List of Abstracts

3 topics, 40 years, and lots of fun

Ugo Bruzzo **1**

Scuola Internazionale Superiore di Studi Avanzati (SISSA), Trieste, Italy
Universidade Federal da Paraíba, João Pessoa (PB), Brazil

I will highlight the main research topics on which I have collaborated with Claudio over the last 36 years.

Instanton sheaves, the next frontier

Marcos Benevenuto Jardim **2**

Universidade Estadual de Campinas (SP), Brazil

I will talk about the history of instanton sheaves on threefolds, from the Atiyah-Ward correspondence in the late 1970's to the most recent developments involving derived categories and Bridgeland stability conditions.

How to integrate the quantum differential equations of varieties?

Giordano Cotti **3**

Universidade de Lisboa, Portugal

Enumerative geometry sinks its roots many centuries back in time. In the last decades, ideas coming from physics brought innovation to this research area, with both new techniques and the emergence of new rich geometrical structures. As an example, Gromov–Witten theory, focusing on symplectic invariants defined as counting numbers of curves on a target space, led to the notion of quantum cohomology and quantum differential equations (qDEs). The qDEs define a class of ordinary differential equations in the complex domain, whose study represents a challenging active area in both contemporary geometry and mathematical physics. These equations, indeed, encapsulate information not only about the enumerative geometry of varieties but even (conjecturally) of their topology and complex geometry. The way to disclose such a huge amount of data is through the study of the asymptotics and monodromy of their solutions.

In this talk, the speaker will address the problem of explicitly integrating the quantum differential equations of varieties. Focusing on the case of complete intersections and projectivizations of vector bundles, he will first introduce a family of integral transforms and special functions (the integral kernels). Then he will show how to use these tools to find explicit integral representations of solutions. Based on arXiv:2005.08262 (Memoirs of the EMS, to appear) and arXiv:2210.05445.

Poisson quasi-Nijenhuis manifolds and integrable systems

Marco Pedroni **4**

Università degli Studi di Bergamo, Italy

The notion of Poisson quasi-Nijenhuis manifold, introduced by Stiénon and Xu, generalizes that of Poisson-Nijenhuis manifold. The relevance of the latter in the theory of completely integrable systems is well established since the birth of the bi-Hamiltonian approach to integrability. In this talk, we discuss the relevance of the notion of Poisson quasi-Nijenhuis manifold in the context of finite-dimensional integrable systems. Moreover, we show that the closed (or periodic) n-particle Toda lattice, along with its relation with the open (or non periodic) Toda system, can be framed in such a geometrical structure. This is a joint work with G. Falqui, I. Mencattini, and G. Ortenzi.

Unitarization of the Radon transform

Filippo De Mari Casareto Dal Verme **5**

Università degli Studi di Genova, Italy

We consider the Radon transform associated to pairs (X, Ξ), a variant of Helgason's notion of dual pair, where $X = G/K$ and $\Xi = G/H$, G being a locally compact group and K and H closed subgroups thereof. Under some technical assumptions, we prove that if the quasi regular representations of G acting on $L^2(X)$ and $L^2(\Xi)$ are irreducible, then the Radon transform admits a unitarization intertwining the two representations. If, in addition, the representations are square integrable, we provide an inversion formula for the Radon transform based on the voice transform associated to these representations. The general assumptions (in particular irreducibility and square integrability of the representations) fail in the case when X is either a noncompact symmetric space or a homogeneous tree and Ξ is the corresponding space of horocycles. Nonetheless, a unitarization theorem holds true in both cases and the outcoming unitary operator does intertwine the quasi regular representations. This is joint work with G. Alberti, F. Bartolucci, E. De Vito, M. Monti and F. Odone.

On the amenability problem for path and loop groups

Vladimir Pestov **6**

Universidade Federal da Paraíba, João Pessoa (PB), Brazil
University of Ottawa, Canada

We will survey the open problem of amenability of groups of gauge transformations, as well as some recent progress in the case of groups of paths and loops of finite energy.

Fukaya category of surfaces and pants decomposition

Nicolò Sibilla **7**

Scuola Internazionale Superiore di Studi Avanzati (SISSA), Trieste, Italy

In this talk I will explain some results joint with James Pascaleff on the Fukaya category of Riemann surfaces. I will explain a local-to-global principle which allows us to reduce the calculation of the Fukaya category of surfaces of genus g greater than one to the case of the pair-of-pants, and which holds both in the punctured and in the compact case. The starting point are the sheaf-theoretic methods which are available in the exact setting, and which I will review at the beginning of the talk. This result has several interesting consequences for HMS and geometrization of objects in the Fukaya category. The talk is based on 1604.06448 and 2103.03366.

How theoretical models may shape sociality: the case of multi-messenger astrophysics

Luca Guzzardi **8**

Università degli Studi di Milano, Italy

Since Lakatos (1978) and Laudan (1978), it is almost common place that, particularly in the natural sciences, theoretical models offer guidance to the scientists involved in certain research programs or research traditions in a variety of ways. Based on cases from recent developments in astrophysics, I will argue that they model researchers' expectations in the first place, enriching the evidence and thus guiding them to expand the models themselves; therefore, they must be sufficiently flexible, susceptible of being progressively integrated, adjusted, slightly changed. In addition, models guide researchers' course of action, suggesting which objects they should look for, in which order the observations should be made to be significant, and who (what kind of facility) could perform the search. Finally, I will claim that under peculiar conditions, which are fulfilled by multi-messenger astrophysics, they also narrow the scientists' choice about what kind of social behavior they should adopt in order to do their research.

SUSY Nori Motives

Daniel Hernández Ruipérez **9**

Universidad de Salamanca, Spain

We define stable supercurves and stable supermaps, and based on these notions we develop a theory of Nori motives for the category of stable supermaps of SUSY curves with punctures. This will require several preliminary constructions, including the development of a basic theory of supercycles.

Atiyah sequences of braided Lie algebras and their splittings

Giovanni Landi **10**

Università degli Studi di Trieste, Italy

TBA

1) Generalized Wronskians, 2) Schubert Calculus and 3) integrals on the Sato Universal Grassmann Manifold

Letterio Gatto **11**

Politecnico di Torino, Italy

The purpose of the talk, dedicated to the long-standing friendship with Claudio Bartocci on the occasion of his birthday, is to sketch the close relationship between 1), 2) and 3) in the title.

The moduli space of linear control systems

Alberto Tacchella **12**

Fondazione Bruno Kessler, Trento, Italy

It is well known that the theory of linear control systems is essentially geometric in character. What is not similarly well known is that the geometry in question has a non-commutative nature. In this talk I will review the construction of the moduli space of linear control systems and show some applications to the problem of controller design.

Hamiltonian integrable systems: methods and results

Gregorio Falqui **13**

Scuola Internazionale Superiore di Studi Avanzati (SISSA), Trieste, Italy
Università degli Studi di Milano-Bicocca

The talk will dwell upon the interplay of Hamiltonian structures (as well as their reduction) and the integrability properties of systems of evolutionary equations, both in the ODE and the PDE case.

Remarks on Penrose integral transform

Jean-Jacques Szczeciniarz **14**

Histoire et Philosophie des Sciences, Laboratoire SΦHERE, Paris, France

In 1966, Penrose discovered that solutions of massless field equations on Minkowski space could be expressed as contour integrals of free holomorphic functions over lines in 3 dimensional complex projective space. This idea of using contour integral formulae to obtain solutions of field equations can in fact be traced back to Whittaker in 1902. The freedom in the function for a fixed solution was later realized to be exactly the freedom of a Čech representative of a sheaf cohomology class, and a formal mechanism was set up by Eastwood, Penrose and Wells to prove isomorphisms between sheaf cohomology groups over a region of the projective space and solutions of massless field equations over a region of spacetime.[1] In this text we return to the origins and we develop the basic characteristics of the Penrose transformation by focusing on the construction from the complex framework of $\mathbb{C}^4$ and in the introduction of the complexified projective Minkowski space, which is compactified by its embedding in the Gr(3, 1) Grassmannian. We detail some calculations according to Shabat's methods and explain why the transformation allows to describe some solutions of systems of current linear PDE's of physics.

The main question appeared with the new frame developed by the theory of integral functor and the transform given by the Fourier Mukai transform. The core idea in the definition of an integral functor is simple. If we have two varieties X and Y we can take some "object" on X, pull it back to the product $X \times Y$, twist him by some object "kernel" in $X \times Y$ and then push it down to Y. (i.e. we integrate on X). (See "Fourier-Mukai and Nahm Transforms in Geometry and Mathematical Physics" (Claudio Bartocci, Ugo Bruzzo, Daniel Hernàndez Ruipérez (Birkhäuser) 2009).

It is what happens with the Fourier Transform on functions: one takes one function $f(x)$ on $\mathbb{R}^n$, pulls it back to $\mathbb{R}^n \times \mathbb{R}^n$, multiplies it by the kernel

$$e^{<xy>}$$

and then integrate on the first copy of $\mathbb{R}^n$ thus obtaining a function $\hat{f}(y)$ on the second copy. This is the CT frame for this transformation.

I will present the general frame for the Penrose transformation. Gindikin and Henkine gave a survey of the representation of solutions of system of massless equations in terms of the Cauchy Riemann equations on the space of Penrose twistors. I will present their exposition of the theory of twistors. The Penrose transformation iis investigated by means of of complex integral geometry. The manifold of lines G_1 in $P\mathbb{C}^3$ can be embedded as a quadric of $P\mathbb{C}^5$. The Penrose construction starts from the fact that among real forms of G_1 there is the conformal compactification of Minkowski space (see below). A conformal class of complex metrics on G_1 is distinguished by the condition that $u, v \in G_1$ (lines in $P\mathbb{C}^3$) are at zero distance if the corresponding lines in $P\mathbb{C}^3$ intersect. The manifold M is distinguished as follows (up to automorphism of G_1). Let (z^0, z^1, z^2, z^3) be the homogeneous coordinates of a point z in $\mathbb{P}^3$, let $H(z)$ be a Hermitian quadratic form of signature (2,2) - that means $H(z) = Im(z^0\bar{z}^2 z^1\bar{z}^3)$. Suppose that a hyperplane N is prescribed by the condition $H(z) = 0$ (the surface of null twistors). The set of complex lines lying on N is them taken as M. The group of

[1] R. Penrose and W. Rindler, Spinors and Space-Time vol. 1, Cambridge University Press 1984

projective transformations preserving N is isomorphic to SU(2, 2) and then induces automorphisms of M together with the conformal class of metrics associated with the intersection of lines (see G and H). Considering $N \setminus l$ (l fixed line) and the lines lying in $N \setminus l$ we arrive at non compactified Minkowski space M_0 and the group of automorphisms of $N \setminus l$, and then M_0 isisomorphic to the Poincaré group. points of N can be interpreted as "light rays" (isotropic lines) on M.

There exists among real form of E of a manifold E isomorphic to the sphere S^4, on which a conformally Euclidean metrics induced. For E is possible (GH) to take all lines connecting the points $(z^0, z^1, z^2, z^3)(-z^0, -z^1 - z^2, -z^3)$ in $\mathbb{P}^3$.

I will resume the main characteristic of the Penrose transformation: absence of a kernel. It has been found that under minor restrictions on the linearly concave domain the transformation P^0 has no kernel, i.e. if the integral is equal to zero for a $\bar{\partial}$-form then ω is a ∂-exact form.

This assertion is proved in the context of the integral geometry of (nq)- concave domains, Wells informed us of a considerably simpler proof. (See Lerner and Ward.)

I will describe in some details the Penrose transform and the fact that it gives us the self-dual solutions of Maxwell's equations. and place these properties in a more general and perhaps epistemological framework. The question that I am beginning to ask ,given the recent nature of the work that I have carried out, is whether the algebraic point of view of integral functors can make a contribution to the understanding of integral transformation (Penrose's one f.e.).

Lagrangian fibrations on hyper-Kähler fourfolds

Emanuele Macrì **15**

Université Paris-Saclay, France

I will present joint work with Olivier Debarre, Daniel Huybrechts and Claire Voisin on the SYZ hyper-Kähler conjecture for fourfolds under certain topological assumptions. As application, this proves a conjecture by O'Grady that a hyper-Kähler fourfold whose cohomology ring is isomorphic to the one of the Hilbert square of a K3 surface is a deformation of a Hilbert square.