsymbol{u}_f^{\alpha})^T$

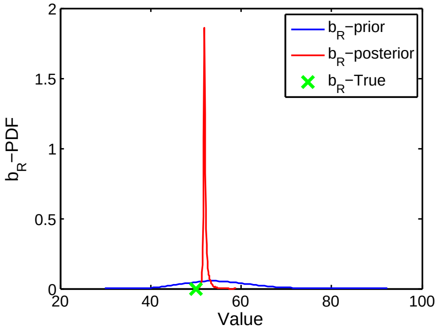
[Matthies et al., 2016]

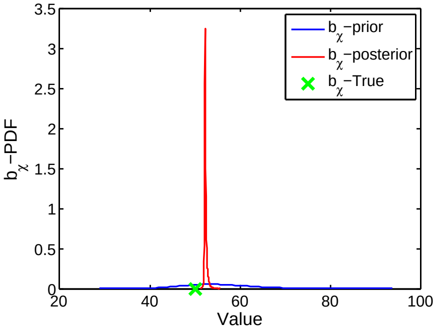


Numerical Results



Numerical Results





Numerical Results